THEORY AND METHODS OF SIGNAL PROCESSING

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STATISTICAL ANALYSIS OF OUTPUT SIGNAL IN SIGNAL PROCESSING SYSTEM FOR MULTIPLICATIVE COMPLEMENTARY GENERALIZED BINARY BARKER SEQUENCES

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Abstract—*Mathematical expressions, which are regular deterministic rules for synthesis of generalized binary Barker binary sequences, were proposed in the scientific literature. Sequences, which can be synthesized within the framework of these mathematical expressions, generalize the structural features of known Barker binary sequences. Sets of multiplicative complementary generalized binary Barker sequences allows obtaining a low equivalent peak sidelobe level after joint signal processing, which is* $1/N_{max}$, *where* N_{max} *is a maximum length of sequence in a set of sequences. One feature of considered sequences is that the multiplication of results of matched filtering of signal components forms a non-stationary output noise in the case of stationary input noise. This fact affects the noise immunity, detection and other characteristics in signal processing system. The aim of the article is a statistical analysis of output signal in signal processing system for multiplicative complementary generalized binary Barker sequences. The results of the analysis show that the value of variance of the output noise is larger in the main lobe of output signal in comparison with values of variance in its sidelobes. The structure of output signal in the case of signal processing of sets of generalized binary Barker sequences can be represented by some number of separately taken partial lobes, each of which is characterized by constant mean value and variance of signal. Statistical characteristics (probability density functions, mean values, variances) of output signal are also shown and analyzed in the article using an example of signal processing.*

Index Terms—Generalized binary Barker sequences; noise immunity; statistical analysis; signal analysis; signal processing; signal detection.

I. INTRODUCTION

The set of mathematical models, which are regular deterministic rules for synthesis of generalized binary Barker binary sequences, were proposed in [1], [2]. Sequences, which can be synthesized within the framework of the above-mentioned mathematical models, generalize the structural features of known Barker binary sequences and have the same structural characteristics. Unfortunately, correlation features of separately taken generalized Barker binary sequences are characterized by a high value of the peak sidelobe level of the autocorrelation function. However, sets of multiplicative complementary generalized binary Barker sequences allows obtaining a low equivalent peak sidelobe level after joint signal processing, which is $1/N_{\max}$, where $N_{\max}$ is a maximum length of sequence in a set of multiplicative complementary generalized binary Barker sequences [3]. In this sense, generalized binary Barker sequences supplement in the theory of signal processing both known Barker binary sequences and Golay complementary series (sequences) [4], which are additive complementary code constructions.

II. ANALYSIS OF PUBLICATIONS

The statistical analysis of signals, which are processed at different stages of a signal processing technique, is an important theoretical and practical step to provide necessary or high noise immunity, detection features, accuracy of range or time measurement etc.

Among the latest publications, which deal with different kinds of statistical analysis of signals in spread-spectrum and pulse compression signal processing techniques, can be distinguished the blind pulse compression technique, which uses an information on the statistical characteristics of the signal being processed for purposes of ultrasonic testing [5], the signal processing scheme with a principal component analysis as a post-processing technique for frequency-modulated thermal wave imaging, which allows obtaining a better detection sensitivity, resolution and signal-to-noise ratio for purposes of signal and data processing in infrared thermography [6], the radar signal processing technique based on the joint use of the linear frequency modulation signal and a signal with the random phase along with denoising processing, which allows to control the signal-to-noise ratio and

http://jrnl.nau.edu.ua/index.php/ESU, http://ecs.in.ua

obtain a high-quality detection characteristics [7], and other publications. Research efforts are continuing in the field of synthesis of sequences with good correlation properties, in particular the intensive scientific search for finding solutions to the low autocorrelation binary sequence problem [8], [9], e.g., the heuristic algorithm [10] for finding binary sequences with a very low autocorrelations.

III. PROBLEM STATEMENT

One feature of multiplicative complementary signal constructions, including generalized binary Barker sequences, is that the multiplication of results of matched filtering of signal components, which constituent a multiplicative complementary set, forms a significantly non-stationary output noise in the case of stationary input noise. This fact affects the noise immunity, detection and other characteristics in signal processing system. In this regard, the aim of the article is a statistical analysis of output signal in signal processing system for generalized binary Barker sequences, which are multiplicative complementary code constructions.

