THEORY AND METHODS OF SIGNAL PROCESSING

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[1]**I. G. Prokopenko,**
[2]**I. P. Omelchuk,**
[3]**K. I. Prokopenko,**
[4]**A. A. Osipchuk**

SYNTHESIS AND EFFECTIVITY ANALYSIS OF ROBUST ALGORITHMS FOR RANDOM SIGNAL DETECTION IN NON-GAUSSIAN INTERFERENCES

[1,2,4]Aviation Radioelectronic Complexes Department, National Aviation University, Kyiv, Ukraine
[3]Air Navigation Systems Department, National Aviation University, Kyiv, Ukraine
E-mails: [1]igorprok48@gmail.com ORCID 0000-0003-4169-3774, [2]omelip@ukr.net,
[3]kprok78@gmail.com, [4] alina.osipchuk2012@gmail.com

Abstract*—The article deals with the problem of synthesis of robust post-detection algorithms of random radar signal on the background of uncorrelated noise and chaotic pulse interference. Two cases of aprioristic uncertainty are considered: 1) random modulated harmonic radar signal with random amplitude, distributed by Gaussian law with known parameters; 2) random modulated harmonic radar signal with unknown law of amplitude distribution. The problem is solved with the use of Wald's reduction approach. Synthesis of the robust signal detection algorithm on background of chaotic pulse interference was don using Tukey model of "pollution". The effectiveness and robustness of the several synthesized robust detection algorithms is investigated by Monte–Carlo method.*

Index Terms—Signal processing; robust algorithms; radar signal detection; haotick puls interference; aproristick uncertainity.

I. INTRODUCTION

The widespread use of radar, navigation and communication in various fields (aviation, including unmanned vehicles, medicine, vehicle control, weapons control, etc.) puts forward a wider range of tasks related to improving their effectiveness in complex applications [1] – [4].

Typically, radar is used in complex interference situations (terrain and type of background surface, various meteorological conditions, man-made and organized interferences, etc.). To work effectively under these conditions, signal processing channels are equipped with a plurality of signal detection means against the background of various interferences and their informative parameters. In particular, synthesis of algorithms for estimating the frequency of a harmonic useful signal was performed in [5]. The complex problem of detection and measurement is also characteristic in studies of real random processes, such as physical, biological, measurement in education [6], etc.

Signal processing theory allows us to obtain optimal, in some sense, algorithms for detecting signals in the parametric description of signal-jamming situations, when certain statistical models of probability distributions of the mixture of signals and noise are laid at the design stage. The most popular is the Gaussian model, for which a number of optimal solutions are obtained. However, in most cases signal and interference distributions are different from the Gaussian model. This leads to a significant decrease in the real performance of radar compared to theoretical.

Recently, more and more developers are paying attent ion to a robust approach to the synthesis of signal processing algorithms. Robust methods are useful when it is necessary to consider the loss of efficiency of processing algorithms when deviating the real probability distributions of signals and interference from the distributions used in the process of algorithm synthesis. In a robust approach, the signal-to-noise situation is set not by one, but by the set of distributions in the neighborhood of a certain parametric distribution. The theoretical foundations of a robust approach in statistics were laid down by the works of Huber [7], Hampel [8], Hogg [9] in the 60–70 years of the last century. The introduction of a robust approach to the synthesis of radar signal processing algorithms should be traced back to the 1970s. These are the works of E. Korniliev [10] – [11], H. Rohling [12], I. Prokopenko [13] – [18], O. Zelensky [19]. Recently, the flow of work on robust signal and data processing techniques has increased significantly [20] – [21].

The problem of synthesis of post-detector algorithms for detecting a harmonic signal with fluctuating amplitude against the background of uncorrelated Gaussian noise is considered in this

http://jrnl.nau.edu.ua/index.php/ESU, http://ecs.in.ua

paper. They have been synthesized using two approaches: the Wald reduction [22] – [23], which allows one to reduce a complex parametric hypothesis to a simple one by integrating the parametric probability distribution over the a priori parameter distribution; approach to the synthesis of robust algorithms using the "polluted" Tukey distribution [7]. A comparative analysis of the efficiency and robustness of the obtained algorithms under the influence of chaotic impulse interference is made.

