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SYNTHRSIS OF QUADROTOR ROBUST GUIDANCE AND CONTROL SYSTEM VIA PARAMETERIZATION OF ALL STABILIZING H-INFINITY STATE-FEEDBACK GAINS

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Abstract*—The main purpose of the research is to develop the quadrotor robust control system adapted to the rejection of external disturbances. The methodology of control system synthesis is based on the parameterization of all stabilizing H-infinity static state feedback gains with applications to output feedback design. The main feature of the article is the development of the above-mentioned method relative to the quadrotor. The main results of the article are synthesized control laws and simulation of the closed-loop dynamics. The basic practical implication is the usage of synthesized control laws in guidance of the quadrotor motion in path following of circular and linear-piecewise reference tracks. Originality and value of the article are caused by the necessity to improve the quality of control by quadrotor motion*.

Index Terms—Quadrotor; guidance; linear matrix inequality approach; Riccati equation; robust control system; state-feedback gain; output feedback.

I. INTRODUCTION

Recently, among a large number of papers, devoted to the quadrotor flight control, the LQR control strategies take a significant place [1] – [6]. It can be explained by the fact that practical implementation of this strategy allows application of static state feedback (SSF) as the control law, which is very simple for implementation; therefore, it is preferable for the application in the small quadrotors. However, the LQR control strategies in these publications are based on the complete measurements of the quadrotor state vector components, meanwhile, in the real quadrotor control systems, it is necessary to take into account the inertia of the quadrotor's motors [7] – [11]. In this case, an additional state variable appears in the quadrotor's mathematical model, which is not measured in practice.

Therefore, in order to build a flight controller using output feedback, it is necessary to use corresponding methods based on the linear matrix inequalities (LMI) approach [12], [13], [15], where further references can be found. This approach produces control laws as static output feedback (SOF) and guarantees H_∞-suppression of external disturbances. These control laws were successfully applied to the flight control of various types of aerial vehicles [11] – [14]. At the same time, further development of this approach brought about new results based on the solution of the algebraic Riccati equation (ARE) [14], [16], producing the similar control strategy (SOF) with the same property of the external disturbances H_∞-suppression. Inasmuch as, from our viewpoint, obtaining ARE solution is simpler than LMI solution, we have chosen approach proposed in [14], [16] as the method of synthesis of the quadrotor guidance and control system.

This article is composed as follows. The first section is devoted to the problem statement and the brief description of the quadrotor's mathematical model including the brief theoretical background of the method of synthesis based on the [16] is given. The second section is devoted to the synthesis of the flight control law and simulation of the closed-loop system dynamics. The third section is devoted to the application of this control law to the guidance of the quadrotor motion in the horizontal plane and its simulation of the guidance system in the mode of path following of linear – piecewise reference track. Note, that the procedure of synthesis was performed for a linear model; meanwhile, the simulation was based on a full nonlinear 2D-model for planar motion.

II. MATHEMATICAL MODEL OF THE QUADROTOR AND THE PROBLEM STATEMENT

Nowadays a huge number of mathematical models exists in literature [1] – [10], where each paper begins from these models. That is why, for the sake of brevity, we avoid the explanation of principles of quadrotor operation due to the large number of publications devoted to this topic [1] – [10], and we represent here simplified model of the quadrotor, which is quite relevant for the solution of aforementioned guidance and control problems. References [5] – [11] give substantiation of this model based on the following assumptions:

1) Small values of Euler angles in the controlled flight allow us the linearization of basic equations.

2) The construction of quadrotor is symmetric with respect to the vertical *Z*-axis; therefore, it is possible to consider four motion control systems independently:

2.1. Rotation with respect to the *X*-axis by roll angle and linear motion along the *Y*-axis.

2.2. Rotation with respect to the *Y*-axis by pitch angle, and linear motion along the *X*-axis.

2.3. Rotation with respect to the *Z*-axis by yaw angle.

2.4. Linear motion along the *Z*-axis for height control.

Later, for the sake of brevity, we will simply refer to the numbers of cases in this list for indication of concrete partial mode.