IV. ANALYSIS OF OUTPUT SIGNAL AFTER SIGNAL PROCESSING OF MULTIPLICATIVE COMPLEMENTARY GENERALIZED BINARY BARKER SEQUENCES

Let's analyze the output signal after signal processing for a set of multiplicative complementary signals based on generalized binary Barker sequences using an example of two sequences of lengths N_1 and N_2:

a = {-1; 1; 1} ($N_1 = 3$) and

b = {-1; 1;-1;-1; 1;-1;-1;-1;-1; 1; 1; 1} ($N_2 = 12$).

The duration $T = 24$ µs of each signal $a(t)$ and $b(t)$ is taken for the time range $t \in [24,\ 48]$ µs in the considered example. Therefore, the durations of elements are: $\tau_1 = T/N_1 = 8$ µs for signal $a(t)$, and $\tau_2 = T/N_2 = 2$ µs for signal $b(t)$.

Signals $a(t)$ and $b(t)$ at the signal to noise ratio $SNR = 20$ dB are shown in Fig. 1.

Note also that input $SNR = 10\log(P_S/P_\eta) = 10\log(1/\sigma_\eta^2)$ dB, where P_S – average power of signal $a(t)$ or $b(t)$, P_η – average power of input noise, σ_η^2 – variance of input noise.

The starting point and the end point of the time range, where components of input signals in signal processing scheme exist, are marked as "SP" and "EP" in Fig. 1 respectively ($SP_1 = 24$ µs and $EP_1 = 48$ µs). These marks are used in further analysis for related starting and end time points of signal components during signal processing.

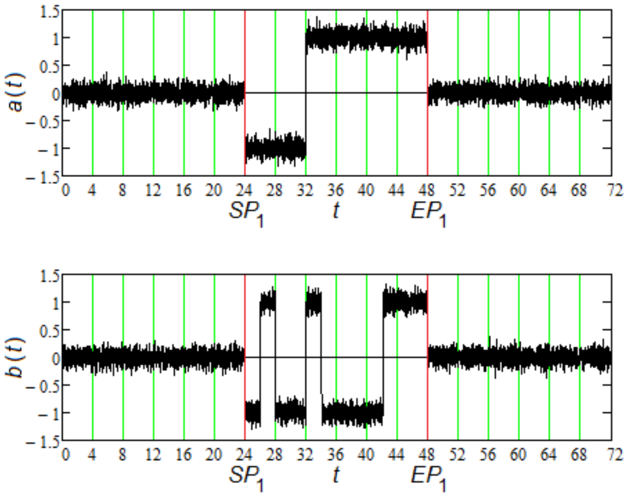

Fig. 1. Signals $a(t)$ and $b(t)$ at input $SNR = 20$ dB

Signal processing of $a(t)$ and $b(t)$ in a pulse compression technique is based on matched filtering. Matched filters for signals $a(t)$ and $b(t)$ are shown in Fig. 2 and Fig. 3 respectively.

