TRANSPORT SYSTEMS

UDC 303.725.36:159.9.015:159.964.21:519.86 (045)
DOI:10.18372/1990-5548.61.14220

A. V. Goncharenko

HYBRID RELATIVE COMBINED PSEUDO-ENTROPY FUNCTION AS A TOOL FOR A TRANSPORT SYSTEM MANAGEMENT

Aerospace Faculty, National Aviation University, Kyiv, Ukraine
E-mail: andygoncharenco@yahoo.com ORCID 0000-0002-6846-9660

Abstract—*It is made an attempt to discover an explainable plausible reason for the existence of the transport system's model production function, or a number of transportation means function, as a hyperbolic type function of the available transportation mean capacity, or a linear type function for two types of transportation means. Substantiation is made in terms of the calculus of variations theory with the help of the special hybrid-optional effectiveness functions uncertainty measure, which includes the hybrid functions entropy of the traditional Shannon's style. In the studied cases, the simplest variational problems solutions, which are the numbers of the transportation means, are stipulated by the specified quadratic forms. It is proposed to evaluate the magnitude and direction of the transport system's managerial certainty with the use of the combined hybrid relative pseudo-entropy function. This is a new insight into the scientific explanation of the well-known dependency derived in another way. The developed theoretical contemplations and mathematical derivations are finalized with a simplest numerical example for the traditional entropy weakness.*

Index Terms—Transport system; production function; multi-optionallity doctrine; conditional optimality; hybrid-optional effectiveness function; pseudo-entropy; prevailing index; domination factor; variational problem.

I. INTRODUCTION

Through the last decades, organizational factors have been playing more and more significant role in the successful management of the transport systems effective operation.

Aviation transport system effectiveness and safety management is a bright example.

It is always an actual task to find an optimal number and capacity for transportation means. Such optimality must take into account economical indexes somehow [1].

Subjective analysis approaches [2] enable us considering individual preferences in an explicit view; and the preferences functions certainty (or uncertainty) measures need applicable adjustments.

Promising innovations have been proposed in papers [3], [4] and reported at a conference [5]. They need further improvements and modifications.

Aim. The presented paper is aimed at modeling an optimal choice for the transport systems abilities based upon the developed combinations of the proposed for considerations quadratic forms and calculus of variations theory methods with an outcome into the hybrid combined relative pseudo entropy function, like in [6] – [9].

II. PROBLEM STATEMENT

Contemplations likewise the described above has instigated the search of a certain kind optimality in the substantiations of the transport systems abilities.

The Problem Setting. The problem statement for the current state would be as to find a value extremized with the known view expression used as a production function. Then, assessing the intensity and inclinations of the options' functions certainty.

III. PROBLEM SOLUTION

A. Hyperbolic Solution

Generally, considering the problem stated above, one can try to look for an optimal in some sense value.

Calculus of variations is a good theoretical tool for searching such optimality.

Consider a quadratic value

$$\frac{n(s)^2}{2} + k\ln(s)n'(s), \tag{1}$$

where $n(s)$ is the number of the required transportations means, as a function of the means capacity s, both being necessary to carryout the amount of the needed transportation work of k, on the conditions of the available transportation means capacity s and with the derivative of

$$n'(s) = \frac{d}{ds}\left[n(s)\right] = n'_s. \tag{2}$$

The proposed for consideration expression (1) with respect to equation (2) has a significance of the under-integral function creating an objective functional

$$\Phi[n(s)] = \int_0^{s_1} \left[\frac{n(s)^2}{2} + k \ln(s) n'(s) \right] ds \,. \quad (3)$$

The value of s_1 in the purpose functional (3) symbolizes the marginal value of the transportation mean's capacity s varying through the possible diapason of $[0 \ldots s_1]$.