3) It is possible to neglect Coriolis accelerations and gyro-effects for engineering design purposes.

We know [1] – [10], that control moments and forces, which can achieve quadrotor's main rotational and translational motions, itemized in the 2.1 – 2.4 statements, look as follows

$$\begin{aligned} Q_\theta &= K_T l(\Omega_2^2 - \Omega_4^2), \\ Q_\phi &= K_T l(\Omega_3^2 - \Omega_1^2), \\ Q_\psi &= K_Q\left[(\Omega_2^2 + \Omega_4^2) - (\Omega_3^2 + \Omega_1^2)\right], \\ T_z &= \sum_{n=1}^{4} T_n = K_T \sum_{n=1}^{4} \Omega_n^2, \end{aligned} \tag{1}$$

where $Q_\theta, Q_\varphi, Q_\psi$ are control moments with respect to corresponding axes, T_z is the total thrust force, K_T, K_Q are the thrust and the reactive moment coefficients respectively, l is the length of the arm between each motor $M_n, n = 1, ..., 4$ and the center of gravity of quadrotor, T_n and Ω_n are the thrust force and angular rotation rate of each motor M_n respectively. It is also useful to mention that the thrust force and the reactive moment of each motor look like $T_n = K_T\Omega_n^2$, $Q_n = K_Q\Omega_n^2$, and θ, φ, ψ are the pitch, roll and yaw angles respectively.

Taking in account aforementioned assumptions, block-diagram of linearized flight control system of quadrotor looks like Fig. 1, where "*X*- cont, *Y*- cont" – controllers for governing motions of quadrotor in the horizontal plane (see items 2.1, 2.2), "ψ -cont" – controller for rotational motion of quadrotor with respect to the vertical axis, "Z -cont" – controller for altitude control, $U_\theta, U_\varphi, U_\psi, U_Z$ are control inputs, X_c, Y_c, ψ_c, Z_c are command signals.

This block-diagram complies with the control structure described in [6]. Note also, that "Transformation" block is designated for the determination of the well-known system (1) solution with respect to the squared angular rates Ω_n^2 of each motor M_n [1] – [10] as functions of control inputs $U_\theta, U_\varphi, U_\psi, U_Z$. Actually, these control inputs generate the increments $\Delta\Omega_n$ of the corresponding angular rates with respect to some equilibrium value Ω_0, which provides hovering mode of quadrotor at the given height. The increments $\Delta\Omega_n$ are produced by the increments of current in each rotor of motors M_n by ESC – electronic speed controllers.

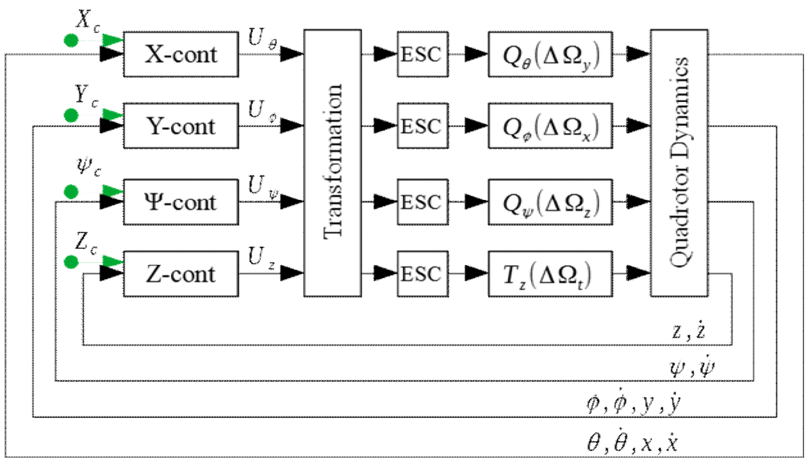


Fig. 1. Quadrotor position control system

Due to the mentioned above assumptions, it is possible to synthesize control law separately for basic partial quadrotor motions mentioned above. As far as the goal of this article is the guidance and control law synthesis for planar motion in the horizontal plane at the constant altitude, we will consider the partial motion of quadrotor, including rotation with respect to X-axis at the angle φ and linear motion along axis Y (item 2.1 in the aforementioned list). Taking into account that linearized models in cases 2.1 and 2.2 are very similar [3] – [9]; results of control law synthesis for case 2.1 will be used for case 2.2 as well.