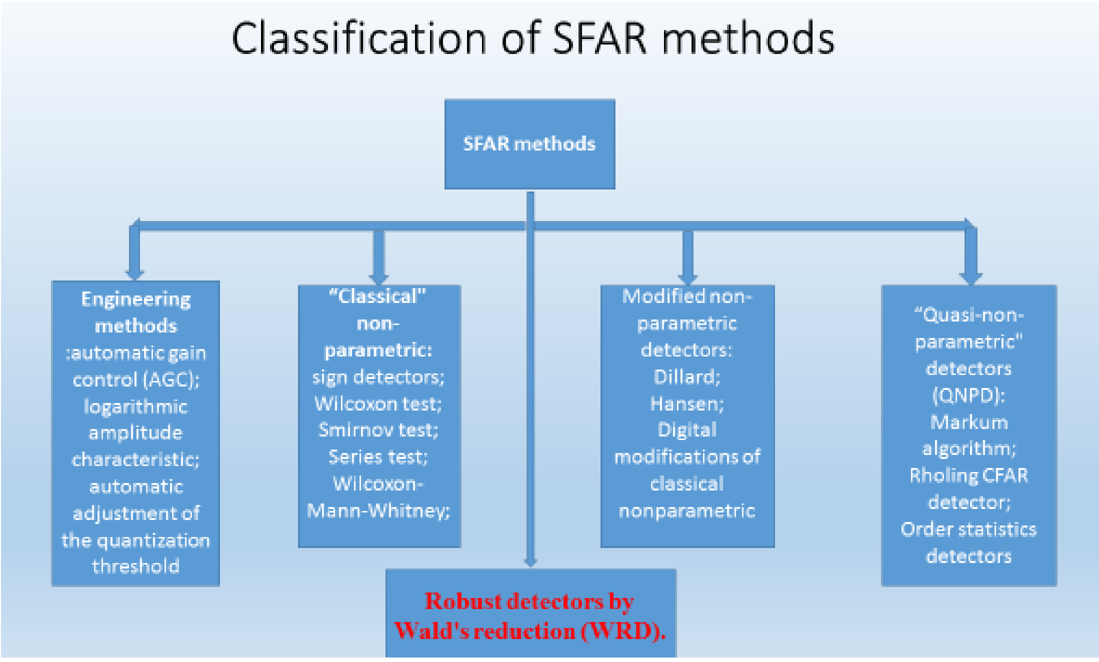

Fig. 1. Classification of Stable Fals Alarm Ratio (SFAR) methods

II. Signal and Interference Models

The problem of detecting a harmonic signal against the background of a Gaussian uncorrelated noise at the output of the amplitude detector in a parametric formulation was considered in the works of Markum [24] and Sverling [25]. The main results are reduced to cases of linear and quadratic amplitude detectors.

In the first case, N samples

$$\overline{x} = x_1,..,x_N \tag{1}$$

of the envelope of the signal and noise are distributed by Rayleigh law and the detection algorithm calculates the sum of sample squares

$$T_1 = \sum_{n=1}^{n} x_n^2 > V_d\left(\alpha, \sigma_\eta^2, N\right), \tag{2}$$

and compare it with decision threshold $V_1(*)$.

In the second case, the samples of the envelope mixture of signal and noise are distributed according to Rice's law and the algorithm calculates their sum

$$T_2 = \sum_{n=1}^{n} x_n > V_d\left(\alpha, \sigma_\eta^2, N\right), \tag{3}$$

$V_1\left(\alpha, \sigma_\eta^2, N\right)$ and $V_2\left(\alpha, \sigma_\eta^2, N\right)$ depend on the given probability of false alarm α, the variance of Gaussian noise σ_η^2, the sample size N.

The cases considered do not exhaust many of all possible situations. In particular, a typical situation is when harmonic signal amplitude fluctuations occur due to fading or radio countermeasure. It is advisable to apply the Wald reduction principle in the synthesis of signal detection algorithms.

Formulation of the problem is following.

Let the output of the linear amplitude detector obtain a sample of the values of the harmonic signal S and noise η mixture (1).

In what follows, we call the sample (1) a pack.

Relative sample (1), two hypotheses are tested. Hypothesis H_0 – no signal: samples of envelope are distributed by Rayleigh's law $f_\eta\left(x_n\right)$. N-dimensional probability distribution density (PDF) of the pack, which is simultaneously the likelihood function (LF) $l_{H_0}\left(\vec{x}\right)$ for H_0 hypothesis is

$$l_{H_0} \triangleq \prod_{n=1}^{N} f_\eta(x_n)$$
$$= \prod_{n=1}^{N} \frac{x_n}{\sigma_\eta^2} \exp\left\{\frac{-x_n^2}{2\sigma_\eta^2}\right\} = \frac{\prod_{n=1}^{N} x_n}{\sigma_\eta^{2N}} \exp\left\{\frac{-\sum_{n=1}^{N} x_n^2}{2\sigma_\eta^2}\right\}, \quad (4)$$

where σ_n^2 is the scale parameter.

Hypothesis H_1 – signal present: the envelope samples of the signal with noise are distributed according to Rice's law, unconditional PDF of the envelope:

$$f_{\Pi,s}(\vec{x}) \triangleq \prod_{n=1}^{N} \frac{x_n}{\sigma_\eta^2} \exp\left(-\frac{x_n^2 + s^2}{2\sigma_\eta^2}\right) I_0\left(\frac{s x_n}{\sigma_\eta^2}\right), \quad (5)$$

where s is the known signal amplitude; $I_0(*)$ is the Bessel function of zero order.