The problem turns into the simplest problem of the calculus of variations:

$$\Phi[n(s)] = \int_0^{s_1} F(n'_s, n, s) ds \to \underset{n \in N}{\text{extr}} \,, \quad (4)$$

$$n(s) = n_s = \arg \underset{n \in N}{\text{extr}} \{ \Phi[n(s)] = \Phi_n \} \,, \quad (5)$$

$$F(n'_s, n, s) = \frac{n^2}{2} + k \ln(s) n'_s \,. \quad (6)$$

The proposed consideration (1) – (6) is solved with the help of the Euler-Lagrange equation:

$$\frac{\partial F}{\partial n} - \frac{d}{ds} \left(\frac{\partial F}{\partial n'_s} \right) = 0 \,, \quad (7)$$

which determines the necessary conditions for a possible extremum existence.

Here, in equation (7)

$$F = F(n'_s, n, s) = \frac{n^2}{2} + k \ln(s) n'_s \,. \quad (8)$$

Thus,

$$\frac{\partial F}{\partial n} = n \,, \quad (9)$$

$$\frac{\partial F}{\partial n'_s} = k \ln(s) \,, \quad (10)$$

$$\frac{d}{ds} \left(\frac{\partial F}{\partial n'_s} \right) = \frac{k}{s} \,, \quad (11)$$

$$n - \frac{k}{s} = 0 \,. \quad (12)$$

The solution through the method of (7) – (12) yields

$$n(s) = \frac{k}{s} \,. \quad (13)$$

One more feasible extremum, for the hyperbolic dependence (13), can be obtained with regards to the additional constraints imposed upon the variables of $n(s)$ and s.

For example, the wanted solution for $n(s_{\text{opt}})$ can be of the economical parameters optimization problem domain.

B. Linear Solution

In general sense, it is possible to model the sustainable development, as for example, for economic (industry, agriculture, construction, transport etc.) development, basing upon the principles of economical optimality [1]. On the other hand, the different types of the considered economic activity levers (optional production sources), or for the presented illustrative needs optional economical (industrial, agricultural, constructional, transportation) means, in the context of their "multi-optional" use (as well as that of the competing kinds/sorts of the sources as a whole) could be compared in pairs, [5].

Thus, it becomes possible to optimize the distribution of the productive work inside the chosen kind of the sources, for instance, between the economical means of types $\tilde{a}$ and $\tilde{b}$. Also, that same approach is applicable for dividing between the kinds of the sources, for example, between the industrial and agricultural (or road/water transportation sorts of delivery) optional sources.

Suppose the specified productive work could be done by the two types (sorts/kinds) of the production (transportation) means; conventionally these are $\hat{a}$ and $\hat{b}$. Now they are just the abstract designations basically used for any class of economical sources as they are (let us say the vehicles of the ground, water, and air, whatever of the same kind/nature etc.); the number of the sources (vehicles) are a and b correspondingly.

The definite amount of the productive work $\hat{D}$ can be executed by the sources of a only; and in such case, the determined number of the sources is n_1 with respect to their productivity. In its turn the sources of the group of b can perform the same definite amount of the productive work $\hat{D}$ alone too. However, because of their other productivity the required number of the sources will be n_2.

Also suppose that for the work of the sources of a in conjunction with the sources of b there might be existed a certain function of $a(b)$ because the productivity of the economical means a might vary due to the presence or change (deviation) of the b option.

For the presented research, it is proposed to consider an objective value (a functional) related to the two types of the economical means; and the value depends upon the numbers of the each type sources in the following way, [5, p. 52, (1)]:

$$\Phi[a(b)]=\int_0^{n_2}\left[\left(\frac{a(b)}{2}+b\frac{n_1}{n_2}\right)a(b)+n_1ba'(b)\right]db\,. \quad (14)$$

Here, in equation (14), $\Phi[a(b)]$ represents the objective functional depending upon the variated parameters of the model and first of all upon $a(b)$ and b within the number of b possible range of alterations $[b_0=0\ldots b_1=n_2]$.

The essential part of the model objective functional (14) is the assumptions and simplifications that the numbers of the each type vehicles (as well as, for instance, in the above considered with the expressions of (1) – (13) problem setting) are of the continuous nature (which is not truth of course but); this illusion helps using the analytical apparatus of the calculus. The mathematical result then is just simply picked up the closest to the integer value.