We will use the model of dynamics of partial motion 2.1 proposed in [2] – [5], which is augmented by the terms describing drag force and the inertia of motors. The parts of the model produced by drag force were omitted in [3] – [5], but in paper [10] it was proposed the estimation of these terms and their incorporation in the linearized model. Also, it is necessary to include the term, which determines the motor inertia. It can be approximated by the transfer function of the 2nd order [8] or the 1st order [11]. Taking into account small inductance of the rotor circuit; we can use the simplest 1st order model of electromotor [11]

$$W_{em}(s) = \frac{K_{\Sigma}}{\tau_{em}s + 1}, \tag{2}$$

where K_{Σ} is the static gain and τ_{em} is a time constant of the electromotor. The input of this block is the increment of currently produced by ESC, and the output is the increment of the motor's angular rate $\Delta\Omega$, which produces control moment applied to the X-axis of quadrotor due to the 2nd equation in the system (1).

In accordance with [3] – [5], [11], it is possible to represent the linearized model for case 2.1 in the following standard form [12] – [16]

$$\begin{aligned} &\frac{d\mathbf{x}}{dt}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_u\mathbf{u}(t) + \mathbf{B}_d(t)\mathbf{d}(t), \\ &\mathbf{v}(t) = \mathbf{C}\mathbf{x}(t), \end{aligned} \tag{3}$$

where the state vector $\mathbf{x}$ and matrices $\mathbf{A}, \mathbf{B}_u, \mathbf{B}_d$ have the following forms

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} y & \dfrac{dy}{dt} & \varphi & \dfrac{d\varphi}{dt} & \Delta\Omega \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \dfrac{-\mu}{m_Q} & g & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & j^{-1} \\ 0 & 0 & 0 & 0 & -\tau_{em}^{-1} \end{bmatrix}, \quad \mathbf{B}_d = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \mu \end{bmatrix}, \tag{4}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

In system (3) $\mathbf{u}(t)$ is the control moment applied to the Y-axis, $\mathbf{d}$ is the disturbance (instantaneous velocity of turbulent wind). In expressions (4) m_Q is the mass of quadrotor, μ is the drag force coefficient [10], having dimension kg·m·s^{-1}, and j is the moment of inertia with respect to the X-axis; other parameters are defined by the transfer function (2). As can be seen from the matrix $\mathbf{C}$ in (4) the increment of the motor's angular rate is not measured. That is why it is expedient to find a solution to control problem synthesis using static output feedback (SOF) [12] – [16], because it guarantees the simplest structure of the controller. For this case, flight control law for quadrotor, described by the model (3), looks like

$$\mathbf{u}(t) = -\mathbf{F}\mathbf{v}(t) = -\mathbf{F}\mathbf{C}\mathbf{x}(t), \tag{5}$$

where $\mathbf{F} \in \mathbf{R}^{1\times 4}, \mathbf{C} \in \mathbf{R}^{4\times 5}$. Now it is possible to formulate the problem of suppressing external disturbance $\mathbf{d}$ by bounded L_2-gain in the standard form [12] – [15]. Let we have well-known integral-quadratic performance index

$$I = \int_0^{\infty} \|z(t)\|^2 = \int_0^{\infty} (\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u})dt, \tag{6}$$

which defines desired output signal

$$\mathbf{z} = \begin{bmatrix} \sqrt{\mathbf{Q}} & \mathbf{0} \\ \mathbf{0} & \sqrt{\mathbf{R}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \end{bmatrix}. \tag{7}$$

Then we define the bounded system L_2-gain as follows [12] – [16]

$$\frac{\int_0^{\infty} \|\mathbf{z}(t)\|^2 dt}{\int_0^{\infty} \|\mathbf{d}(t)\|^2 dt} = \frac{\int_0^{\infty} (\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u})dt}{\int_0^{\infty} (\mathbf{d}^{\mathrm{T}}\mathbf{d})dt} \le \gamma^2. \tag{8}$$

