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## APPLIED A CONDITIONS OF SMOOTHNESS OF CAUSAL RELATIONSHIPS IN THE PROBLEM OF CONSTRUCTING OF MATHEMATICAL MODELS

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***Abstract*—*The publication, based on the fundamental property of all real processes, namely the smoothness of nonlinear causal relationships between the variables of the state of the object of identification and the variables themselves, presents a few of new identification methods that are simpler and more accurate than traditional. These methods are illustrated by examples of estimating the aerodynamic coefficients and balancing characteristics of aircraft, building a single mathematical model of the magnetization curve.*

**Index Terms**—Identification; smoothness; Hammerstein model; natural experiment.

### I. INTRODUCTION

Building mathematical models of the behavior of real-world objects is certainly an urgent task. However, its effective solution requires a correct approach. Based on the fundamental laws of the universe, such as the inseparability of matter and motion, the general relationship, we must take into account such fundamental properties of real objects as multidimensionality (not autonomy), nonstationarity, nonlinearity of state variables, infinity, etc. Therefore, the correct construction of a mathematical model (MM) is possible only by reducing the dimensionality of state variables from infinite to finite in space essential for solving a specific problem variables, narrowing the range of variables and time even with such a statement, the task of identifying the behavior of a real object, its MM will be nonlinear

$$\dot{X}(t) = f\left(X(t)\right), \tag{1}$$

where $X$ and $\dot{X}$ is the vector-functions of state variables and the rate of change, respectively, limited in size and range; $t$ is the limited time.

An identification is the definition of the mapping $f$ of variables $X(t)$ to their derivatives. This task can be significantly simplified if one very important property of processes (1) in a real object is taken into account. Real objects have a finite mass $m$, if $X(t)$ could change instantly, then its derivative $\dot{X}(t)$, like the rate of $X(t)$ change, would be infinitely large. Then the power, as a product $m\dot{X}(t)$, would be infinite. However, all real-world objects have finite power, therefore, both $X(t)$ and $f\left(X(t)\right)$ should be and smooth functions. That is, the norm should be limited

$$\left\| \frac{df}{dX^{\mathrm{T}}} \right\| < \infty, \ \left\| \frac{dX}{dt} \right\| < \infty .$$

The smoothness of $f\left(X(t)\right)$ for a given approximation error guarantees the finiteness of its decomposition into multiple Taylor series (Weierstrass's theorem [1]); the smoothness of the derivative $\frac{dX}{dt}$ of $t$ guarantees the limited order of the differential equations of MM.

### II. THE EXAMPLES OF THE USE OF SMOOTHNESS PROPERTIES TO SOLVE A SERIES OF PRACTICAL IDENTIFICATION PROBLEMS

*A. Determination of aerodynamic coefficients of the aircraft according to natural tests*

As a rule, the aerodynamic coefficients (ADC) is the first component of the MM (1) decomposition in the Taylor series:

$$\Delta\dot{X}(t) \cong \Delta X(t)\cdot A + \Delta X(t)\cdot B\cdot \Delta X^{\mathrm{T}}(t) \quad (2)$$

or

$$\Delta\dot{X}(t) \cong \Delta X(t)\left[A + B\cdot \Delta X^{\mathrm{T}}(t)\right] = \Delta X(t)\cdot C, \quad (3)$$

where $C = A + B\cdot \Delta X^{\mathrm{T}}(t)$. (4)

Therefore, the estimates of the coefficients $C_{ij}$ determined by the least squares method will depend on the norm $\|\Delta X^{\mathrm{T}}\|$. When conducting a natural experiment with different $\|\Delta X^{\mathrm{T}}\|$ we obtain an offset value for each $\|\Delta X^{\mathrm{T}}\|$ relative to the valid ADC. Next, by regression analysis methods we approximate the each ADC $C_{ij}$ by smooth dependence of on the function $\|\Delta X\|$, for example

$$C_{ij} = C_{ij}(0) + K_{ij}\|\Delta X\|, \quad (5)$$

where $C_{ij}(0)$ is the unbiased appraisal ADC $C_{ij}$.

In the Figure 1 shows the oscillograms of the longitudinal short-period motion of the aircraft M-17 in the form of different amplitude of the "transfers" of a handlebar height and the reactions to it angular velocity $\omega_{z1}$ and overload $\eta_y$.

