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CARTESIAN VECTOR DIRECTION COSINES AS THE MULTI-OPTIONAL HYBRID FUNCTIONS OPTIMAL DISTRIBUTION

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Abstract—*It is made an attempt to discover an explainable plausible reason for the existence of the conditions of optimality for Cartesian vector direction cosines, having importance in energy mechanical engineering, with the help of the multi-optional hybrid functions entropy conditional optimality doctrine. Substantiation is made in terms of the calculus of variations theory with the help of the special hybrid-optional effectiveness functions uncertainty measure, which includes the hybrid functions entropy of the traditional Shannon's style. In the studied cases, the simplest variational problems solutions, which are the numbers known as the direction cosines of a Cartesian vector, are stipulated by the specified natural logarithmic quadratic forms. It is proposed to evaluate the uncertainty/certainty degree of the magnitude and direction of a Cartesian vector with the use of the objective functional. This is a new insight into the scientific explanation of the well-known dependency derived in another way. The developed theoretical contemplations and mathematical derivations are finalized with a simplest numerical example for the variated value of the multi-optional hybrid function resulting in the objective functional.*

Index Terms—Mechanical engineering; multi-optionallity doctrine; conditional optimality; hybrid-optional effectiveness; Cartesian vector; direction cosine; maximal uncertainty; variational problem.

I. INTRODUCTION

The subject area studied in energy mechanical engineering is fairly broad and rather diverse [1] – [4]. Energy industry deals with the design requirements development and operation of the wide range of energy machines, starting from the fossil fuel burning ones [1] – [6], hydroelectric, nuclear and ending with the so popular nowadays renewable, ecosystem, natural system power plants.

The described aspects of the diversity bring to being the multi-alternativeness characterized in the Subjective Analysis Theory [7], with regards to the subjective effectiveness functions, then subjective individuals' preferences functions optimal distributions based upon the preferences entropy (uncertainty) measure conditional extremum principle. The concept of the multi-optional hybrid functions doctrine [8] – [11] uses the objectively existed categories rather than operates with the someone's subjectively preferred estimations likewise developed in [7].

There are direction cosines that are used for vector values in Cartesian space [12] and [13]. Some elements of the multi-optional hybrid functions doctrine [8] – [11], [14], and [15] can help find those well known parameters based upon the newest ideas of the multi-optional hybrid functions uncertainty, expressed through their entropy, conditional optimality.

Aim. The presented paper is aimed at revealing the conditions of optimality for Cartesian vector direction cosines with the help of the multi-optional hybrid functions entropy conditional optimality doctrine.

II. PROBLEM STATEMENT

The research in the framework of the optional hybrid functions entropy conditional optimality doctrine activates the closer look at the traditionally known magnitudes, which however could be explained based upon some principle of optimality.

The Problem Setting. For now the problem statement is to find the value which extremum provides the numbers known as the direction cosines of a Cartesian vector. Such magnitudes are used for energy and work calculations.

III. PROBLEM SOLUTION

A. General Approach

Consider a functional value similar to the postulated in [7] and used in [8] – [11], [14], [15]:

$$\Phi_h = -\sum_{i=1}^{n} h_i^2 \ln h_i^2 + k\sum_{i=1}^{n} h_i^2 \ln x_i + \gamma\left(\sum_{i=1}^{n} h_i^2 - 1\right), \quad (1)$$

where Φ_h is the functional given on the set of the multi-optional hybrid functions h_i, $i=\overline{1\ldots n}$, where n is the number of options; k is the magnitude expressing the weight coefficient or uncertain Lagrange multiplier (analog to many interpretations, likewise fines function etc.); x_i are the multi-optional effectiveness parameters influencing and determining the sought optimal distribution of the multi-optional hybrid functions h_i; γ is one more weight coefficient.

The first member of the functional sum (1):

$$H_h = -\sum_{i=1}^{n} h_i^2 \ln h_i^2 , \tag{2}$$

is the measure of the squares of the multi-optional hybrid functions h_i uncertainty, i.e. the entropy of the h_i^2.