The problem consists of finding static output feedback (SOF) gain $\mathbf{F} \in \mathbf{R}^{1\times 4}$, which guarantees satisfying inequality (8). Note, that minimal possible value of γ is denoted as $\gamma *$. We used here the following basic result concerning parameterization of all stabilizing H_∞ SOF controllers [16]: "Consider a specified matrix $\mathbf{Q} \geq 0$ such that $\left(\mathbf{A}, \mathbf{Q}^{1/2}\right)$ is detectable, $(\mathbf{A}, \mathbf{B}_u)$ is stabilizable, and a specified value $\gamma > \gamma *$. Then there exists a SOF gain $\mathbf{F}$ such that $\mathbf{A}_0 \equiv (\mathbf{A} - \mathbf{B}_u\mathbf{FC})$ is asymptotically stable with bounded L_2-gain, if and only if there exists a parameter matrix $\mathbf{L}$ such that

$$\mathbf{FC} = \mathbf{R}^{-1}(\mathbf{B}_u^{\mathrm{T}}\mathbf{P} + \mathbf{L}), \tag{9}$$

where $\mathbf{P} = \mathbf{P}^{\mathrm{T}} \geq 0$, is a solution to

$$\begin{gathered}\mathbf{PA} + \mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} + \frac{1}{\gamma^2}\mathbf{PB}_d\mathbf{B}_d^{\mathrm{T}}\mathbf{P} - \\ \mathbf{PB}_u\mathbf{R}^{-1}\mathbf{B}_u^{\mathrm{T}}\mathbf{P} + \mathbf{L}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{L} = 0\text{''}.\end{gathered} \tag{10}$$

Let $\mathbf{H}_{zd}(s)$ will be a matrix of transfer functions from disturbance $\mathbf{d}$ to the desired output $\mathbf{z}$. It is known from [12] – [15], that in this case, $\|\mathbf{H}_{zd}(s)\|_\infty \leq \lambda$. The strict proof of the aforementioned statement is given in [16].

III. Synthesis of the Quadrotor Control System Based on the Parameterization of All Stabilizing H-infinity Static State-Feedback (SSF) Gains with Application to SOF Design

The basic idea of H_∞-SOF design, proposed in [16], is the parameterization of all H_∞-SSF gains at the 1st stage and its application to the SOF – design at the 2nd stage. That is why it is possible to find solutions of the algebraic Riccati equations (ARE) instead of LMI solution, which essentially alleviates computational problems. Let $\mathbf{K}$ will be SSF-gain obtained after application of the LQR-procedure to the system (3), (4) with given matrices $\mathbf{Q}, \mathbf{R}$ in (6). In our case, this SSF-gain has size $\mathbf{K} \in \mathbf{R}^{1\times 5}$. On the other hand, in our case SOF –gain has the size $\mathbf{F} \in \mathbf{R}^{1\times 4}$ (see (4), (5)), and from (9) it follows

$$\mathbf{F} = \mathbf{R}^{-1}(\mathbf{B}_u^{\mathrm{T}}\mathbf{P} + \mathbf{L})\mathbf{C}^{\mathrm{T}}(\mathbf{CC}^{\mathrm{T}})^{-1}. \tag{11}$$

It is proposed in [16] to use the singular value decomposition (SVD) of the matrix $\mathbf{C}$. It should be noted that $\mathbf{C} = \mathbf{USV}^{\mathrm{T}} = \; = \mathbf{U}\begin{bmatrix}\mathbf{S}_0 & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{\mathrm{T}} \\ \mathbf{V}_2^{\mathrm{T}}\end{bmatrix}$ for the pseudo-inverse matrix $\mathbf{C}^+ = \mathbf{C}^{\mathrm{T}}(\mathbf{CC}^{\mathrm{T}})^{-1}$ calculation. Using this SVD, we obtain

$$\mathbf{C}^+ = \mathbf{V}_1(\mathbf{S}_0)^{-1}\mathbf{U}^{\mathrm{T}}. \tag{12}$$

In order to recalculate gain matrix $\mathbf{K}$ in the gain matrix $\mathbf{F}$, it is proposed [16] to use the projection onto null space perpendicular of $\mathbf{C}$ using matrix

$$\mathbf{f} = \mathbf{I} - \mathbf{V}_2\mathbf{V}_2^{\mathrm{T}}. \tag{13}$$

After these preliminary remarks, we will present the algorithm of synthesis proposed [16], which is based on (9), (10) and used here for quadrotor flight control design.