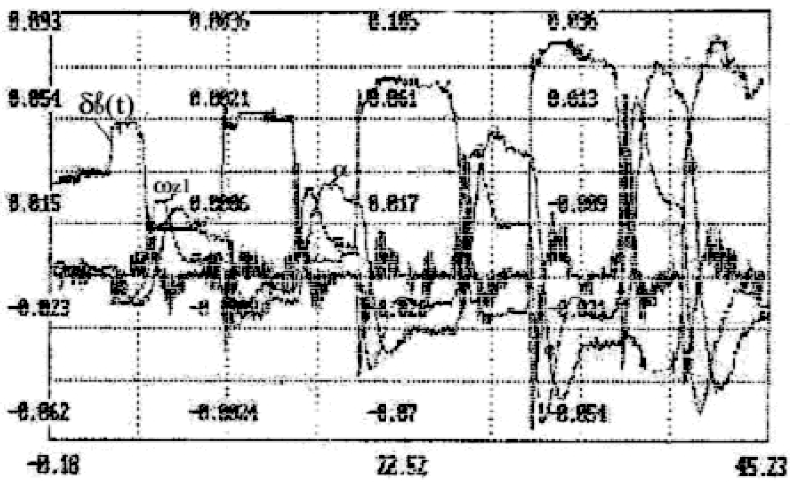

Fig. 1. Oscillograms of regimes of change of handlebar height, angle of attack and angular velocity

From linear models

$$\Delta\dot{\omega}_{z1}(t) = a_{11}\Delta\omega_{z1} + a_{12}\Delta\eta_y + a_{13}\Delta\delta_h, \quad (6)$$

coefficients $a_{ij}$ were estimated by the least squares method for each "transfers" $\Delta\alpha_h(t)$. The margin of safety $\sigma_n$ further calculated. The unbiased estimate $\hat{\sigma}_n$ was obtained by linear approximation (5) (Fig. 2) and extrapolation of MM (5) to the zero point of $\|\Delta\alpha_h\|$. It is due to the smoothness of dependences (1) and (4) the difficult problem of determining of ADC by approximating the shifted (due to the simplification of the model (6)) estimates of ADC by the regression model (5) was correctly solved.

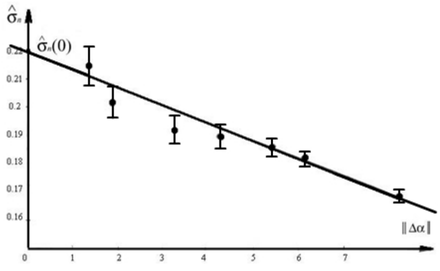

Fig. 2. The dependence of ratings $\hat{\sigma}_n$ from $\|\alpha\|$

*B. Structural-parametric identification of complex multidimensional nonlinear dependences according to the natural experiment*

In the process of building the model of the aircraft TU-144 were determined by the values of the aerodynamic correction "*y*" depending on the three variables $x_1, x_2, x_3$. The experiment was performed in such a way that one of the variables changed for the fixed two others. Therefore, taking into account the property of the smoothness of the dependence, in order to automatically determine the structure and parameters, according to tabular data, local models were built using least squares method

$$y(x_i) = \alpha_{1ik}x_i + \alpha_{2ik}x_i^2, \quad i = 1,2,3; \quad k = 1,2,3. \quad (7)$$

The coefficients $\alpha_{jik}$ were determined from the model

$$\alpha_{jik} = \alpha'_{jk}x_2 + \alpha''_{jk}x_2^2, \quad j = 1,2. \quad (8)$$

In turn, the coefficients $\alpha'_{jk}, \alpha''_{jk}$ were from the model

$$\alpha'_{jk}(x_3) = \beta'_j x_3 + \beta''_j x_3^2, \quad \alpha''_{jk}(x_3) = \gamma'_j x_3 + \gamma''_j x_3^2. \quad (9)$$

After substitution (9) in (8), and then (8) in (7) and neglect of insignificant components, the expression (Fig. 3) of MM having simplified structure was received

$$y(x) = -0,61\cdot 10^{-2} x_1 x_2^2 x_3^2 + 0.9\cdot 10^{-2} x_2^2 x_3^2 + 0.23 x_1^2 x_2 x_3^2 + 0.04 x_1 x_2 x_3^2 - \\ -1.72 x_2 x_3^2 - 0.0232 x_2^2 x_3 - 0.0126 x_1^2 x_2 x_3 + 0.133 x_1 x_2 x_3 + 1.02 x_2 x_3. \tag{10}$$

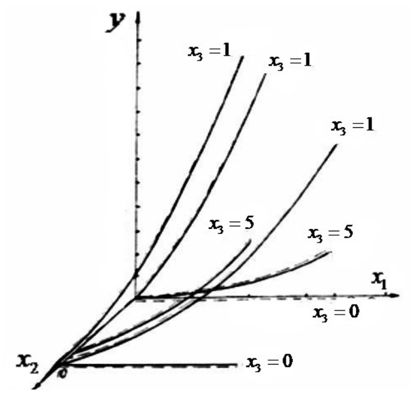


Fig. 3. The dependence $y(x_1, x_2, x_3)$ (dotted line) and its model $\hat{y}(x_1, x_2, x_3)$ (continuous line)

The error was not more than 5%.