By $s >> \sigma_\eta$ an asymptotic approximation of the Bessel function can be used

$$I_0\left(\frac{s x_n}{\sigma_\eta^2}\right) \sim \frac{\sigma_\eta}{\sqrt{2\pi s x_n}} \exp\left\{-\frac{(x_n - s)^2}{2\sigma_\eta^2}\right\}. \quad (6)$$

Then expression (5) can be represented as a Gaussian multivariate PDF

$$f_{\Pi,s}(\vec{x}) \triangleq \prod_{n=1}^{N} f_s(x_n \mid s)$$
$$= \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left\{-\frac{[x_n - s]^2}{2\sigma_\eta^2}\right\}. \quad (7)$$

In the future, the studies are performed under partial conditions, when the amplitudes of the signal at different points in time are a sequence of random numbers $\{s_1,..,s_n\}$, unlike expression (5). Then expression (7) takes the form

$$f_{\Pi,s}(\vec{x}) \triangleq \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left\{-\frac{[x_n - s_n]^2}{2\sigma_\eta^2}\right\}. \quad (8)$$

Is assumed that all signal components have a Gaussian distribution with a known variance σ_s^2, where m is the same mathematical expectation (ME) for all packet samples:

$$f_s(s_n \mid m) \triangleq \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left\{-\frac{(s_n - m)^2}{2\sigma_s^2}\right\}, \quad (9)$$

Two cases will be considered:

1) m is known;
2) m is random with some known PDF $f_m(m)$.

III. Methodology of Detection Algorithms Synthesis

The synthesis of detection algorithms in the work is based on the Neumann–Pearson criterion [22] using the likelihood ratio

$$\Lambda(\vec{x}) = \frac{l_{H_1}(\vec{x})}{l_{H_0}(\vec{x})} > V(\alpha), \quad (10)$$

where $l_{H_1}(\vec{x})$ is the LF for hypothesis H_1; $l_{H_0}(\vec{x})$ is the LF for hypotheses H_0, defined by expression (4); V is the threshold for deciding whether a signal is present.

Likelihood function for complex hypothesis $l_{H_1}(\vec{x})$, can be obtained by applying the Wald reduction principle [23]. According to the statistical model defined in (8), the principle of reduction is as follows. First, a statistical averaging over the distribution of the signal component (9) is made for each reference. And the statistical averaging over the distribution of ME is carried out in the second step with respect to the conditional multivariate PDF, namely:

$$l_{H_1}(\vec{x})$$
$$= \int_{-\infty}^{\infty}\left[\prod_{n=1}^{N}\int_{-\infty}^{\infty} f_{s+\eta}(x_n \mid s_n) f_s(s_n \mid m)\, ds_n\right] f_m(m_n)\, dm. \quad (11)$$

In the following, we focus on the two a priori conditions relative signal ME (m): 1) m is distributed by improper probability distribution dm and 2) m is known value.

IV. Synthesis of Detection Algorithm with a Random ME of Pack

The synthesis conditions for this variant are generally defined by expressions (3) – (11). The first step is to integrate the variable s. The internal integral (we denote $\int_{s,n}(m)$) is tabular [28], after its identical transformation we obtain

$$\int_{s,n}(m) = \sqrt{\frac{\pi}{2}} \frac{\sigma_\eta \sigma_s}{\sqrt{\sigma_s^2 + \sigma_\eta^2}} \exp\left\{-\frac{x_n^2}{2(\sigma_s^2 + \sigma_\eta^2)}\right\} \cdot \ldots$$
$$\ldots \cdot \exp\left\{-\frac{m^2}{2(\sigma_s^2 + \sigma_\eta^2)} + \frac{m x_n}{(\sigma_s^2 + \sigma_\eta^2)}\right\}. \quad (12)$$

According to expression (11), in the second step of the synthesis, using formula (12), we write the multivariate conditional LF of the pack as

$$f_{\Pi}\left(\vec{x}\,|\,m\right) \triangleq \frac{1}{\left(2\pi\sigma_{\eta}\sigma_{s}\right)^{N}} \prod_{n=1}^{N} \int_{s,n}(m)$$
$$= \frac{\sigma_{s\eta}^{-N}}{2^{\frac{3N}{2}} \pi^{\frac{N}{2}}} \exp\left\{ -\frac{1}{2\sigma_{s\eta}^{2}} \sum_{n=1}^{N} x_n^2 - \frac{m^2 N}{2\sigma_{s\eta}^{2}} + \frac{1}{\sigma_{s\eta}^{2}} m \sum_{n=1}^{N} x_n \right\}, \quad (13)$$

where marked for simplification $\sigma_{s\eta}^2 \triangleq \sigma_s^2 + \sigma_\eta^2$.