The under-integral function of the objective functional (14):

$$\left(\frac{a(b)}{2}+b\frac{n_1}{n_2}\right)a(b)+n_1ba'(b) \quad (15)$$

expresses the specific idea of a quadratic form application which implies the members responsible for the half current number of the vehicles of the $\hat{a}$ group of $a(b)$ squared, their number in conjunction with the corresponding current value of the $\hat{b}$ group transportation means b number assessed with the ratio of n_1/n_2, and the rated value of $a(b)$ with respect to b number $[a'(b)]$ evaluated through the coefficients of proportionality b and n_1.

For this reason, in the considered case-study, it is investigated the model constructed of the expressions of (14) and (15), being also dependent upon the fist complete derivative of

$$a'(b)=\frac{d}{db}[a(b)]=a'_b\,. \quad (16)$$

"Prognostically" the model objective functional (14) must have an extremal value. Therefore it is important to find a plausible substantiation for the objective functional (14) to undergo such an extremum with respect to the accepted suppositions and simplifications reflected in expressions of (14) and (15).

Mathematically, formulated in the view of the equations of (14) – (16), this particularly given problem setting is stated as the simplest problem of the calculus of variations (likewise the problem of (4) considered above) for the objective functionals similar to (3):

$$\Phi_a=\int_0^{n_2}F(b,a,a'_b)db\,, \quad (17)$$

where $F(b,a,a'_b)$ is the under-integral function (15) of the stated problem objective functional (14).

That is

$$F(a'_b,a,b)=\left(\frac{a}{2}+b\frac{n_1}{n_2}\right)a+n_1ba'_b\,. \quad (18)$$

The purpose is to minimize the objective functional (14) on conditions of (15) by finding such function of $a(b)$ that delivers the wanted minimum for the supply efforts formation (this is the crucially different thing with respect to the reference [4] ideology); and for the general view integral of (14) there are the necessary conditions for the extremum existence in the view of the well-known Euler-Lagrange equation (7) again, but for the considered case it gets the view of

$$\frac{\partial F}{\partial a}-\frac{d}{db}\left(\frac{\partial F}{\partial a'_b}\right)=0\,. \quad (19)$$

Here, in equation (19), [5, p. 53, (4)]:

$$F=F(a'_b,a,b)=\left(\frac{a}{2}+b\frac{n_1}{n_2}\right)a+n_1ba'_b\,. \quad (20)$$

Applying the conditions of equations (15) – (20), and the methods analogous to the procedures of the formulas of (9) – (11), for solving the objective functional (14) it yields [5, p. 53, (5)]:

$$a(b)=n_1-b\frac{n_1}{n_2}\,. \quad (21)$$

Thus, we have got a linear solution (21), which likewise above (13) might have an optimum as a "point on the curve".

C. Hybrid Combined Relative Pseudo-Entropy Function

It is possible to apply an entropy paradigm approach, likewise initiated in publication [2], especially discussed in the reference [3].

In order to assess the "valuableness" and certainty/uncertainty of the considered options, some special hybrid-optional functions can be introduced.

For an evaluation of the hybrid-optional functions h_i (either probabilities or subjective preferences [2] from subjective analysis) certainty/uncertainty degree relating with the positive or negative options (or alternatives); the number of the options (alternatives) is i; it is proposed to use the hybrid combined relative pseudo-entropy function $\overline{H}_{\max - \frac{\Delta h}{|\Delta h|}}$, [3]:

$$\overline{H}_{\max - \frac{\Delta h}{|\Delta h|}} = \frac{H_{\max} - H_h}{H_{\max}} \cdot \frac{\Delta h}{|\Delta h|} . \qquad (22)$$

Here in expression (22) $H_{\max}$ is the maximal possible entropy (uncertainty) of the hybrid-optional functions h_i, H_h is the factual entropy

$$H_h = -\sum_{i=1}^{n} h_i \ln h_i , \qquad (23)$$

$$\Delta h = \sum_{j=1}^{M} h_j^+ - \sum_{k=1}^{L} h_k^- , \qquad (24)$$

where h_j^+ and h_k^- are positive and negative properties hybrid-optional functions respectively, M and L are numbers of the positive and negative properties options:

$$M + L = n . \qquad (25)$$

The point is that, unfortunately, such measure of uncertainty as expression (23) (the traditional view entropy of the Shannon's type) does not show the direction of the certainty/uncertainty neither its relative value.