Algorithm for quadrotor flight control synthesis includes following steps [16].

1) Input: Quadrotor state space model $\mathbf{A}, \mathbf{B}_u, \mathbf{B}_d, \mathbf{C}$; matrices – $\mathbf{Q}, \mathbf{R}$, $\mathbf{L}_0 = 0$, projecting matrices $\mathbf{C}^+$, $\mathbf{f}$ defined by (12) and (13) respectively; constants $\gamma *$, $tol(\approx 0.01)$, $n = 0$

2) Solve LQR for given $(\mathbf{A}, \mathbf{B_u}, \mathbf{Q}, \mathbf{R})$ and obtain SSF-gain $\mathbf{K}_0$ and ARE-solution for $\mathbf{P}_0$.

3) Define matrix $\tilde{\mathbf{A}}_0 = \mathbf{A} - \mathbf{BK}_0$ for close-loop system at *n-th* iteration: solve ARE for $\mathbf{P}_n$

$$\mathbf{P}_n(\tilde{\mathbf{A}}_n) + (\tilde{\mathbf{A}}_n^{\mathrm{T}})\mathbf{P} + \mathbf{Q} + \mathbf{K}_n^{\mathrm{T}}\mathbf{RK}_n + \frac{1}{\gamma^2}\mathbf{P}_n\mathbf{B}_d\mathbf{B}_d^{\mathrm{T}}\mathbf{P}_n = 0.$$

4) Update $\mathbf{K}_n$: $\mathbf{K}_{n+1} = \mathbf{R}^{-1}(\mathbf{B}_U^{\mathrm{T}}\mathbf{P}_n + \mathbf{L}_n)\mathbf{f}$,

5) Update $\mathbf{L}_n$: $\mathbf{L}_{n+1} = \mathbf{RK}_n - \mathbf{B}^{\mathrm{T}}\mathbf{P}_n$,

6) Update $\tilde{\mathbf{A}}_n$: $\tilde{\mathbf{A}}_{n+1} = \tilde{\mathbf{A}}_n - \mathbf{BK}_{n+1}$.

7) Check convergence: if $\|\mathbf{P}_{n+1} - \mathbf{P}_n\| < tol$, go to the step 8, otherwise go to the step 2 and set $n = n+1$.

8) End.

9) Set $\mathbf{K} = \mathbf{K}_{n+1}$ and compute SOF gain $\mathbf{F} = \mathbf{KC}^+$.

10) Compute $\|\mathbf{H}_{zd}(j\omega)\|_\infty$ and $\|\mathbf{H}_{zd}(j\omega)\|_2$.

IV. Case Study

Consider the application of the aforementioned results to the H_∞-SOF design of the guidance and control system for the quadrotor, which was designed in the National Aviation University (Kyiv, Ukraine). It has the following parameters: the total mass of quadrotor together with batteries and payload $m_Q = 5.5$ kg, including the mass four electrical engines Foxtech X5010 KV288 with $m = 0.213$ kg each; the length from the center of mass to the engine $l = 0.343$ m.

Using results devoted to the quadrotor's mathematical model design [6] – [11], the following numerical values of matrices in system (3), (4) were computed:

$$\mathbf{A}=\begin{bmatrix}0 & 1 & 0 & 0 & 0\\ 0 & -0.28 & 9.81 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 13.51\\ 0 & 0 & 0 & 0 & -8.33\end{bmatrix},\quad \mathbf{B}_u=\begin{bmatrix}0\\0\\0\\0\\10.42\end{bmatrix},\quad \mathbf{B}_d=\begin{bmatrix}0\\1\\0.0285\\0\\0\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{bmatrix}. \tag{14}$$