The next step in the synthesis is to obtain an unconditional LF hypothesis, by statistically averaging the PDF $f_{\Pi}\left(\vec{x}\,|\,m\right)$ over the distribution of the signal ME:

$$l_{H_1}\left(\vec{x}\right) = \frac{\exp\left(\dfrac{-\sum_{n=1}^{N} x_n^2}{2\sigma_{s\eta}^2} \right)}{2^{\frac{3N}{2}} \pi^{\frac{N}{2}} \sigma_{s\eta}^{N}} \int_{-\infty}^{\infty} \exp\left(\frac{-m^2 N}{2\sigma_{s\eta}^2} + \frac{m \sum_{n=1}^{N} x_n}{\sigma_{s\eta}^2} \right) dm. \quad (14)$$

The integral in (14) is tabular. Thus, the multivariate unconditional LF of hypothesis H_1 is defined as

$$l_{H_1} = \frac{\sigma_{s\eta}^{1-N} N^{-\frac{1}{2}}}{2^{\frac{3N+1}{2}} \pi^{\frac{N-1}{2}}} \exp\left\{ \frac{1}{2N\sigma_{s\eta}^2} \left(\sum_{n=1}^{N} x_n \right)^2 - \frac{1}{2\sigma_{s\eta}^2} \sum_{n=1}^{N} x_n^2 \right\}. \quad (15)$$

In the last step of the synthesis, substituting LF (4) and (15) for formula (10) we obtain the likelihood ratio, the logarithm and the identity transformation of which leads to the decision rule

$$\sum_{n=1}^{N} x_n^2 + \frac{\sigma_\eta^2}{N\sigma_s^2} \left(\sum_{n=1}^{N} x_n \right)^2 - \frac{2\sigma_\eta^2\left(\sigma_\eta^2 + \sigma_s^2\right)}{\sigma_s^2} \sum_{n=1}^{N} \ln x_n > \widehat{V}_1, \quad (16)$$

where $\widehat{V}_1\left(\sigma_\eta^2, \sigma_s^2, N\right)$ is a detection threshold that depends on the dispersions and the size of the pack.

V. Synthesis of Detection Algorithm with the Known ME of Pack

In this case, the general LF (11) of the hypothesis H_1, after the substitution of the delta function $f_m(m) = \delta(M)$, where M is the known value of ME, loses the external integral. Then we can use formula (14), substituting m for known M. Substituting the following LF into (10), taking into account the LF (4), we have a likelihood ratio which, after logarithm and identical transformations, leads to a decision rule for detecting a signal of the following form:

$$\sum_{n=1}^{N} x_n^2 + \frac{2M\sigma_\eta^2}{\sigma_s^2} \sum_{n=1}^{N} x_n - \frac{2\sigma_\eta^2\left(\sigma_\eta^2 + \sigma_s^2\right)}{\sigma_s^2} \sum_{n=1}^{N} \ln x_n > \widehat{V}_2. \quad (17)$$

VI. Synthesis of a Robust Detection Algorithm

The a priori uncertainty of the mixture of signal and interference PDF at the output of the linear detector is given in the form of a Tukey model [7]: PDF of the mixture of noise and chaotic pulse interference (HPI) envelope

$$l_{H_0}(\overline{x}) = \prod_{n=1}^{N} \left(\frac{(1-p)x_n}{\sigma_\eta^2} \exp\left(-\frac{x_n^2}{2\sigma_\eta^2} \right) + \frac{p}{a} \exp\left(-\frac{x_n}{a} \right) \right), \quad (18)$$

where a is the scale parameter of pulse interference; p is the probability of occurrence of exponential distribution HPI.

Probability distribution density of envelope mixture of signal, noise and HPI

$$l_{H_1} = \prod_{n=1}^{N} \left[\frac{(1-p)x_n}{\sigma_\eta^2} \exp\left(-\frac{x_n^2 + m^2}{2\sigma_\eta^2} \right) I_0\left(\frac{m x_n}{\sigma_\eta^2} \right) + \frac{p}{a} \exp\left(\frac{-x_n}{a} \right) \right].$$

Using the Gaussian Rice distribution approximation (6), and synthesis technic [18] we obtain a locally optimal decision rule as a derivative of the logarithm of the likelihood ratio at a point $m = 0$.

$$\frac{\partial}{\partial m} \ln \frac{l_{H_1}(\overline{x})}{l_{H_0}(\overline{x})}\bigg|_{m=0} = \sum_{n=1}^{N} \frac{x_n}{1 + \dfrac{p}{(1-p)a} \sqrt{2\pi\sigma_\eta^2} \exp\left(\dfrac{x_n^2}{2\sigma_\eta^2} - \dfrac{x_n}{a} \right)}. \quad (19)$$

VII. Efficiency Analysis of Detection Algorithms

The efficiency of the synthesized algorithms were analyzed using the Monte Carlo method. Figures 1 and 2 present the operational characteristics of the four detection algorithms.