The second member of the functional sum (1):

$$k \sum_{i=1}^{n} h_i^2 \ln x_i , \tag{3}$$

is the multi-optional effectiveness function taking into account the logarithmic values of the optional parameters of x_i: $\ln x_i$, moreover assessed with the squares of the multi-optional hybrid functions h_i^2 as well as the weight coefficient k.

The third member of the functional sum (1):

$$\gamma \left(\sum_{i=1}^{n} h_i^2 - 1 \right), \tag{4}$$

is the function of the normalizing condition for the squares of the multi-optional hybrid functions h_i^2 estimated with the uncertain Lagrange multiplier γ.

The suspected conditional extremal distribution of the squared multi-optional hybrid functions h_i^2 for the functional of (1) with the elements of (2) – (4) can be obtained from

$$\frac{\partial \Phi}{\partial h_i^2} = 0 . \tag{5}$$

The necessary condition (5) yields

$$-\ln h_i^2 - 1 + k \ln x_i + \gamma = 0 . \tag{6}$$

From (6)

$$\ln h_i^2 = \gamma - 1 + k \ln x_i , \tag{7}$$

and (7) gives

$$h_i^2 = e^{\gamma - 1 + k \ln x_i} = e^{\gamma - 1} e^{k \ln x_i} = e^{\gamma - 1} x_i^k . \tag{8}$$

The normalizing condition of (4) applied to (8) will result in

$$\sum_{i=1}^{n} h_i^2 = e^{\gamma - 1} \sum_{i=1}^{n} x_i^k = 1 , \tag{9}$$

which means

$$e^{\gamma - 1} = \frac{1}{\sum_{i=1}^{n} x_i^k} , \tag{10}$$

Therefore, after the relations of (9) and (10) application, the possible extremum of the multi-optional hybrid functions h_i distribution will be in the view of

$$h_i^2 = \frac{x_i^k}{\sum_{j=1}^{n} x_j^k} . \tag{11}$$

B. Special Solutions

Considering a Cartesian vector, one can use the squared values.

For the case when

$$k = 2 , \qquad n = 3 , \tag{12}$$

from the general expression of (11), on conditions of (12), it can be easily found that

$$h_i = \frac{x_i}{\sqrt{\sum_{j=1}^{3} x_j^2}} . \tag{13}$$

For the three-dimensional orthogonal coordinate system, the projections of the vector upon the coordinate axes are

$$x_1 = F_x , \qquad x_2 = F_y , \qquad x_3 = F_z , \tag{14}$$

where F_x, F_y, and F_z are the orthogonal projections of the Cartesian vector $\vec{F}$ upon the coordinate axes of $(O - x)$, $(O - y)$, and $(O - z)$ respectively.

In such circumstances, the direction cosines of $\vec{F}$: the cosines of the coordinate direction angles between $\vec{F}$ and axes: $(\angle \vec{F}, Ox)$, $(\angle \vec{F}, Oy)$, and $(\angle \vec{F}, Oz)$ have the expressions of the multi-optional hybrid functions h_i (13) obtained from (11) in the special case described with (12):

$$h_1 = \cos(\angle \vec{F}, Ox) = \frac{F_x}{\sqrt{F_x^2 + F_y^2 + F_z^2}} , \tag{15}$$

$$h_2 = \cos(\angle \vec{F}, Oy) = \frac{F_y}{\sqrt{F_x^2 + F_y^2 + F_z^2}} , \tag{16}$$

$$h_3 = \cos(\angle \vec{F}, Oz) = \frac{F_z}{\sqrt{F_x^2 + F_y^2 + F_z^2}} . \tag{17}$$

For the problems of planimetry, conditions (12) are converted into

$$k = 2, \qquad n = 2. \tag{18}$$

Then, for (13) and (15) – (17), it will be

$$h_i = \frac{x_i}{\sqrt{\sum_{j=1}^{2} x_j^2}}. \tag{19}$$

In trigonometric problem, when

$$x_1 = a, \qquad x_2 = b, \tag{20}$$

where a and b are catheti,

$$h_1 = \frac{a}{\sqrt{a^2 + b^2}}. \tag{21}$$

Or for the second option in the squared view

$$h_2^2 = \frac{b^2}{a^2 + b^2}. \tag{22}$$

C. Simulation Results

In general sense, the solution (11) delivers the maximum to the functional (1). It can be visualized in three-dimension space with the help of variation.

