MATHEMATICAL MODELLING OF PROCESSES AND SYSTEMS

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MODELLING OF ROBUST INERTIALLY STABILIZED PLATFORMS

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Abstract—*The article focuses on problems of development of the robust systems for control of platforms motion. The main goal is to consider structure and features of data ware, hardware, and software necessary for the efficient modelling of inertially stabilized platforms with payload assigned for operation on moving vehicles of the wide class. To solve this problem methods of robust parametrical optimization and robust structural synthesis are used. The proposed approaches to modelling of inertially stabilized platforms are based on MATLAB. The analysis of appropriate software tools is represented. The advantages of MATLAB are described. The basic stages of the robust control system modelling are given. The list of models necessary for the design of the robust inertially platforms is represented. The model of the stabilization plant was developed. The model includes models of the actuator and the measuring system. The basic features of modelling of the robust inertially stabilized platforms are given. The results of modelling are shown. The results have been obtained in conditions of parametrical uncertainty. The presented example was based on the inertially stabilized platform assigned for the ground moving vehicle operated in difficult conditions of the real operation accompanied with parametrical disturbances. Proposed ways to modelling of inertially stabilized platforms allow us to decrease time and cost losses of design. The presented results can be useful for inertially stabilized platforms operated on moving objects of the wide class.*

Index Terms—Inertially stabilized platforms; modelling; robustness; Simulink; stabilization.

I. INTRODUCTION

Nowadays the inertially stabilized platforms are widely used for stabilization of navigation sensors, photo and video cameras, antennas, telescopes, and weapon systems operated at moving vehicles of the various types. Requirements to systems of control of platform angular motion depend on an application area. At the same time, these platforms have a common goal such as control of the line-of-sight of the equipment mounted at a vehicle [1]. The equipment stabilized by means of control systems can be used at the ground vehicles, unmanned aerial vehicles, aircraft, spacecraft, and ships for solving different problems such as tracking, mapping, and imaging.

In accordance with the generally accepted terminology, the gimbal represents a mechanism, which is used for control by the inertial orientation of the measuring axes of devices mounted at the platform [1]. The appropriate control system is believed to be included in the concept of the inertially stabilized platforms. These systems are usually integrated into stabilization, tracking or pointing system.

The inertially stabilized platforms provide solving important problems such as:

1) stabilization of payload during angular motion of a moving vehicle in conditions of parametrical uncertainty;

2) pointing line-of-sight of the observation devices to the given reference point;

3) tracking a given reference point by means of keeping a constant orientation of a line-of-sight in the given direction.

The wide area of application of the inertially stabilized platforms and the new achievements in inertial sensor technologies makes new researches in this area necessary for instrument making.

Operation of the inertially stabilized platforms is implemented under influence of parametrical and coordinate disturbances. To provide accuracy of stabilization and tracking processes is possible using robust control. Design of robust control systems requires the development of mathematical models and modelling.

Modelling is an effective tool for researching complex control systems. Using modelling it is possible to estimate the quality of the control processes under the influence of reference and disturbing signals and also changing of dynamic objects characteristics. Such an approach to design of control systems allows accelerating the design time. In contrast to the mode of real operation, modelling provides more information about the operation of system components in different modes. Moreover, modelling gives possibilities to imitate faults and difficult situations of system operation.

II. REVIEW OF LITERATURE

General principles of development of the inertially stabilized platforms are represented in [1]. Essentials of the robust control are given in [3]. Principles of design of robust control systems are given in many textbooks, for example, [2]. Usage of MATLAB in the automated designed procedures of the robust control systems is represented in [4].

The algorithm of the robust parametrical optimization of the inertially stabilized platform for the moving ground vehicle is given in [5]. The principles of the robust structural synthesis of the inertially stabilized platforms are described in [6]. Features of stabilized platform modelling require further research.

III. PROBLEM STATEMENT

The process of design of the robust inertially stabilized platforms of the general type is illustrated by the block-scheme in Fig. 1. The basic stages of the design of the robust inertially stabilized platforms including robust parametrical optimization and robust structural synthesis are shown here.