Operational characteristics were calculated for the following algorithms: (1) – classical; (16) – is invariant to the mathematical expectation m; (17) – is an algorithm with a known value of the mathematical expectation of the signal *M*; (19) – is a robust algorithm. Sampling size *N* = 64, probability of false alarm $\alpha = 0.01$, scale parameter of HPI $a = 10$, probability of HPI occurrence $p = 0.05$.

The calculations made for two situations. In the first, the mathematical expectation of the signal amplitude was constant for all pack pulses and different for different packets (together fluctuating pack). Probability distribution density of ME is Gaussian. In the second situation, the ME of the signal amplitude was different at different points in time (fluctuating pack).

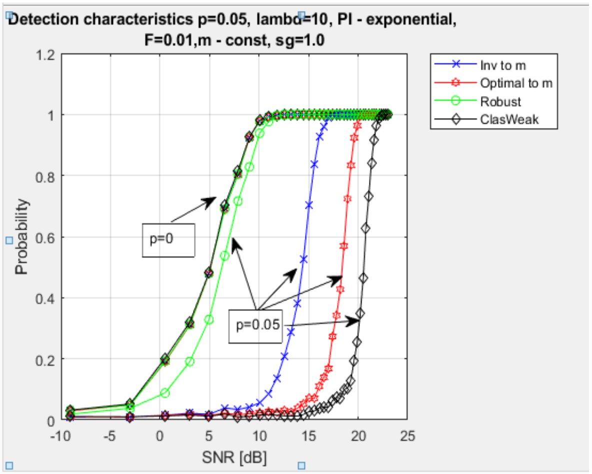

Fig. 2. Operational characteristics of signal detection algorithms in situation with known mathematical espectation *m*. Strong amplitude fluctuation $\sigma_s^2 = \sigma_\eta^2$

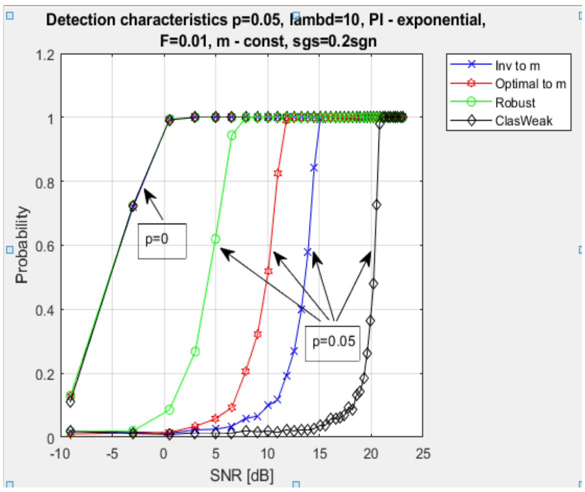

Fig. 3. Operational characteristics of signal detection algorithms in situation with known mathematical espectation *m*. Weak amplitude fluctuation $\sigma_s^2 = 0.2\sigma_\eta^2$

Figures 1 and 2 show the operational characteristics of the investigated detectors for the situation without HPI (*p* = 0). In this case, the characteristics are almost identical. But with the action of HPI (*p* > 0), the efficiency of detectors differs.

VIII. Conclusion

The problem of finding a harmonic signal with random amplitude against the background of Gaussian uncorrelated noise is considered. For three signal interference situations, three algorithms were synthesized: 1) an algorithm using the Wald reduction principle for a signal with known Gaussian amplitude distribution; 2) algorithm for detecting a signal with a randomized mathematical expectation of a Gaussian amplitude distribution; 3) robust algorithm using heavy-tailed “polluted” Tukey distribution, describing the effect of chaotic impulse interference.

The analysis of efficiency of the obtained algorithms showed their practical equivalence in the situation of absence of impulse interference and different resistance to its action. Robust algorithm (19) is the most robust to HPI. At 5% of the sample pollution, its efficiency (signal level at which a detection probability of 0.5 is reached) drops by 1dB (Fig. 1, case of strong fluctuations) compared to the unpolluted sample. The efficiency of algorithms (16) and (17) decreases by 9 dB and 13 dB, respectively. The efficiency of the classical parametric algorithm (1) under these conditions decreases by 15 dB. By weak amplitude fluctuation (Fig. 2) algorithm (17) is more effective then (16) and vice versa in case of strong fluctuation (Fig. 1).