Although the hybrid-optional functions entropy (23) serves as a measure of uncertainty of the hybrid-optional functions h_i, it will be the same, for example, if $h_1 = 0.77$ and $h_2 = 0.23$ or vice versa $h_1 = 0.23$ and $h_2 = 0.77$ or for any other symmetrical distributions. Moreover, the entropy's absolute (irrelative) value does not say much of the relative degree of certainty/uncertainty either.

That is why in order to bypass such a difficulty it seems helpful to apply the entropy-based measure (22) including (23) – (25).

IV. CONCLUSIONS

It is discovered the explanations for the transport systems economic activity dependences known for a long period. Classical relations are obtained based upon the specified combinations of the proposed for considerations quadratic forms and calculus of variations theory methods.

The hybrid combined relative pseudo entropy function gives a possibility to study the relative measure of assuredness and inclinations towards the available options. Therefore, the named function can be proposed as a tool for a transport system management.

Parameters of the hybrid combined relative pseudo entropy function need further investigation.

REFERENCES

[1] E. Silberberg and W. Suen, *The Structure of Economics. A Mathematical Analysis*, New York: McGraw-Hill Higher Education, 2001, 668 p.

[2] V. Kasianov, *Subjective Entropy of Preferences*, *Subjective Analysis*: Monograph, Warsaw, Poland: Institute of Aviation Scientific Publications, 2013, 644 p.

[3] A. V. Goncharenko, "Measures for Estimating Transport Vessels Operators' Subjective Preferences Uncertainty," *Scientific Bulletin of Kherson State Maritime Academy*. vol. 1(6), 2012, pp. 59–69.

[4] A. V. Goncharenko, "Airworthiness support measures analogy to the prospective roundabouts alternatives: theoretical aspects," *Journal of Advanced Transportation*. 2018. vol. 2018, Article ID 9370597, 7 pages.

[5] A. V. Goncharenko, "Relative Pseudo-Entropy Functions and Variation Model Theoretically Adjusted to an Activity Splitting," *2019 9th International Conference "Advanced Computer Information Technologies (ACIT'2019)" Proceedings*. June 5–7, 2019, Ceske Budejovice, Czech Republic, 2019, pp. 52–55.

[6] A. V. Goncharenko, "Subjective Entropy Maximum Principle for Preferences Functions of Alternatives Given in the View of Logical Conditions," *Artificial Intelligence*. vol. 4(62), 2013, pp. 1 G, 4–9.

[7] A. V. Goncharenko, "Horizontal Flight for Maximal Distance at Presence of Conflict Behavior (Control) of the Aircraft Control System Active Element," *2013 11th International Conference "AVIA-2013" Proceedings*. May 21–23, 2013, Kyiv, Ukraine, 2013, pp. 22.30–22.33.

[8] V. Kasjanov and K. Szafran, "Some Hybrid Models of Subjective Analysis in the Theory of Active Systems," *Transactions of the Institute of Aviation*. vol. 3(240), pp. 27–31, 2015.

[9] K. Szafran and I. Kramarski, "Safety of Navigation on the Approaches to the Ports of the Republic of Poland on the Basis of the Radar System on the Aerostat Platform," *The International Journal on Marine Navigation and Safety of Sea Transportation*. vol. 1(9), pp. 131–136, 2015.

Received August 26, 2019.