The role and place of modelling in the process of design of the robust inertially stabilized platform is of great importance. The goal of the research is to describe the features of this process. For this, it is necessary to choose tools for modelling, to develop mathematical model and to carry out modelling of the inertially stabilized platform.

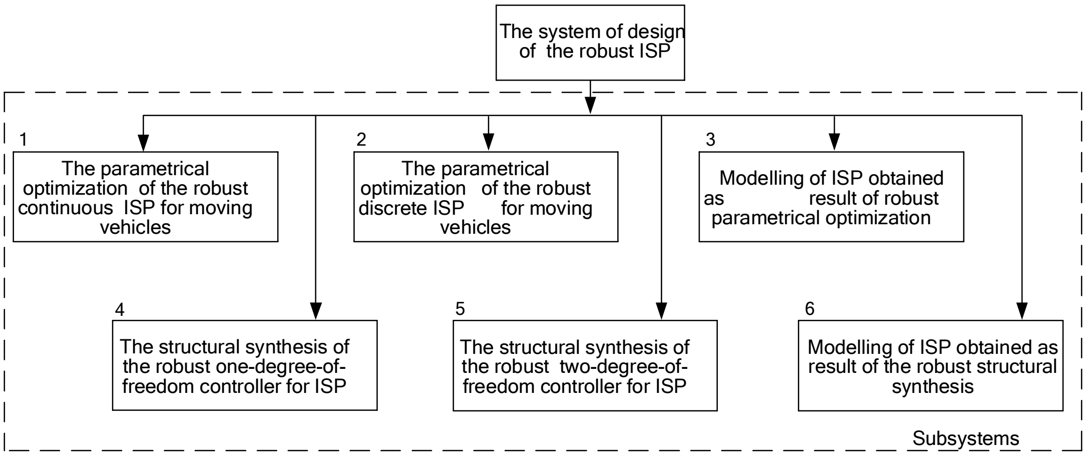


Fig. 1. Block-scheme of the algorithm of design of the robust inertially stabilized platform: ISP is the inertially stabilized platform

IV. CHOICE OF SOFTWARE FOR MODEL DEVELOPMENT

The design of robust systems requires the implementation of large amount of transformations of matrix transfer functions that describe the system. Nowadays, it is possible to overcome these difficulties by using software that allows you to automate complex functional and analytical transformations, such as Maple, MathCAD, SCILAB, and MATLAB. MATLAB is especially convenient among the listed computing systems. It contains specialized software packages for the design of optimal, robust control and stabilization systems. The design of control systems is greatly simplified when using additional specialized toolboxes of the MATLAB system [7].

This way, the Control System Toolbox application suite is designed to simulate, analyze, and synthesize complex control systems. The advantages of this toolbox include the possibility to use traditional frequency methods of creating control systems based on transfer functions. Also, it allows using the modern theory of control based on the representation of models in the state space. The tools provide creating optimal design procedures for both continuous and discrete systems. The Control System Toolbox contains a large number of software-implemented algorithms for analyzing and synthesizing control systems. In addition, it has a custom environment, as it allows you to use original own-development algorithms.

The Optimization Toolbox has the option of choosing an optimization method that takes into account the specifics of a problem and provides the opportunity to obtain the optimal solution. For the robust synthesis of the inertially stabilized platforms, it is most appropriate to use the Nelder–Mead method or the genetic algorithm.

Robust Control Toolbox is a powerful tool for designing robust systems, which provides the complex calculations required for the structural synthesis of controllers based on the optimization criteria that are based on H-norms of the sensitivity functions.

Finally, for the analysis of the synthesized system, it is expedient to use models that take into account all the typical nonlinearities inherent in real systems. The results of modelling with the use of such mathematical descriptions should confirm the effectiveness of the optimization. The MATLAB software has ample opportunities for creating such models based on the use of the Simulink.

