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ORTHOGONAL MULTIPLES IN DIGITAL PROCESSING PROBLEMS

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Abstract*—The paper deals with the application of classical orthogonal Jacobi and Chebyshev–Lagerra polynomials to solving digital information processing problems and solving Volterra convolution integral equations, used to solve the problem of remote sensing of the Earth and the problem of identification of natural objects. The presence of two free parameters in Jacobi polynomials satisfies the conditions under which the problem of approximation of signals is solved, and the use of Chebyshev–Lagerra polynomials avoids the sampling procedures for solving Voltaire type integral equations.*

Index Terms—Orthogonal polynomials; spectral methods; data processing, identification tasks.

I. INTRODUCTION

The solution of a number of important scientific and applied problems of the theory of digital information processing, mass transfer is based on the effectiveness of existing analytical and numerical methods of solving the corresponding problems and processing of real experimental data taking into account a priori information.

One of the promising approaches to solving the formulated problems is the use of spectral methods. In computational experiments, these methods have a number of advantages that are associated with the parametric (symbolic) solution of problems at its various stages. These solutions allow us to investigate the effect of physical process parameters on its flow and to study the stability of the solution of relatively small perturbations of the input data. In cases where a symbolic solution to the problem fails, you can construct a solution in the basis of the so-called structural elements.

Structural elements are understood to mean functions that, with the precision of a normalizing factor, are given in a functional basis by a finite set of integers or rational numbers. This class of elements includes, for example, classical orthogonal polynomials, each of which has integer coefficients. Using the basis of structural elements together with the ability to find the values of the images, in turn, allows you to organize the computational process of solving the problem with guaranteed accuracy. In this case, computation with guaranteed accuracy refers to the organization of the computational process, in which at all intermediate stages errors of machine (hardware) implementation of arithmetic operations in such an order that ensures the accuracy of the final result are eliminated. According to D.K. Fadeev [1], such a numerical method of solving a problem is called regular.

The numerical finding of the solution of the problem with the guaranteed accuracy of calculations has, in general, only the errors of the method. In these cases, when it is not possible to organize the calculations with guaranteed accuracy, there are additional complications associated with the accumulation of machine errors.

The essence of spectral methods for solving problems is to represent known functions and sought solutions by orthogonal series and to build algorithms for calculating the coefficients of these series (generalized spectra) [2].

Comparative analysis of the spectral methods in the Fourier, Haar, and Walsh bases revealed that often not all the criteria relating to the desired solutions are satisfied. Therefore, the spectral methods in other orthogonal bases - Jacobi, Chebyshev–Lagerra, Hermite, etc. [2].

The purpose of this work is to build a technique for using classical orthogonal polynomials to solve applied problems, including problems of identification, multiplicative noise filtering, solving integrodifferential equations..

II. USE OF JACOBI POLYNOMIALS IN REMOTE SENSING OF VEGETATION

Remote spectrometric surveys are effectively used to detect certain deviations in the spectra of reflection from their background values [2] – [5], in particular of plants. areas of definition.

1) To illustrate identification problems, consider the use of Jacobi polynomials in remote sensing of vegetation, which makes it possible to map vegetation sections with anomalous spectral characteristics, knowing the data of ground-based calibration works [2], [3]. To do this, we approximate the reflected signal by the *N*th partial sum of the Fourier–Jacobi series [2]

$$f(x)=\omega(x)\sum_{n=0}^{\infty}\frac{f_n}{r_n}P_n^{(\alpha,\beta)}(x),\qquad x\in[-1,1],$$
$$f_n=\int_{-1}^{1}P_n^{(\alpha,\beta)}(x)\,f(x)\,dx. \tag{1}$$

Since signal values are known mainly in discrete points when processing information, it is advisable to use formulas to calculate unknown coefficients f_n.