Andriy Goncharenko. orcid.org/0000-0002-6846-9660 Doctor of Engineering Science. Professor.
Aircraft Airworthiness Retaining Department, Aerospace Faculty, National Aviation University, Kyiv, Ukraine.
Education: Odessa's Institute of Marine Engineers, Odesa, Ukraine (1984).
Research area: Multi-optional optimality, Variational principles, Operation and control in active systems, Flight safety.
Publications: more than 150.
E-mail: andygoncharenco@yahoo.com

А. В. Гончаренко. Гібридна відносна комбінована псевдо-ентропійна функція як інструмент для керування транспортною системою

Здійснено спробу відкрити правдоподібну причину, що пояснює існування моделі виробничої функції транспортної системи, або функції кількості транспортних засобів, як функції гіперболічного типу від наявної спроможності транспортного засобу, або лінійного типу функції для двох типів транспортних засобів. Обґрунтування здійснено в термінах теорії варіаційного обчислення за допомогою спеціальної міри невизначеності функцій гібридно-опційної ефективності, що включає ентропію тих гібридних функцій традиційного Шеннонівського стилю. У випадках, які вивчаються, розв'язки найпростішої варіаційної задачі, що є кількостями транспортних засобів, обумовлені специфікованими квадратичними формами. Пропонується оцінювати величину та спрямування визначеності керування транспортної системи із використанням комбінованої гібридної відносної псевдо-ентропійної функції. Це є новим поглядом на наукове пояснення добре відомої залежності виведеної іншим шляхом. Теоретичні міркування, які розвиваються, а також математичні викладки завершуються найпростішим числовим прикладом слабкості традиційної ентропії.

Ключові слова: транспортна система; виробнича функція; доктрина багатоопційності; умовна оптимальність; гібридно-опційна функція ефективності; псевдо-ентропія; індекс превалювання; фактор домінування; варіаційна задача.

Андрій Вікторович Гончаренко. orcid.org/0000-0002-6846-9660. Доктор технічних наук. Професор.
Кафедра збереження льотної придатності авіаційної техніки, Аерокосмічний факультет, Національний авіаційний університет, Київ, Україна.
Освіта: Одеський інститут інженерів морського флоту, Одеса, Україна (1984).
Напрям наукової діяльності: багатоопційна оптимальність, варіаційні принципи, експлуатація та керування в активних системах, безпека польотів.
Кількість публікацій: понад 150.
E-mail: andygoncharenco@yahoo.com

А. В. Гончаренко. Гибридная относительная комбинированная псевдо-энтропийная функция как инструмент для управления транспортной системой

Осуществлена попытка открыть правдоподобную причину, поясняющую существование модели производственной функции транспортной системы, или функции количества транспортных средств, как функции гиперболического типа от наличной производительности транспортного средства, или линейного тапа функции для двух типов транспортных средств. Обоснование осуществлено в терминах теории вариационного исчисления с помощью специальной меры неопределенности функций гибридно-опционной эффективности, включающей энтропию этих гибридных функций традиционного Шенноновского стиля. В изучаемых случаях, решения простейшей вариационной задачи, являющиеся количествами транспортных средств, обусловлены специфицированными квадратичными формами. Предлагается оценивать величину и направленность определенности управления транспортной системы с использованием комбинированной гибридной относительной псевдо-энтропийной функции. Это является новым взглядом на научное пояснение хорошо известной зависимости выведенной другим путем. Развиваемые теоретические соображения, а также математические выкладки завершаются простейшим численным примером слабости традиционной энтропии.

Ключевые слова: транспортная система; производственная функция; доктрина многоопционности; условная оптимальность; гибридно-опционная функция эффективности; псевдо-энтропия; индекс превалирования; фактор доминирования; вариационная задача.

Андрей Викторович Гончаренко. orcid.org/0000-0002-6846-9660. Доктор технических наук. Профессор.
Кафедра сохранения летной годности авиационной техники, Аэрокосмический факультет, Национальный авиационный университет, Киев, Украина.
Образование: Одесский институт инженеров морского флота, Одесса, Украина (1984).
Направление научной деятельности: многоопционная оптимальность, вариационные принципы, эксплуатация и управление в активных системах, безопасность полетов.
Количество публикаций: более 150.
E-mail: andygoncharenco@yahoo.com