The MATLAB software provides the combined use of Control System Toolbox, Robust Control Toolbox, and Simulink, which extends the capabilities of each one and improves the effectiveness of procedures assigned for the optimal design of robust stabilization systems.

To develop the optimal design procedure, you need to choose an optimization method. The most common methods of optimization that have program implementation in the MATLAB are the golden section method, the quadratic approximation method, the Nelder–Mead method, the fastest descent method, and genetic algorithms [8].

The golden section method is used to minimize within a given interval, with the objective function to be unimodal. The essence of the method of quadratic approximation consists in approximating the objective function by a quadratic function. The Nelder-Mead method [8] is used to minimize the multi-objective objective function when methods of the golden section and quadratic approximation cannot be used. The algorithm of the Nelder–Mead method is implemented in the MATLAB system by the built-in *fminsearch* function, which allows minimizing the multi-variable objective function. The fastest descent method provides the search for an *n*-dimensional objective function in the direction of a negative gradient [8].

All of the above optimization methods are effective in finding a minimum if their initial conditions are sufficiently close to him. In addition, the optimization point, which is the result of the method, may be one of the local minima, but not the global minimum. In the general case, it is necessary to change the initial conditions and determine the global among the obtained local minima. This task is very complex in terms of computational cost since there is no systematic approach to determining the sequence of the corresponding initial conditions that lead to finding all local minima, among which it is possible to determine the global minimum.

The genetic algorithm is a method of controlled random search, which is based on the modelling of evolution-selection processes in the direction of survival of the best [8]. Genetic operators have been dealing with individuals in a population for several generations with a view to substantially improving them. Individuals from possible solutions are often equated with chromosomes and represented by binary strings. The genetic algorithm allows searching the global minimum, even when the objective function has several extremes, including local maxima and minima.

The task of the optimal design of any system is characterized by the presence of constraints on design parameters. In this case, the most well-known approaches to determining such constraints are the method of indeterminate Lagrange multipliers and the method of penalty functions [8]. The method of indeterminate Lagrange multipliers is very effective if the constraints are given in the form of equalities. More practical significance is given to the method of penalty functions, which can be used for a wide range of optimization tasks, including constraints in the form of both equality and inequalities. This method is effective for optimization problems with fuzzy and free constraints. The method of penalty functions is realized in two stages [8]. In the first stage, a new objective function is defined with the inclusion of such components, which are defined by large values in violation of the given constraints, while the implementation of permissible constraints will not affect the appearance of the original objective function. At the second stage, the search for the extremum of a new objective function is carried out using the method of unconditional optimization.

Gradient methods cannot be used for solving problems of optimization of systems of the specified type. The most expedient is to use the Nelder–Mead method or genetic algorithm. The kind of the penalty function and the weighting coefficients are determined depending on the features of the designed system.

V. DEVELOPMENT OF MATHEMATICAL MODEL

Mathematical models of a stabilization system can be represented by the hierarchical structure consisting of two levels. Such a structure is shown in Fig. 2. The first level corresponds to models of stabilization systems and the second level – to mathematical models of the system's devices.

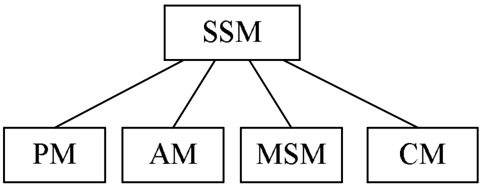


Fig. 2. The hierarchical structure of mathematical models: SSM is the stabilization system model; PM is the plant model; AM is the actuator model; MSM is the measuring system model; CM is the controller model

There are three basic methods, which are the most convenient for the development of mathematical models of control systems. These methods are based on transfer functions and frequency characteristics, variables in the state space, and structure-topological connections [9].

The mathematical model of an inertially stabilized platform includes mathematical models of separate devices taking into consideration structural connections. It is convenient to represent mathematical descriptions of separate devices as state space models and transfer functions. Such an approach provides the representation of the model of the stabilization system in the form of the structural scheme. In this case, it is convenient to implement the optimal design of stabilized platforms, for example, by means of the robust parametric optimization and robust structural synthesis. It should be noted that modern software tools provide the possibility to formalize representation of the structural connections between units of the general mathematical model. Modelling of complex control systems requires using mathematical models of different typical disturbances.