$$f_n\sim W_N\sum_{i=1}^{N+1}\eta_{N,i}P_n^{(\alpha,\beta)}(x_i),$$
$$P_{N+1}^{(\alpha,\beta)}(x_i)=0,\quad i=\overline{1,N+1}, \tag{2}$$

where marked:

$$\eta_{N,i}=\frac{(1-x_i)^{1-\alpha}(1+x_i)^{1-\beta}}{\left[P_N^{(\alpha,\beta)}(x_i)\right]^2}\varphi(x_i),$$

$$W_N=2^{\alpha+\beta-1}\frac{\Gamma(N+\alpha+2)\Gamma(N+\beta+2)}{(N+1)!\Gamma(N+\alpha+\beta+2)}\cdot\frac{(2N+\alpha+\beta+2)^2}{(N+\alpha+1)^2(N+\beta+1)^2},$$

$$r_n=2^{\alpha+\beta+1}\frac{\Gamma(N+\alpha+1)\Gamma(N+\beta+1)}{N!(2N+\alpha+\beta+1)\Gamma(N+\alpha+\beta+1)}.$$

Plant contamination with nitrates leads to the perturbation of the reflected spectrum at wavelengths from 450 nm to 650 nm [2], [3], and the amount of nitrates can be determined after calibration by chemical samples deviation $\Delta f=f_{f\max}(x_m)-f_{a\max}(x_m)$ at the maximum point $x=x_m$ (Fig. 1). Here f_f and f_a are the values of the background and anomalous reflected spectra, respectively. Note that in this case, the fact that the points of maximum anomalous and background spectra must coincide is important. Practice shows that failure to comply with this requirement leads to a significant inaccuracy in the determination of perturbation Δf. The presence of two free parameters in Jacobi polynomials allows to solve this problem.

The algorithm for determining the value Δf will be as follows:

a) According to formula (2) and the values of the background $f_f(\lambda_i)$ and anomalous $f_a(\lambda_i)$ spectra, the Fourier–Jacobi coefficients are calculated f_{fn} and f_{an} $\left(n=\overline{0,NV}\right)$.

b) By setting the values of α and β, there are points of maxima x_{fm} and x_{am} for the background and anomalous reflected spectra, as the solution of equations $f'(x)=0$ in the interval 450–650 nm, or

$$\sum_{n=0}^{NV}\frac{f_n}{r_n}\left\{(\beta-\alpha-(\alpha+\beta)x)P_n^{(\alpha,\beta)}(x)+\frac{1}{2n+\alpha+\beta}\left((\alpha-\beta+(2n+\alpha+\beta)x)P_n^{(\alpha,\beta)}(x)-2(n+\alpha)(n+\beta)P_{n-1}^{(\alpha,\beta)}(x)\right)\right\}=0.$$

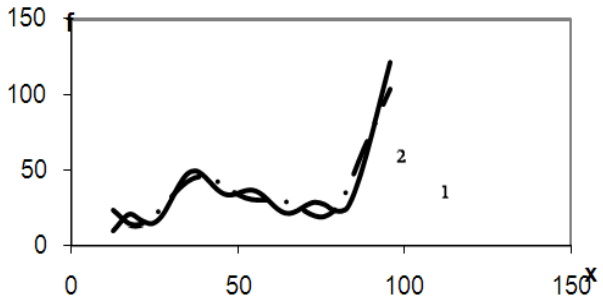

Fig. 1. The amount of nitrate that is determined after calibration for deviation of chemical samples

Through a computational experiment, values α are β chosen so that equality occurs $x_m=x_{fm}=x_{am}$.

c) The values of the background f_f and anomalous f_a reflected spectra and their difference $\Delta f=f_a(x_m)-f_f(x_m)$ are calculated at the maximum point found. Table I shows the values x_m and Δf for different values α and β. The number I corresponds to the case where before the sum in equality (1) the factor is one and the digit II – $\omega(x)=(1-x)^{\alpha}(1+x)^{\beta}$.