As we can see, due to the smoothness of the dependence, the decomposition into local models (7) and the composition of the local into the global (10), the structure and parameters of the complex (Fig. 3) dependence $y(x_1, x_2, x_3)$ are automatically determined.

*C. Identification of the smooth static component of the dynamic Hammerstein MM*

The dynamics of a nonlinear object can be represented by a composition of linear dynamic and nonlinear static images

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \ldots + x(t) = f(U(t)), \tag{11}$$

where $x$ is the smooth consequential; $U$ is the causal variable; $f$ is the unambiguous smooth static nonlinear dependence $x = f(U)$.

Filtering the variables $\alpha_h(t)$ and $\eta_y(t)$ (see Fig. 1) by low-pass filter, we ensure the smoothness $\hat{x}(t)$ and its derivatives $\hat{x}'(t), \hat{x}''(t)$ and the proximity to zero of higher-order derivatives in (11). As a result, we obtain a simplified structure (12) of model (11):

$$a_2 \frac{d^2 \hat{x}}{dt^2} + a_1 \frac{d\hat{x}}{dt} + \ldots + \hat{x}(t) = f(\hat{U}(t)), \tag{12}$$

with unknown $a_2, a_1$ and *f*.

Traditionally, the dependence $x = f(U)$ was decomposed into a series and estimated from equation (11) or even simplified (12) a fairly large number of unknown coefficients under the condition of the minimum mean square of the error between the left and right (in the form of a series) parts. This approach did not take into account the smoothness and for short series of decomposition did not give the desired approximation accuracy, and for long led to the pulsation of the model, for example (Fig. 4) [5].

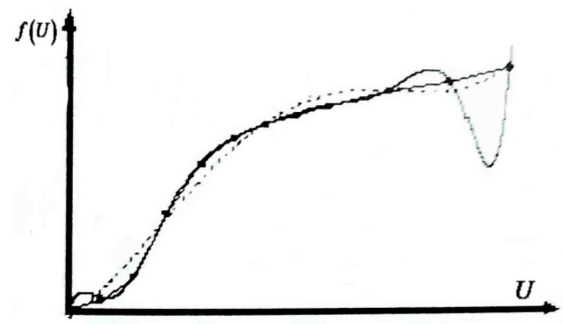


Fig. 4. Estimate pulsation $f(U)$ approximated by a polynomial of 9th degree

The availability of a priori information about smoothness has greatly simplified the task. For this purpose, the coefficients in (12) were estimated by the Pukhov–Hatiashvili smoothness criterion [6], i.e., under the condition of the minimum of the *r*th derivative of the adjusted variable

$$a_2 \frac{d^2 \hat{x}}{dt^2} + a_1 \frac{d\hat{x}}{dt} + \ldots + \hat{x}_{cor}(t_k) = \\ = \hat{x}(t_k) - \hat{a}_1 \left.\frac{d\hat{x}}{dt}\right|_{t=t_k} - \hat{a}_2 \left.\frac{d^2\hat{x}}{dt^2}\right|_{t=t_k},$$

on the $\hat{U}(t_k)$

$$\hat{a}_1, \hat{a}_2 = \arg\min \sum_{k=1}^{N} \left. \left( \frac{d^r \hat{x}_{cor}(t)}{dt^r} \right)^2 \right|_{t=t_k}.$$

Then the nonparametric estimate of static nonlinearity is determined by the graph $\hat{x}_{cor}(U)$, where the variable $U(t)$ is ordered in ascending and ordered accordingly $\hat{x}_{cor}$. Thus, one of the balancing (static) dependences obtained by this method from the graphs of Fig. 1 for LA M-17 shown in the Fig. 5. Test examples of accurate recovery from dynamic noisy data are given in [2], [3].

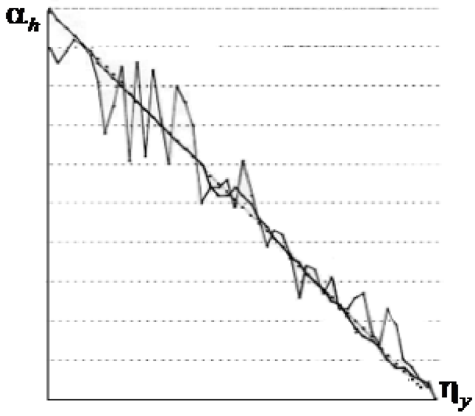


Fig. 5. The dependence $\alpha_h(\eta_y)$ for LA M-17

*D. A simple analytical description of piecewise analytical dependences*

Often, based on the physics of processes in the object, the nonlinearity can be piecewise analytical. For example, nonlinearity in Fig. 4 may represent a magnetization curve of a ferromagnetic medium It is characterized by two areas. The portion of the ferromagnet can be represented by MM

$$f_1(U)=\beta_1 U^2+\beta_2 U^3, \qquad (15)$$

where $U\in[0,U_1)$; the section of the diamagnet, according to the physics of processes, is represented by a linear model

$$f_2(U)=\beta_3+\beta_4 U, \qquad (16)$$

where $U\in[U_1,\infty)$.