So, modelling of the robust inertially stabilized platforms in conditions of uncertainty can be successfully done by means of the following stages.

1) Development of the full mathematical descriptions of the control system components including stabilization plant, controller, actuator, the measuring system based on the principles of operation. This model takes into consideration nonlinearities inherent in real operating systems.

2) Development of the full mathematical model of the stabilization system using models of components. The structural connections are also taking into consideration.

3) Development of the models of different disturbances.

4) Linearization of the model of the control system and its components. Such an approach provides a representation of models in the state space and in the form of transfer functions.

5) Modelling during carrying out procedures of the robust parametrical optimization and robust structural synthesis.

6) Modelling for checking results of optimal control system synthesis.

All the models it is convenient to realize in MATLAB software. The mathematical models taking into account nonlinearities can be developed by Simulink.

Development of the mathematical model of the inertially stabilized platform can be considered on the example of the two-axial stabilization system assigned for operation on the ground moving object.

The dynamics and kinematics of the platform can be described by Euler's equations. The appropriate full relationships and sequence of platform rotations in the space for the two-axis platform with payload are represented in [10]. In accordance with [10], the mathematical model of the platform with payload can be represented in the following form

$$
\begin{aligned}
&\dot{\alpha} = \omega_x \cos\beta + \omega_z \sin\beta ,\\
&\dot{\beta} = \omega_y ,\\
&\dot{\alpha}_e = \omega_{e\alpha} ,\\
&\dot{\beta}_e = \omega_{e\beta} ,\\
&\dot{U}_{\omega\alpha} = U_{\omega d\alpha} ,\\
&\dot{U}_{\omega\beta} = U_{\omega d\beta} ,\\
&\dot{\omega}_x = [-(J_z - J_y)\omega_y\omega_z - M_{frx}\mathrm{sign}\omega_x - M_{unbx}\cos\alpha\\
&\qquad\qquad + c_r(\alpha_g - \alpha)/n_r]/J_x ,\\
&\dot{\omega}_y = [-(J_y - J_x)\omega_x\omega_z - M_{fry}\mathrm{sign}\omega_y - M_{unby}\cos\beta\\
&\qquad\qquad + k_{spr}(A - \beta) + \frac{c_r(\beta_g - \beta)}{n_r}]/J_y ,\\
&\dot{\omega}_{e\alpha} = \left[-M_{fre}\mathrm{sign}\omega_{e\alpha} + \frac{c_m}{R_w}U_\alpha + \frac{c_r(\alpha_g - \alpha)}{n_r}\right]/J_e ,\\
&\dot{\omega}_{e\beta} = \left[-M_{fre}\mathrm{sign}\omega_{e\beta} + \frac{c_m}{R_w}U_\beta + \frac{c_r(\beta_g - \beta)}{n_r}\right]/J_e ,\\
&\dot{U}_\alpha = [-U_\alpha + k_{PWM}U_{PWM\alpha} - c_{ed}\omega_{e\alpha}]/T_{arm} ,\\
&\dot{U}_\beta = [-U_\beta + k_{PWM}U_{PWM\beta} - c_{ed}\omega_{e\beta}]/T_{arm} ,\\
&\dot{U}_{\omega d\alpha} = [-2\nu T_0 U_{\omega e\alpha} - U_{\omega\alpha} + k_{ars}\omega_x]/T_0^2 ,\\
&\dot{U}_{\omega d\beta} = [-2\nu T_0 U_{\omega e\beta} - U_{\omega\beta} + k_{ars}\omega_y]/T_0^2 ,
\end{aligned}
\tag{1}
$$