Therefore, using the results of (12) – (22), it will be supposedly

$$F_x = 2, \qquad F_y = 10, \qquad F_z = 11, \tag{23}$$

of the corresponding measurement units accepted in calculations.

Then

$$|\vec{F}| = 15. \tag{24}$$

The direction cosines (15) – (17)

$$h_1 \approx 0.133, \quad h_2 \approx 0.667, \quad h_2 \approx 0.733, \tag{25}$$

and

$$h_1^2 \approx 0.018, \quad h_2^2 \approx 0.444, \quad h_3^2 \approx 0.538. \tag{26}$$

Normalizing condition (4) yields

$$h_1^2 + h_2^2 + h_3^2 = 1. \tag{27}$$

Now, in order to demonstrate the maximal value of (1) for the distribution of the direction cosines (15) – (17) with the data of (23) – (27), consider a variation of

$$\delta = -0.001, -0.0009 \ldots 0.001. \tag{28}$$

The variated function, for example, will be

$$h_1(\delta) = h_1^2 + \delta. \tag{29}$$

Then, for instance

$$h_3(\delta) = 1 - h_1(\delta) - h_2^2. \tag{30}$$

Applying these values of (29) and (30) for the objective functional of (1), its magnitude

$$\Phi_h(\delta) \tag{31}$$

will vary due to (28) – (30).

The results of simulation with (22) – (31) are shown in Fig. 1.

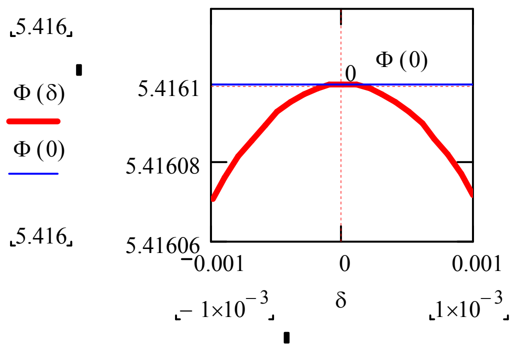

Fig. 1. Variated value of the objective functional

D. Discussion

The conditional optimum (maximal value) of (1) or (31) with respect to the multi-optional hybrid functions h_i^2 of h_i distribution can be verified with the second order conditions (sufficient). It is obtained from (6):

$$\frac{\partial^2 \Phi}{\partial \left(h_i^2\right)^2} = -\frac{1}{h_i^2} < 0. \tag{32}$$

In case of non-existed options, it is the relevant number of the options n that is taken into consideration rather than the operation of the absent multi-optional effectiveness parameters x_i equalization to zero value is performed. This allows bypassing the impossibility of mathematical logarithm operations of zero magnitudes.

Possible Cartesian vectors in engineering mechanics (forces, accelerations, velocities etc.), as for example, (1) – (22), are important values for the energy mechanical engineering estimations of the energy production efficiency and at the conversion of energy from one form to another.

Concerning rectangular triangle calculations, in style of Pythagorean theorem (15) – (22), especially visible in (18) – (22), it is obvious that $a^2 + b^2$ is the square magnitude of the hypotenuse and h_i is the cosine of the angle between the correspondent cathetus and hypotenuse.