Combining both sections with one polynomial (as shown in Fig. 4) does not give the desired result in terms of accuracy and simplicity of the mathematical model

The use of the traditional piecewise analytical description (15), (16) with selective signal functions $\eta(U)$:

$$\eta_1(U)=\begin{cases}1, 0\le U\le U_1,\\ 0, U\ge U_1,\end{cases}\quad \eta_2(U)=\begin{cases}1, U>U_1,\\ 0, U\le U_1,\end{cases} \qquad (17)$$

i.e. the model

$$\hat{f}(U)=\eta_1(U)\cdot f_1(U)+\eta_2(U)\cdot f_2(U), \qquad (18)$$

leads to a violation of the natural condition of smoothness at the point $U_1$. To ensure smoothness and accuracy in the whole range, it was proposed [4] to use pseudosignum functions $\hat{\eta}$ instead of relay signal functions (17). For example, for Fig. 4 pseudosignum functions will look like

$$\hat{\eta}_1(U)=\frac{1}{1+\left(\frac{U}{0.7}\right)^n},\quad \hat{\eta}_2(U)=\frac{1}{1+\left(\frac{0.7}{U}\right)^n}. \qquad (19)$$

Then MM (18) takes the form

$$\hat{f}(U)=\frac{\beta_1 U^2+\beta_2 U^3}{1+\left(\frac{U}{0.7}\right)^n}+\frac{\beta_3+\beta_4 U}{1+\left(\frac{0.7}{U}\right)^n}, \qquad (20)$$

where the root mean square error $\overline{\varepsilon}^2$ of the approximation is extreme from the indicator "*n*" (Fig. 6): the error $\overline{\varepsilon}^2$ is large for small *n* due to poor selectivity, for very large *n* is also large due to the transition of functions (19) to relay (17).

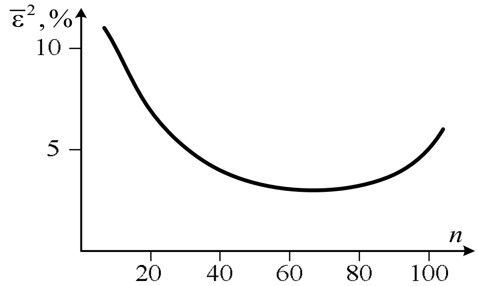


Fig. 6 The dependence of mean square error $\overline{\varepsilon}^2$ on indicator *n*

## III. Conclusion

Thus, taking into account the smoothness of the dependences and allowed to build simple and fairly accurate methods of identification of nonlinear dynamic object (1) on the basis of natural experiment with finite samples in time and in the ratio of "signal-to-noise". The considered methods can be successfully applied to the decision of problems of identification of behavior of real objects in various industries.

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**А. М. Сільвестров, Л. Ю. Спінул, А. А. Сердюк. Застосування умови гладкості причинно-наслідкових зв'язків у задачі побудови математичних моделей**

У публікації, спираючись на фундаментальну властивість всіх реальних процесів, а саме гладкість нелінійних причинно-наслідкових зв'язків між змінними стану об'єкта ідентифікації і самих змінних, наведено низку нових методів ідентифікації, які є більш простими і точними на відміну від традиційних. Ці методи проілюстровано на прикладах оцінювання аеродинамічних коефіцієнтів і балансировочних характеристик літальних апаратів, побудови єдиної математичної моделі кривої намагнічування.

**Ключові слова**: ідентифікація; модель Гамерштейна; натурний експеримент.

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**А. Н. Сильвестров, Л. Ю. Спинул, А. А. Сердюк. Применение условия гладкости причинно-следственных связей в задаче построения математических моделей**

В публикации, опираясь на фундаментальное свойство всех реальных процессов, а именно гладкость нелинейных причинно следственных связей между переменными состояния объекта идентификации и самих переменными, приведен ряд новых методов идентификации, которые являются более простыми и точными в отличие от традиционных. Эти методы проиллюстрированы на примерах оценки аэродинамических коэффициентов и балансировочных характеристик летательных аппаратов, построения единой математической модели кривой намагничивания.
**Ключевые слова**: идентификация; модель Гаммерштейна; натурный эксперимент.

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