where α, β are the angles of the platform rotations; ω_x, ω_y are the platform angular rates in the horizontal and vertical planes; $\omega_{e\alpha}$, $\omega_{e\beta}$ are the angular rates of the motors mounted at the axes x, y; α_e, β_e are the rotation angles of the motors mounted at the axes x, y; $U_{\omega\alpha}$, $U_{\omega\beta}$ are the output signals of the rate gyros; $U_{\omega d\alpha}$, $U_{\omega d\beta}$ are the derivatives of the rate gyro signals; J_x, J_y, J_z are the inertia moments of the platform relative to axes x, y, z; M_{frx}, M_{fry} are the nominal dry friction moments acting by the gimbals axes x, y; M_{unbx}, M_{unby} are the unbalanced moments acting by the axes x, y; k_{spr} is the rigidity coefficient of the spring compensator; A is the initial angle of spring resetting; c_r is the reducer rigidity; α_g, β_g are the angles of the platform rotation taking into account presence of the drive hysteresis; M_{frex}, M_{frey} are the nominal dry friction moments of motors mounted at the gimbals axes x, y; c_m is the constant of the load moment at the motor shaft; R_w is the resistance of the motor armature winding; U_α, U_β are the armature voltages of motors; n_r is the reducer gear ratio; T_{arm} is the time constant of the motor armature circuit; k_{PWM} is the transfer constant of the linearized pulse-width-modulator; U_{PWM} is the voltage at the pulse-width-modulator input; c_{ed} is the coefficient of proportionality between the motor angular rate and the electromotive force; ν is the relative damping coefficient; T_0 is the time constant of the rate gyro, k_{ars} is the transfer constant of the rate gyro.

In the represented set of the nonlinear equations (1) the angles α_g, β_g may be defined in accordance with the expressions, which take into consideration the drive hysteresis

$$\begin{aligned}
&\alpha_g = \alpha_e / n_p, \text{ if } |\alpha_e / n_p - \alpha| \geq 0.5\Delta, \\
&\alpha_g = \alpha, \text{ if } |\alpha_e / n_p - \alpha| < 0.5\Delta, \\
&\beta_g = \beta_e / n_p, \text{ if } |\beta_e / n_p - \beta| \geq 0.5\Delta, \\
&\beta_g = \beta, \text{ if } |\beta_e / n_p - \beta| < 0.5\Delta,
\end{aligned} \tag{2}$$

where Δ is the experimentally determined value.

For further researches, it is necessary to implement linearization of the equations (1) relative to the nominal values of the phase coordinates. Such linearization must include the following steps:

1) linearization of the expressions determining friction and unbalanced moments of motor and stabilization plant;

2) neglect drive hysteresis described by the expressions (2);

3) neglect rate gyros errors;

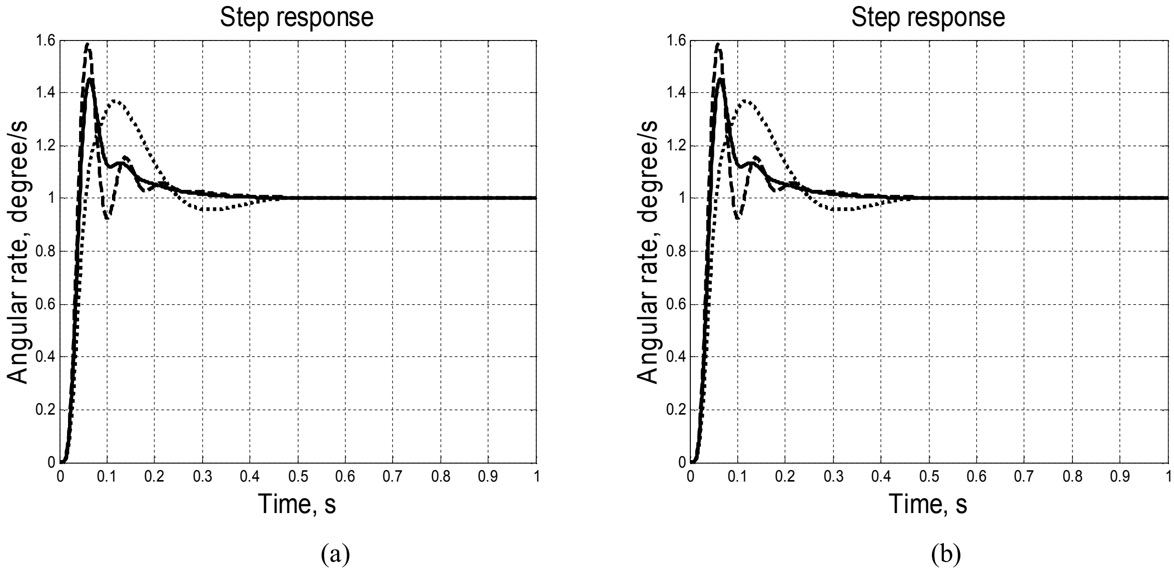
4) assumption of the smallness of the platform turn angles for linearization of the trigonometric functions.