TABLE I. THE PERTURBATIONS OF THE BACKGROUND SPECTRUM Δf OF THE REFLECTION OF PLANTS AT THE MAXIMUM x_m AT DIFFERENT VALUES OF PARAMETERS α AND β

β		–0.6		–0.28		0.04		0.36		0.68	
α		x_m	Δf	x_m	Δf	x_m	Δf	x_m	Δf	x_m	Δf
–0.5	I	577.6	3.84	578.1	3.89	578.5	3.70	578.9	3.54	579.3	3.43
	II	543.8	6.18	596.9	3.53	543.7	6.33	545.6	6.58	533.0	9.87
0.0	I	577.4	4.17	577.9	3.97	578.3	3.77	578.7	3.59	579.1	3.46
	II	544.4	6.47	597.4	0.43	544.2	6.42	545.3	6.58	548.8	6.85
1.0	I	577.0	4.32	577.5	4.16	577.9	3.98	578.3	3.80	578.7	3.64
	II	570.9	7.16	569.8	6.73	529.5	5.98	529.2	5.90	529.1	5.84

The results in the table show that for each α there are such values β at which Δf it is constant. The perturbation constant of the background spectrum Δf is the criterion for selecting the

parameters α and β under which the plant spectrometry data are processed.

III. Use of Jacobi Polynomials in EPR Dosimetry

The method described above was used in object identification tasks by processing discrete data in electron paramagnetic resonance dosimetry (EPR dosimetry). The task is to find the maximum differences in the intensity of the reflected rays and to compare them with the reference data according to the given discrete data. The results of the numerical experiment are presented in Table II and Fig. 2, where the measured data (their number 4096) is their approximation according to the algorithm described above, the maximum and minimum values at given intervals are calculated and the difference between the maximum and minimum values is found.

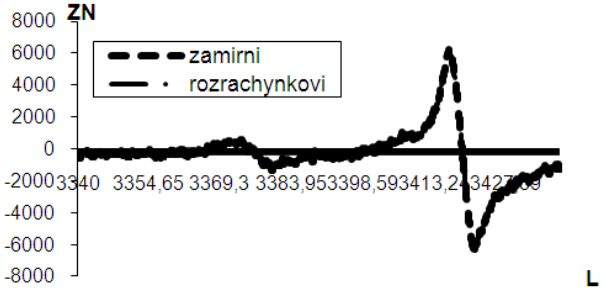

Fig. 2. The results of the numerical experiment are presented

Table II. The values of the beginning and end of the intervals, which are the extreme values of the signal, the corresponding values of the arguments and the difference between the maximum and minimum values

	Beginning	End	Value	Argument
max	3410.75	3411.41	1166.763	3411.01
max	3416.29	3417.05	5924.285	3416.69
min	3377.15	3386.9	-1470.92	3381.69
min	3390	3396.02	-927.662	3390.38
min	3420.44	3423.95	-6276.58	3421.32
Signal difference		2637.68		
Signal difference		2094.425		
Signal difference		12200.86		

IV. The Chebyshev–Lagerra Polynomials and their Generalizations

Jacobi polynomials are useful when signals are defined in a finite region. However, there is a certain class of signals whose arguments are the time that is defined at a half-boundary interval. For this class of signals it is advisable to use Chebyshev–Lagerra polynomials $L_m^{\lambda}(t)$, $\lambda > -1$ that are orthogonal to $t \in [0,\infty)$. The distribution of a function in a series by polynomials is [2], [6]

$$f(t) = \sum_{m=o}^{\infty} \frac{m! f_m}{\Gamma(m+\lambda+1)} L_m^{\lambda}(t). \qquad (3)$$

In this case, they are calculated by the formula

$$f_m = \int_0^{\infty} t^{\lambda} e^{-t} L_m^{\lambda}(t) f(t) dt. \qquad (4)$$

The Chebyshev–Lagerra polynomials have the significant disadvantage that as their order increases, they grow exponentially with time. One way out of this provision is to introduce an additional factor that does not always allow you to extend the fast convergence gap. More appropriate is the use of modified Chebyshev–Lagerra polynomials.