Appendix

A1. Synthesis of the detection algorithm in a pack of a random signal with the same random ME in the sample

The peculiarity of the probabilistic model in this formulation of the problem is the following:

– probability distribution density $f_{s+\eta}(x_n \mid s_n)$ of each individual sample of the pack $\{x_n = s_n + \eta_n\}$, where $\{s_n\}$, $\{\eta_n\}$ is the signal and noise components are Gaussian, respectively (8);

– signal components $\{s_n\}$ each of the sample is also described by the same conditional PDF $f_s(s_n \mid m)$ (9);

– the ME of the signal is random and has a nonproper distribution $f_m(m_n) = dm$ on the interval $[-\infty, \infty]$;

– the likelihood function l_{H_0} for the hypothesis H_0 is defined by the formula (4).

The binary detection algorithm is synthesized according to the Neumann–Pearson criterion (10). The likelyhood ratio for a pack by Wald-reduction according to this statistical model is written as

$$\Lambda(\vec{x}) = \frac{\int_{-\infty}^{\infty}\left[\prod_{n=1}^{N}\int_{-\infty}^{\infty} f_{s+\eta}(x_n \mid s_n) f_s(s_n \mid m)\, ds_n\right] f_m(m_n)\, dm}{l_{H_0}(\vec{x})} = \frac{\int_{-\infty}^{\infty}\left[\prod_{n=1}^{N}\int_{-\infty}^{\infty} \exp\left\{-\frac{(x_n - s_n)^2}{2\sigma_\eta^2}\right\}\exp\left\{-\frac{(s_n - m)^2}{2\sigma_s^2}\right\} ds_n\right] f_m(m)\, dm}{\left(\sqrt{2\pi}\pi\sigma_s\right)^N \prod_{n=1}^{N}\exp\left\{-\frac{x_n^2}{2\sigma_\eta^2}\right\}} \quad \text{(a1)}$$

Obviously, significant numerical transformations require the numerator of this expression. After the elementary transformations, the internal integral (by s) is reduced to the standard form (the index n is omitted):

$$\int_{-\infty}^{\infty} \exp\left\{\left(-\frac{x^2}{2\sigma_\eta^2} + \frac{xs}{\sigma_\eta^2} - \frac{s^2}{2\sigma_\eta^2}\right) + \left(-\frac{s^2}{2\sigma_s^2} + \frac{sm}{\sigma_s^2} - \frac{m^2}{2\sigma_s^2}\right)\right\} ds$$

$$\triangleq \int_{-\infty}^{\infty} \exp\left\{-as^2 + bs - c\right\} ds \triangleq \int_{s,n}(m), \quad \text{(a2)}$$

where $a \triangleq \frac{\sigma_s^2 + \sigma_\eta^2}{2\sigma_\eta^2\sigma_s^2}$, $b \triangleq \frac{x\sigma_s^2 + m\sigma_\eta^2}{\sigma_\eta^2\sigma_s^2}$, $c \triangleq \frac{x^2}{2\sigma_\eta^2} + \frac{m^2}{2\sigma_s^2}$.

The integral on the right (a2) is a table

$$\int_{s,n} = \frac{\sqrt{\pi}}{2}\frac{\exp\left\{\frac{b^2}{4a - c}\right\}}{\sqrt{a}} \operatorname{erf}\left\{s\sqrt{a} - \frac{b}{2\sqrt{a}}\right\}_{-\infty}^{\infty}$$

and after identical transformations it is reduced to expression (12) (where the indices are restored).

The product in the numerator of formula (d1) determines the conditional on the m PDF packs $f_\Pi(\vec{x} \mid m)$ (13). Obtaining an unconditional PDF according to expression (14) requires integration

$$\int_m \triangleq \int_{-\infty}^{\infty} \exp\left(\frac{-m^2 N}{2\sigma_{s\eta}^2} + \frac{m\sum_{n=1}^{N} x_n}{2\sigma_{s\eta}^2}\right) dm. \quad \text{(a3)}$$

Like (a2), the integral (a3) is also tabular

$$\int_m \triangleq \int_{-\infty}^{\infty} \exp\left\{-am^2 + bm\right\} dm$$

$$= \frac{\sqrt{\pi}}{2}\frac{\exp\left\{\frac{b^2}{4a}\right\}}{\sqrt{a}} \operatorname{erf}\left\{m\sqrt{a} - \frac{b}{2\sqrt{a}}\right\}_{-\infty}^{\infty},$$

where

$$a \triangleq \frac{N}{2\sigma_{s\eta}^2}, \; b \triangleq \sum_{n=1}^{N}\frac{x_n}{2\sigma_{s\eta}^2}.$$

Also we have

$$\int_m = \frac{\sqrt{\pi}}{2}\frac{\exp\left\{\frac{1}{\sigma_{s\eta}^4}\left(\sum_{n=1}^{N} x_n\right)^2 \frac{\sigma_{s\eta}^2}{2N}\right\}}{\sqrt{\frac{N}{2\sigma_{s\eta}^2}}}$$

$$= \sqrt{\frac{\pi}{2N}}\sigma_{s\eta} \exp\left\{\frac{\left(\sum_{n=1}^{N} x_n\right)^2}{2N\sigma_{s\eta}^2}\right\}.$$

As a result, we get a multivariate unconditional LF l_{H_1} of hipothsis H_1 (15).