After the above-listed transformations the set of equations (1) may be converted to the linearized form and represented in the state space by the quadruple of matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$.

It should be noted, that the modelling during optimal system design is implemented based on the state-space model. And the check of results of the synthesized system simulation is carried out by means of the nonlinear model, which can be developed based on the set of equations (1).

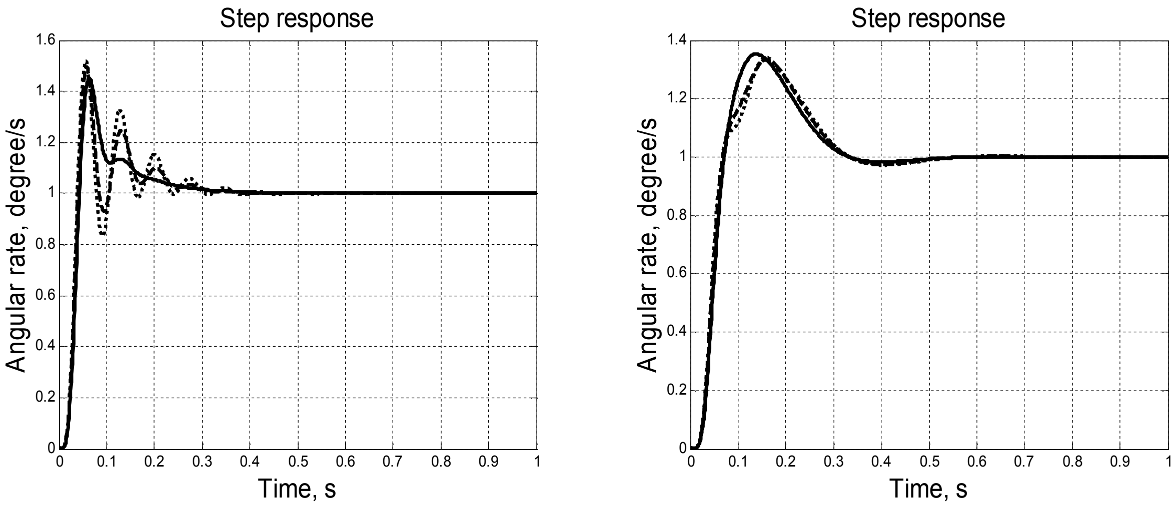
VI. MODELLING EXPERIMENT

The results of modelling stabilization processes in the vertical plane taking into account the parametrical disturbances are represented in Fig. 3. The results of modelling stabilization processes in the horizontal plane taking into account the parametrical disturbances are given in Fig. 4. Transient processes of angular rates of the two-axis inertially stabilized platform were obtained for the different values of the inertia and of the coefficient of rigidity between the actuator and the platform. These parameters were changed in the wide range (± 50%). The modelling was carried out in conditions of the change of unbalanced moment (± 20%) for every channel. Results of modelling prove the possibility of keeping accuracy in conditions of the parametrical and coordinate disturbances.