Let ν $(\nu < 0, \nu \neq 0)$ and $\mu (0 < \mu < \infty)$ – some steel. We define the modified Chebyshev–Lagerra transform by the relation [2]

$$f_m(\nu,\mu) = \int_0^{\infty} t^{\nu\lambda+\nu-1} e^{-\mu t^{\nu}} L_m^{\lambda}(\mu t^{\nu}) f(t) dt. \qquad (5)$$

The original transformation (5) is given by the formula

$$f(t) = \sum_{m=o}^{\infty} \frac{m! f_m(\nu,\mu)}{\Gamma(m+\lambda+1)} L_m^{\lambda}(\mu t^{\nu}). \qquad (6)$$

The properties of integral transformations (4) and (6) are derived based on the relations between the corresponding orthogonal polynomials.

V. Inversion of a One-Dimensional Volterra Convolution

A large number of applied problems (in particular, information processing, lidar equations, derivatives and fractional order integrals, etc.) are reduced to convolution integral equations. The main approaches to numerically solving integral equations of the first type of convolution type are to use Tikhonov type regularization algorithms or to approximate the original equation directly.

The use of Tikhonov-type algorithms results in the loss of voltarosity, which significantly reduces the possibility of restoring the desired functions for the considered areas of their application and the application of small grid steps.

The main disadvantage of the second direction is the lack of accounting for the instability of the numerical solution to the errors of the input information, which leads the solution of the perturbed equation beyond the set of correctness.

Integral transformations, the kernels of which are Chebyshev–Lagerra polynomials, allow us to construct effective methods for solving convolution-type integral equations [7] – [9].

Consider the following integral equations

$$\alpha f(t)+\mu\int_0^t k'(t-\tau)\,f(\tau)\,d\tau=y(t), \qquad (7)$$

$$\mu\int_0^t k(t-\tau)\,f(\tau)\,d\tau=y(t). \qquad (8)$$

Here α,μ are some steels; $f(t)$ is the desired function; $k(t)$ is the kernel of the equation. We assume that the functions in integral equations (7) and (8) satisfy the conditions that allow them to be represented by Fourier–Lagerraire series of type (3). Recovering the desired solution is determined by the unknown coefficients f_n.

If we look for the solution of equation (7) as (3) at $\lambda=0$, then the unknown coefficients f_n of the solution are calculated by the formula

$$f_n=\frac{1}{\alpha+\mu\left(k_0-k(0)\right)}\left(y_n-\mu\sum_{m=1}^{n} k_m\,f_{n-m}\right). \qquad (9)$$

Here, k_m and f_m are the Fourier–Lagerra coefficients of functions $k(t)$ and $f(t)$. Since the coefficients k_m and y_n are known, the formula (9) determines the Lagerra spectrum of an unknown function $f(t)$.

A. Equation Solution

Let the functions $k(t)$ and $f(t)$ be represented by rows by polynomials $L_n^{\lambda_k}(t)$, $\lambda_k>-1$, and $L_n^{\lambda_f}(t)$, $\lambda_f>-1$, respectively, and the right part (8) is filed nearby

$$y(t)=t^{\lambda}\sum_{n=0}^{\infty}\frac{y_n}{r_n}L_n^{\lambda}(t), \quad \lambda=\lambda_k+\lambda_f+1.$$

Then the unknown coefficients f_n are calculated by the formula

$$f_n=\frac{1}{k_0}\left(\frac{1}{\mu}y_n-\sum_{m=0}^{n-1} k_{n-m}\,f_m\right). \qquad (10)$$

The main advantage of this method is that the sampling procedure is eliminated because the integral convolution becomes a series convolution.