Variation δ in type of (28) can be made not only for the squared values of h_i^2 (11), (22), (26) or (29) but also for the multi-optional hybrid functions h_i (13), (15) – (17), (19), (21) or (25). The character of $\Phi_h(\delta)$ will be like (31) (see Fig. 1). Condition (32) will be satisfied. The extremum (maximum) will be at the $\delta = 0$ value. Thus, this proves the extremality (optimality) of the multi-optional hybrid functions h_i (13) distribution. Also, it allows stating that the optimized value is the objective functional (1) with taking into consideration the entropy (2) of the squared multi-optional hybrid functions distribution (11).

IV. Conclusions

It is discovered the explanations for the conditions of optimality for Cartesian vector direction cosines with the help of the multi-optional hybrid functions entropy conditional optimality doctrine. Classical relations of the direction cosines happed to be the multi-optional hybrid functions obtained based upon the specified combinations of the proposed for considerations squared forms and calculus of variations theory methods.

This provides a new vision and establishes theoretical background of optimality for the elements of engineering mechanics engaged at the mechanical work calculations, which is necessary for the estimations of the energy mechanical engineering production efficiency and at the conversion of energy from one form to another.

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А. В. Гончаренко. Направляючі косинуси картезіанського вектора як оптимальний розподіл багатоопційних гібридних функцій

Здійснено спробу відкрити правдоподібну причину, що пояснює існування умов оптимальності направляючих косинусів картезіанського вектору, що є важливим в енергетичному машинобудуванні, за допомогою доктрини умовної оптимальності ентропії багатоопційних гібридних функцій. Обґрунтування здійснено в термінах теорії варіаційного обчислення за допомогою спеціальної міри невизначеності функцій гібридно-опційної ефективності, що включає ентропію тих гібридних функцій традиційного Шеннонівського стилю. У випадках, які вивчаються, розв'язки найпростішої варіаційної задачі, що є величинами знаними як направляючі косинуси Картезіанського вектору, обумовлені специфікованими натуральними логарифмами квадратичних форм. Пропонується оцінювати ступінь невизначеності / визначеності величини та спрямування картезіанського вектору із використанням цільового функціоналу. Це є новим поглядом на наукове пояснення добре відомої залежності виведеної іншим шляхом. Теоретичні міркування, які розвиваються, а також математичні викладки завершуються найпростішим числовим прикладом варійованої величини багатоопційної гібридної функції, результуючій в цільовому функціоналі.

Ключові слова: машинобудування; доктрина багатоопційності; умовна оптимальність; гібридно-опційна ефективність; картезіанський вектор; направляючий косинус; максимальна невизначеність; варіаційна задача.

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Напрям наукової діяльності: багатоопційна оптимальність, варіаційні принципи, експлуатація та керування в активних системах, безпека польотів.
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А. В. Гончаренко. Направляющие косинусы картезианского вектора как оптимальное распределение многоопционных гибридных функций

Осуществлена попытка открыть правдоподобную причину, поясняющую существование условий оптимальности направляющих косинусов картезианского вектора, что является важным в энергетическом машиностроении, с помощью доктрины условной оптимальности энтропии многоопционных гибридных функций. Обоснование осуществлено в терминах теории вариационного исчисления с помощью специальной меры неопределенности функций гибридно-опционной эффективности, включающей энтропию этих гибридных функций традиционного Шенноновского стиля. В изучаемых случаях, решения простейшей вариационной задачи, являющиеся величинами известными как направляющие косинусы картезианского вектора, обусловлены специфицированными натуральными логарифмами квадратичных форм. Предлагается оценивать степень неопределенности / определенности величины и направленности Картезианского вектора с использованием целевого функционала. Это является новым взглядом на научное пояснение хорошо известной зависимости выведенной другим путем. Развиваемые теоретические соображения, а также математические выкладки завершаются простейшим численным примером варьируемой величины многоопционной гибридной функции, результирующей в целевом функционале.

Ключевые слова: машиностроение; доктрина многоопционности; условная оптимальность; гибридно-опционная эффективность; картезианский вектор; направляющий косинус; максимальная неопределенность; вариационная задача.

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