(a) (b)

Fig. 3. Angular rates for changing moments of inertia (a) and coefficient of rigidity (b) in the vertical plane



(a) (b)

Fig. 4. Angular rates for changing moments of inertia (a) and coefficient of rigidity (b) in the horizontal plane

VII. CONCLUSIONS

The mathematical model of the two-axial inertially stabilized platform assigned for operation on the ground vehicle was developed. The features of modelling during the design of the robust inertially stabilized platforms are described. The modelling of the system for both the horizontal and the vertical channels in conditions of the parametrical and coordinate disturbances has been carried out. The obtained results prove the possibility to provide the accuracy and stability of the stabilized platform functioning at the ground vehicle in the difficult conditions of the real operation.

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Сущенко Ольга Андріївна. Моделювання робастних інерціальних стабілізованих платформ

У статті розглянуто основні проблеми розробки робастних систем управління рухом платформ. Головною метою є розгляд структури та особливостей алгоритмічного, апаратного і програмного забезпечення, необхідного для ефективного моделювання інерціальних стабілізованих платформ з навантаженням, призначеним для експлуатації на об'єктах широкого класу. Для розв'язання цієї проблеми використовуються методи робастної параметричної оптимізації і робастного структурного синтезу. Запропоновані підходи до моделювання інерціальних стабілізованих платформ засновуються на системі MATLAB. Представлено аналіз відповідного програмного забезпечення. Описано переваги обчислювального середовища MATLAB. Приведено основні етапи моделювання робастних систем керування. Представлено перелік моделей, необхідних для проектування інерціальних стабілізованих платформ. Розроблено модель об'єкта стабілізації, яка містить моделі виконавчого механізму та вимірювальної системи. Приведено основні особливості моделювання інерціальних стабілізованих платформ. Показано результати моделювання, отримані в умовах параметричної невизначеності. Представлений приклад засновано на інерціальних стабілізованих платформах, призначених для наземних рухомих об'єктів, які функціонують в складних умовах реальної експлуатації, що супроводжуються параметричними збуреннями. Запропоновані підходи до моделювання інерціальних стабілізованих платформ дозволяють зменшити час і витрати на проектування. Представлені результати можуть бути корисними для інерціальних стабілізованих платформ, експлуатованих на рухомих об'єктах широкого класу.

Ключові слова: інерціальні стабілізовані платформи; моделювання; робастність; Simulink; стабілізація.

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Сущенко Ольга Андреевна. Моделирование робастных инерциальных стабилизированных платформ

В статье рассматриваются основные проблемы разработки робастных систем управления движением платформ. Главной целью является рассмотрение структуры и особенностей алгоритмического, аппаратного и программного обеспечения, необходимого для эффективного моделирования инерциальных стабилизированных платформ с нагрузкой, предназначенной для эксплуатации на объектах широкого класса. Для решения этой проблемы используются методы робастной параметрической оптимизации и робастного структурного синтеза. Предложенные подходы к моделированию инерциальных стабилизированных платформ основываются на системе MATLAB. Представлен анализ соответствующего программного обеспечения. Описаны преимущества вычислительной среды MATLAB. Приведены основные этапы моделирования робастных систем управления. Представлен перечень моделей, необходимых для проектирования инерциальных стабилизированных платформ. Разработана модель объекта стабилизации, которая включает модели исполнительного механизма и измерительной системы. Приведены основные особенности моделирования инерциальных стабилизированных платформ. Показаны результаты моделирования, полученные в условиях параметрической неопределенности. Представленный пример основан на инерциальных стабилизированных платформах, предназначенных для наземных подвижных объектов, которые функционируют в сложных условиях реальной эксплуатации, сопровождающихся параметрическими возмущениями. Предложенные подходы к моделированию инерциальных стабилизированных платформ позволяют уменьшить время и затраты на проектирование. Представленные результаты могут быть полезными для инерциальных стабилизированных платформ, эксплуатируемых на подвижных объектах широкого класса.

Ключевые слова: инерциальные стабилизированные платформы; моделирование; робастность; Simulink; стабилизация.

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