Applying to this model aforementioned algorithm with the following parameters

$$\mathbf{Q}=\mathrm{diag}\begin{pmatrix}1000 & 1500 & 400000 & 1000000 & 0\end{pmatrix},\quad R=2500,\qquad \gamma=0.1, \tag{15}$$

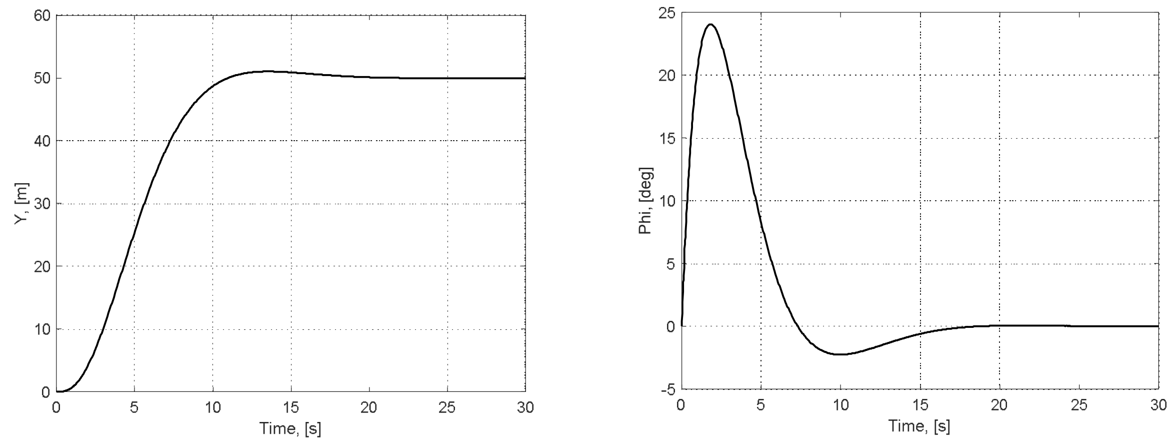
we will obtain the following H_∞-SOF gain matrix: $\mathbf{F}=[-0.6325\quad 1.4566\quad 27.3862\quad 20.7291]$, which produces the following eigen-values of the closed-loop system state propagation matrix $\widetilde{\mathbf{A}}=\mathbf{A}-\mathbf{BFC}$: $eig(\widetilde{\mathbf{A}})=[-3.5\pm 53.81i,\quad -0.7,\quad -0.45\pm 0.47i]$.

The H_∞-norm of the transfer function $\mathbf{H}_{zd}(s)$ is equal to $\|\mathbf{H}_{zd}(j\omega)\|_\infty=0.1291$; and the L_2-gain is evaluated as $\|\mathbf{H}_{zd}(j\omega)\|_2=0.0856$. This completely corresponds to the given value $\gamma=0.1$. The transient processes in this system are represented in Fig. 2.

As can be seen from Fig. 2, the transient time in the closed-loop system is 10 s (Fig. 2 a) and the angle of the roll is below 25 deg (Fig. 2 c). So the dynamics of this system is quite acceptable for practical applications.

Taking into account the symmetry of quadrotor and its mathematical model, we omit the procedure of control system design for rotation with respect to the *Y*-axis by pitch angle, and linear motion along the *X*-axis and we give here only H_∞-SOF gain matrix for this case:

F1 = [0.6325 -1.4566 -27.3862 -20.7291].



(a) (b)

Fig. 2. Transient processes for the components of $\mathbf{x}$ vector: (a) $x_1=y$; (b) $x_2=\varphi$

These control systems for X and Y axes were used for control of planar circular motion in the horizontal plane. It is necessary to note, that in this case, we used the nonlinear model of quadrotor translational dynamics. In accordance with [3] – [5], [10] it will have the following form

$$\frac{d^2x}{dt^2}=-\frac{\mu}{m_Q}\frac{dx}{dt}-g\frac{\tan\theta}{\cos\varphi},\qquad \frac{d^2y}{dt^2}=-\frac{\mu}{m_Q}\frac{dy}{dt}+g\tan\varphi. \tag{16}$$