B. Modification of the constructed scheme

Since the input is a value, it is advisable to modify the formulas so that the known values $y(t_i)$ directly obtain the value of the desired solution at certain points. For this purpose, we write formula (10) in the form

$$K_N F_N=Y_N, \qquad (11)$$

where

$$K_N=\begin{pmatrix} k_0 & 0 & \dots & 0\\ k_1 & k_0 & \dots & 0\\ \vdots & \vdots & \vdots & \vdots\\ k_{N_1} & k_{N-2} & \dots & k_0\end{pmatrix}, F_N=\begin{pmatrix} f_0\\ k_1\\ \vdots\\ f_{N_1}\end{pmatrix}, Y_N=\begin{pmatrix} y_0\\ y_1\\ \vdots\\ y_{N_1}\end{pmatrix}.$$

Then the unknown coefficients f_n are from the solution of the matrix equation (11), i. e. $F_N=K_N^{-1}Y_N$, where K_N^{-1} is the matrix inverted to K_N, or $f_n=\sum_{i=0}^{N-1} y_i\,z_{n,i}$, $n=\overline{0,N-1}$.

Here $z_{n,i}$ are the elements of the matrix K_N^{-1}. If $N-$ to be written as a partial sum of the Fourier–Lagerrau series

$$f_N(t)=\sum_{n=0}^{N-1}\eta_n^{\lambda}(t)\,y_n,$$

$$\eta_n^{\lambda}(t)=t^{\lambda}\sum_{m=n}^{N-1}\frac{n!}{\Gamma(n+\lambda+1)}z_{n+1,m+1}L_m^{\lambda}(t),$$

then the values of the function $f(t)$ in points t_k, $k=\overline{1,K}$, are calculated by the formula

$$\begin{pmatrix} f(t_1)\\ f(t_2)\\ \vdots\\ f(t_k)\end{pmatrix}=\begin{pmatrix} \eta_0(t_1) & \eta_1(t_1) & \dots & \eta_{N-1}(t_1)\\ \eta_0(t_2) & \eta_1(t_2) & \dots & \eta_{N-1}(t_2)\\ \vdots & \vdots & \ddots & \vdots\\ \eta_0(t_k) & \eta_1(t_k) & \dots & \eta_{N-1}(t_k)\end{pmatrix}\begin{pmatrix} y_0\\ y_1\\ \vdots\\ y_k\end{pmatrix}.$$

Note that values $\eta_i(t_j)$ can be calculated with the required accuracy and stored in a database.

VI. Reduction of the Convolution in the Lagerra Basis Function

The exponential growth of Chebyshev–Lagerra polynomials can also be solved by applying the spectral decomposition in the Lagerra basis function

$$\varphi_n(t)=e^{-t/2}\,L_n^{\lambda}(t), \qquad \lambda>-1,$$

which are orthogonal to the interval $[0,\infty)$ and retain all the basic properties of the Chebyshev–Lagerra polynomials. When $n\to\infty$ and $t\to\infty$ Lagerra functions $\varphi_n(t)$ go to zero.

Let the function $f(t)$ be approximated by a partial sum of the series

$$f(t)=t^{\lambda}e^{\gamma\tau}\sum_{n=0}^{\infty}\frac{f_n}{r_n}\varphi_n(t/h), \qquad (12)$$

where $\gamma(|\gamma|<\infty)$ and $h(0<h<\infty)$ are some steels; r_n is the normalizing factor. Then the formulas are used to calculate the coefficients

$$f_n=\int_0^{\infty} e^{-\gamma h\tau} f(h\tau)\,\varphi_n(\tau)\,d\tau,$$

$$f_n\approx\sum_{m=0}^{N}\frac{\lambda_m e^{-\gamma h\lambda_m} f(h\lambda_m)}{[(N+2)\varphi_{N+2}(\lambda_m)]^2}\varphi_n(\lambda_m).$$

The last formula λ_m is the root of the function $\varphi_{N+1}(\tau)$.

Since the Chebyshev–Lagerra basis is effective for solving convolution-type equations, given the relation between Laguerre functions and polynomials, one should expect that similar results hold for Laguerre functions.