Using LF (4) and (15), decision rule (10) in the expanded form can be written as

$$\frac{\exp\left\{\frac{1}{2N\sigma_{s\eta}^2}\left(\sum_{n=1}^{N} x_n\right)^2 - \frac{1}{2\sigma_{s\eta}^2}\sum_{n=1}^{N} x_n^2\right\}}{\exp\left\{-\frac{1}{2\sigma_\eta^2}\sum_{n=1}^{N} x_n^2\right\}\prod_{n=1}^{N} x_n} > V\left(\sigma_\eta^2, \sigma_s^2, N\right),$$

where decision threshold $V\left(\sigma_\eta^2, \sigma_s^2, N\right)$ depends on the variance.

After identical transformations and logarithms of both parts of this inequality, we have a decisive rule of discovery (16) or otherwise as

$$\sum_{n=1}^{N} x_n^2 + \nu_1\left(\sum_{n=1}^{N} x_n\right)^2 - \nu_2\sum_{n=1}^{N}\ln x_n > \widehat{V}_1, \quad \text{(a4)}$$

where coefficients are marked:

$$\nu_1 \triangleq \frac{\sigma_\eta^2}{N\sigma_s^2}, \qquad \nu_2 \triangleq \frac{2\sigma_\eta^2\left(\sigma_\eta^2 + \sigma_s^2\right)}{\sigma_s^2}$$

A2. Synthesis algorithm of detection in a bundle of a random signal with the same known MO in the sample

Given the known ME = M, there is no external integration in the numerator of the likelihood ratio (a1), that is, we have

$$\Lambda\left(\vec{x}\right)=\frac{l_{H1}\left(\vec{x}\right)}{l_{H_0}\left(\vec{x}\right)}=\frac{\prod\limits_{n=1}^{N}\int\limits_{-\infty}^{\infty} f_{s+\eta}\left(x_n \mid s_n\right) f_s\left(s_n \mid M\right) ds_n}{l_{H_0}\left(\vec{x}\right)}. \quad \text{(a5)}$$

Carrying out transformations similar to $A1$, we obtain the numerator of formula (a5) as

$$l_{H1}\left(\vec{x}\right)=\frac{1}{2^{\frac{3N}{2}}\pi^{\frac{N}{2}}\sigma_{s\eta}^{N}}\exp\left\{-\frac{\sum\limits_{n=1}^{N}x_n^2}{2\sigma_{s\eta}^2}\right\}$$

$$\exp\left\{-M^2\frac{N}{2\sigma_{s\eta}^2}+M\frac{\sum\limits_{n=1}^{N}x_n}{\sigma_{s\eta}^2}\right\}. \quad \text{(a6)}$$

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Prokopenko Igor.orcid.org/0000-0003-4169-3774.
Doctor of Engineering Science.Professor.
Aviation Radioelectronic Complexes Department, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Institute of Civil Aviation Engineers, Kyiv, Ukraine, (1972).
Research area: theory of signals and data processing.
Publications: more than 300 papers.
E-mail: prokop-igor@yandex.ru

OmelchukIgor. Candidate of Science (Engineering).Associate Professor.
Aviation Radioelectronic Complexes Department, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Institute of Civil Aviation Engineers, Kyiv, Ukraine (1979).
Research area: process and systems modeling
Publications:more than 30.
E-mail:kwh@ukr.net

Prokopenko Kostiantyn. Candidate of Science (Engineering). Postdoctoral student.
Air Navigation Systems Department, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Institute of Civil Aviation Engineers, Kyiv, Ukraine, (1990).
Research area: process and systems modeling
Publications: 25.
E-mail:kprok78@gmail.com

OsipchukAlina. Assistant Professor.
Aviation Radioelectronic Complexes Department, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine, (2004).
Research area: process and systems modeling
Publications:10.
E-mail:alina.osipchuk2012@gmail.com

І. Г. Прокопенко, І. П. Омельчук, К. І. Прокопенко, А. А. Осипчук. Синтез та аналіз ефективності робастних алгоритмів виявлення випадкових сигналів в негауссових завадах

У статті розглянуто проблему синтезу робастних після детекторних алгоритмів виявлення випадкових радіолокаційних сигналів на тлі некорельованого шуму та хаотичних імпульсних перешкод. Розглянуто два випадки апріорної невизначеності: 1) випадковий модульований гармонічний радіолокаційний сигнал з випадковою амплітудою, розподіленою за законом Гаусса з відомими параметрами; 2) випадковий модульований гармонічний радіолокаційний сигнал з невідомим законом розподілу амплітуди. Проблема вирішується з використанням редукції Вальда. Синтез робастного алгоритму виявлення сигналу на тлі хаотичних імпульсних перешкод здійснювався за допомогою моделі «забруднення» Тьюкі. Ефективність та робастність декількох синтезованих робастних алгоритмів виявлення досліджується методом Монте–Карло.