Figure 3 shows a generalized block diagram of the guidance control system, which includes two separate systems *X-con* and *Y-con* for quadrotor motion control along X and Y axes, and RT – block, generating reference track. In Figure 3 X_C, Y_C stand for command signals, and X_q, Y_q are coordinates of the quadrotor actual position. Figure 4 presents the processes of simulated UAV motion in the horizontal plane

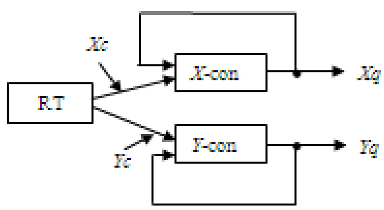


Fig. 3. A generalized block-diagram of the guidance and control system

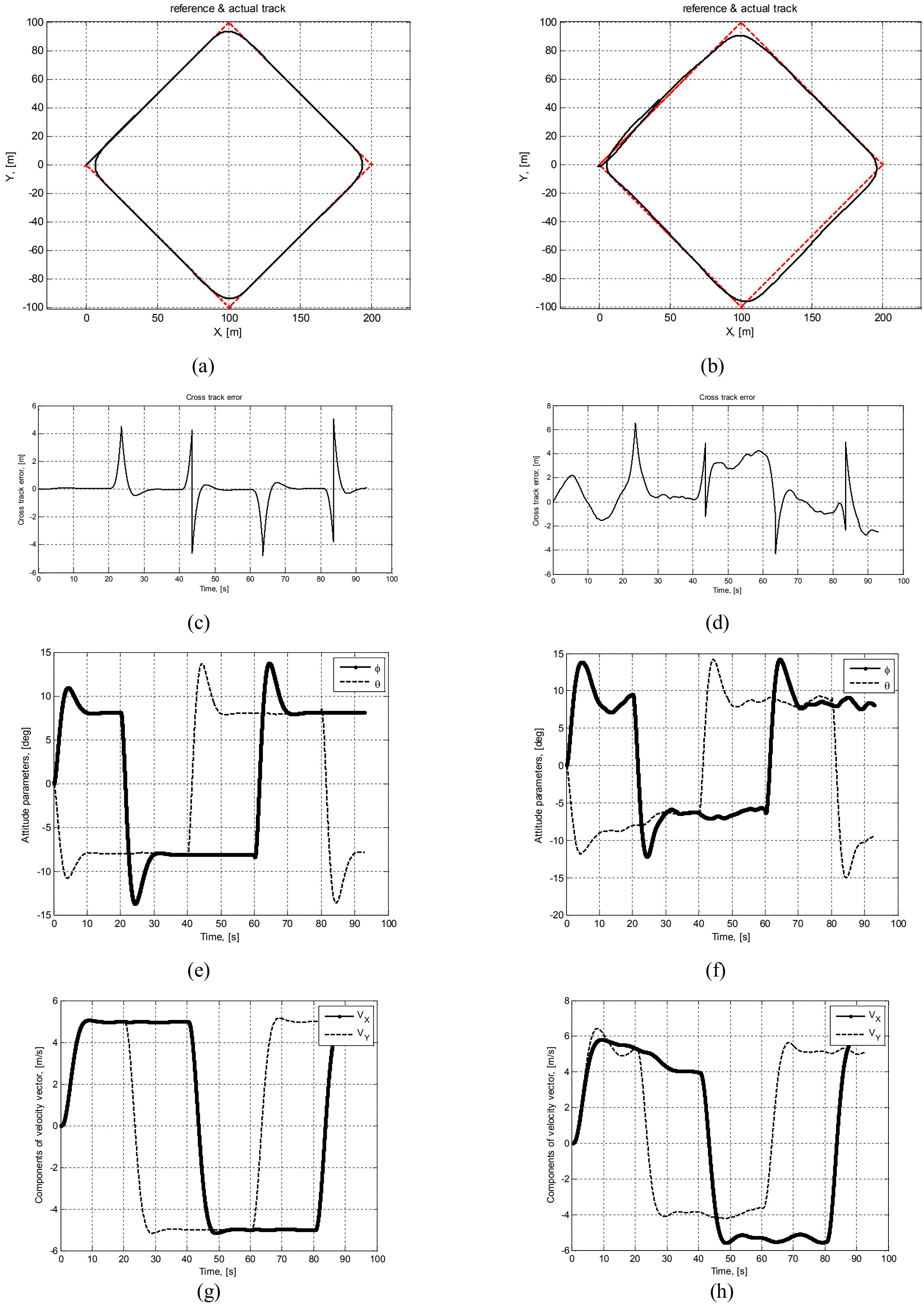


Fig. 4. Results of simulation of the quad-rotor guidance system for reference track in the case of calm and disturbed atmosphere: (a), (b) is the reference and actual tracks in calm and turbulent atmosphere respectively; (c), (d) are cross-track errors in calm and turbulent atmosphere respectively; (e), (f) are attitude parameters such as pitch and roll in calm and turbulent atmosphere respectively; (g), (h) is the component of velocity vectors by axes X, Y in calm and turbulent atmosphere respectively

Simulation of quadrotor path following in the turbulent atmosphere used Dryden model of moderate wind disturbances at low altitude [17]. Obtained results of simulation prove the resilience of synthesized flight control system to atmospheric disturbances.

V. CONCLUSIONS

Taking into account the simplicity and efficiency of the method of synthesis of the quadrotor control system based on the parameterization of all stabilizing H_∞ static state-feedback (SSF) gains with application to SOF design [16], we have chosen it for the design of the quadrotor guidance and flight control system.

The separate control systems for X and Y axes of the quadrotor were designed on the basis of the aforementioned method. The simulation of the transient processes in these systems demonstrated their efficiency.

These control systems were applied to the quadrotor's circular path following in the horizontal plane when the nonlinear model of the planar quadrotor's motion was used. In this case, the results of the simulation are also successful.

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А. А. Тунік, С. І. Ільницька, О. А. Сущенко. Синтез робастної системи наведення та керування квадрокоптера за допомогою параметризації усіх стабілізувальних підсилень H_∞ зворотного зв'язку за станом