Theorem. Let functions $k(t)$, $\varphi(t)$ and $f(t)$ be represented by rows of type (12). Then the unknown coefficients f_n of the desired solution are calculated by the formula

$$f_n=\frac{1}{k_0}\left[\varphi_n-\sum_{m=0}^{n-1} k_{n-m} f_m\right],$$

where k_n and φ_n are the coefficients of the orthogonal series of functions $k(t)$ and $\varphi(t)$ the basis of the Laguerre functions.

This method also applies to the solution of the following convolution-type integral equations:

$$\alpha f(t)+\mu\int_0^t k(t-\tau)\,f(t)\,dt=\varphi(t),$$

$$\alpha f(t)+\mu\frac{d}{dt}\int_0^t k(t-\tau)\,f(t)\,dt=\varphi(t),$$

where α and μ some are constant. In these cases, the Fourier–Lagerra coefficients of the unknown functions $f(t)$ will be calculated by the formulas

$$f_n=\frac{1}{\alpha+\mu k_0}\left[\varphi_n-\mu\sum_{m=0}^{n-1} k_{n-m} f_m\right],$$

$$f_n=\frac{1}{\alpha+\mu(k_0-k(0))}\left[\varphi_n-\mu\sum_{m=0}^{n-1} k_{n-m} f_m\right].$$

VII. CONCLUSIONS

The use of the spectral method of solving convolution-type equations in the basis of the Chebyshev–Lagerra polynomials avoids the sampling procedure. This solves the problem of the instability of the numerical solution to the errors of the input information, which brings the solution of the perturbed equation beyond the bounds of the correctness set.

On the basis of the Chebyshev–Lagerra polynomials, similar results were obtained for the reduction of a semi-boundless convolution and the method for solving two-dimensional convolution-type integral equations.

Spectral methods can be effectively used to solve lidar equations [10] – [13]. It is known that the vast majority of atmospheric optics problems can be reduced to Fredholm first- or second-order integral equations. In relation to the Fredholm integral equation of the first kind, which is the Laplace transform, an adaptive algorithm of its inversion is constructed in the basis of the Jacobi polynomials [2]. Note that in this case the kernel of an integral equation is an exponential function. However, this approach can be extended to the case of an arbitrary kernel of an integral equation.

The Chebyshev–Lagerra polynomials have also been successfully used to solve differential equations in partial fractional derivatives [14], [15].

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Я. Д. П'янило. Ортогональні многочлени в задачах оброблення цифрової інформації
В роботі розглянуто питання застосування класичних ортогональних многочленів Якобі та Чебишева–Лагерра до розв'язування задач оброблення цифрової інформації та розв'язування інтегральних рівнянь Вольтерра типу згортки, використано їх для вирішення задачі дистанційного зондування Землі та задачі ідентифікації природних об'єктів. Наявність двох вільних параметрів у многочленах Якобі дозволяє задовольнити умови, за яких розв'язана задача апроксимації сигналів, а застосування многочленів Чебишева–Лагерра дозволяє уникнути процедури дискретизації при розв'язуванні інтегральних рівнянь Вольтерівського типу.
Ключові слова: ортогональні многочлени; спектральні методи; оброблення даних, задачі ідентифікації.

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Я. Д. Пянило. Ортогональные многочлены в задачах обработки цифровой информации
В работе рассмотрены вопросы применения классических ортогональных многочленов Якоби и Чебышева–Лагерра для решения задач обработки цифровой информации и решения интегральных уравнений Вольтерра типа свертки, использование их для решения задачи дистанционного зондирования Земли и задачи идентификации природных объектов. Наличие двух свободных параметров в многочленах Якоби позволяет удовлетворить условия, при которых решена задача аппроксимации сигналов, а применение многочленов Чебышева–Лагерра позволяет избежать процедуры дискретизации при решении интегральных уравнений вольтеровских типа.
Ключевые слова: ортогональные многочлены; спектральные методы; обработка данных, задачи идентификации.

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