Ключові слова: обробка сигналів; робастні алгоритми; виявлення радіолокаційного сигналу; хаотична імпульсна перешкода; апріорна невизначеність.

Прокопенко Ігор Григорович.orcid.org/0000-0003-4169-3774
Доктор технічних наук. Професор.
Кафедра авіаційних радіоелектронних комплексів, Національний авіаційний університет, Київ, Україна.
Освіта: Київський інститут інженерів цивільної авіації, Київ, Україна (1972).
Напрям наукової діяльності: теорія обробки сигналів і даних.
Кількість публікацій: більше 300 наукових робіт.
E-mail: prokop-igor@yandex.ru

Омельчук Ігор Павлович. Кандидат технічних наук. Доцент.
Кафедра авіаційних радіоелектронних комплексів, Національний авіаційний університет, Київ, Україна.
Освіта: Київський інститут інженерів цивільної авіації, Київ, Україна, (1979).
Напрям наукової діяльності: моделювання систем і процесів.
Кількість публікацій: більше 30 наукових робіт.
E-mail: kwh@ukr.net

Прокопенко Костянтин Ігорович. Кандидат технічних наук. Докторант.
Кафедра аеронавігаційних систем, Національний авіаційний університет, Київ, Україна.
Освіта: КНУ ім. Т. Г. Шевченка, факультет кібернетики. Київ, Україна, (2001).
Напрям наукової діяльності: обробка сигналів і даних.
Кількість публікацій: 25.
E-mail: kprok78@gmail.com

Осипчук Аліна Олександрівна. Старший викладач.
Кафедра авіаційних радіоелектронних комплексів, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна, (2004).
Напрям наукової діяльності: моделювання систем і процесів.
Кількість публікацій: 10.
E-mail: alina.osipchuk2012@gmail.com

И. Г. Прокопенко, И. П. Омельчук, К. И. Прокопенко, А. А. Осипчук. Синтез и анализ эффективности робастных алгоритмов обнаружения случайных сигналов в негауссовских помехах

В статье рассматривается проблема синтеза робастных последетекторных алгоритмов обнаружения случайных радиолокационных сигналов на фоне некоррелированных шумовых и хаотических импульсных помех. Рассматриваются два случая априорной неопределенности: 1) гармонический радиолокационный сигнал со случайной амплитудой, распределенный по закону Гаусса с известными параметрами; 2) –случайный модулированный гармонический радиолокационный сигнал с неизвестным законом распределения амплитуд. Проблема решается с использованием редукции Вальда. Синтез робастного алгоритма обнаружения сигнала на фоне хаотических импульсных помех выполнен с использованием модели «загрязнения» Тьюки. Эффективность и робастность нескольких синтезированных робастных алгоритмов обнаружения исследуется методом Монте-Карло.

Ключевые слова: обработка сигналов; робастные алгоритмы; обнаружение радиолокационного сигнала; хаотическая импульсная помеха; априорная неопределенность.

Прокопенко Игорь Григорьевич.orcid.org/0000-0003-4169-3774
Доктор технических наук. Профессор.
Кафедра авиационных радиоэлектронных комплексов, Национальный авиационный университет, Киев, Украина.
Образование: Киевский институт инженеров гражданской авиации, Киев, Украина, (1972).
Направление научной деятельности: теория обработки сигналов и данных.
Количество публикации: более 300 научных работ.
E-mail: prokop-igor@yandex.ru

Омельчук Ігорь Павлович. Кандидат технических наук. Доцент.
Кафедра авиационных радиоэлектронных комплексов, Национальный авиационный университет, Киев, Украина.
Образование: Киевский институт инженеров гражданской авиации, Киев, Украина (1979).
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Количество публикации: более 30 научных работ.
E-mail: kwh@ukr.net

Прокопенко Константин Игоревич. Кандидат технических наук. Докторант.
Кафедра аэронавигационных систем, Национальный авиационный университет, Киев, Украина.
Образование: КНУ им. Т. Г. Шевченко, факультет кибернетики. Киев, Украина (2001).
Направление научной деятельности: обработка сигналов и данных.
Количество публикаций: 25.
E-mail: kprok78@gmail.com

Осипчук Алина Александровна. Старший преподаватель.
Кафедра авиационных радиоэлектронных комплексов, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина, (2004).
Направление научной деятельности: обработка сигналов и данных.
Количество публикаций: 10.
E-mail: alina.osipchuk2012@gmail.com