Основною метою дослідження є розробка робастної системи керування квадрокоптера, пристосованої до усунення зовнішніх збурень. Методологія синтезу системи керування заснована на параметризації усіх стабілізуючих підсилень H_∞ статичного зворотного зв'язку за станом із застосуванням синтезу зворотного зв'язку за виходом. Основною особливістю статті є розробка вищезазначеного методу щодо квадрокоптера. Основними результатами статті є синтезовані закони керування і моделювання динаміки замкнутої системи. Основне практичне значення – використання синтезованих законів керування для управління рухом квадрокоптера для заданих кругових і лінійно-кускових траєкторій. Оригінальність і цінність статті зумовлено необхідністю підвищення якості керування рухом квадрокоптера.

Ключові слова: квадротор; наведення; метод лінійних матричних нерівностей; рівняння Ріккаті; робастна система керування; підсилення зворотного зв'язку за станом; зворотний зв'язок за виходом.

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А. А. Туник, С. И. Ильницкая, О. А. Сущенко. Синтез робастной системы наведения и управления квадрокоптера посредством параметризации всех стабилизирующих усилений H_∞ обратной связи по состоянию

Основной целью исследования является разработка робастной системы управления квадрокоптера, приспособленной к устранению внешних возмущений. Методология синтеза системы управления основана на параметризации всех стабилизирующих усилений H_∞ статической обратной связи по состоянию с применением синтеза обратной связи по выходу. Основной особенностью статьи является разработка вышеупомянутого метода относительно квадрокоптера. Основными результатами статьи являются синтезированные законы управления и моделирование динамики замкнутой системы. Основное практическое значение – использование синтезированных законов управления для управления движением квадрокоптера по заданным круговым и линейно-кусочным траекториям. Оригинальность и ценность статьи обусловлены необходимостью повышения качества управления движением квадрокоптера.

Ключевые слова: квадротор, наведение, метод линейних матричных неравенств, уравнение Риккати, робастная система управления, усиление обратной связи по состоянию, обратная связь по выходу.

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