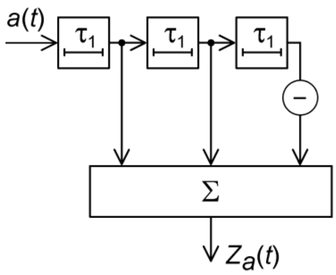

Fig. 2. Matched filter for signal $a(t)$

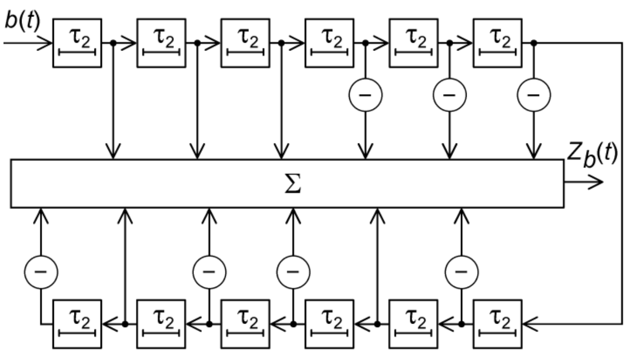

Fig. 3. Matched filter for signal $b(t)$

Signals $Z_a(t)$ and $Z_b(t)$ after matched filters (at input $SNR = 20$ dB) are shown in Fig. 4. Analysis of signal processing in matched filters lead to a conclusion that the variances of noise at the output of matched filters are $\sigma_{Za}^2 = N_1\sigma_\eta^2$ and $\sigma_{Zb}^2 = N_2\sigma_\eta^2$ for the matched filters for $a(t)$ and $b(t)$, respectively, if an input noise is a non-correlated process ("white" noise). It is also worth mentioning that in this case a noise at the output of matched filters is a stationary process in a sense that $\sigma^2_{Za} = \text{const}$, $\sigma^2_{Zb} = \text{const}$.

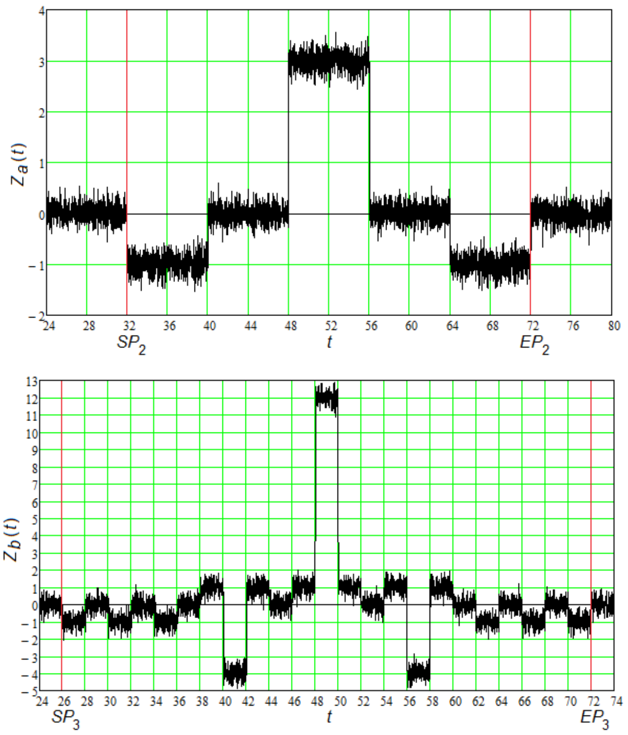

Fig. 4. Signals $Z_a(t)$ and $Z_b(t)$ after matched filters at input SNR = 20 dB

The signal to noise ratio at the outputs of matched filters are (in dB):

$$SNR_{Za} = 10\log\left(N_1^2/\sigma^2_{\eta Za}\right) = 10\log\left(N_1/\sigma^2_{\eta}\right) = 10\log N_1 + 10\log\left(1/\sigma^2_{\eta}\right) = G_a + SNR,$$

$$SNR_{Zb} = 10\log\left(N_2^2/\sigma^2_{\eta Zb}\right) = 10\log\left(N_2/\sigma^2_{\eta}\right) = 10\log N_2 + 10\log\left(1/\sigma^2_{\eta}\right) = G_b + SNR,$$

where $G_a = 10\log N_1 = 4.8$ dB is a processing gain of the matched filters for $a(t)$; $G_b = 10\log N_2 = 10.8$ dB is a processing gain of the matched filters for $b(t)$.

The starting and the end time points of the time range, where components of output signals $Z_a(t)$ and $Z_b(t)$ in signal processing scheme exist, are SP_2 = 32 μs and EP_2 = 72 μs for $a(t)$; SP_3 = 26 μs and EP_3 = 72 μs for $b(t)$. Components, which exist outside of these time ranges, are formed only by input noise and do not contain results of signal processing $a(t)$ and $b(t)$.

The property of multiplicative complementariness of generalized binary Barker sequences boils down to a multiplication of results of matched filtering, which are centered in main lobes, and it is performed for considered signals $a(t)$ and $b(t)$ by signal processing scheme in Fig. 5.

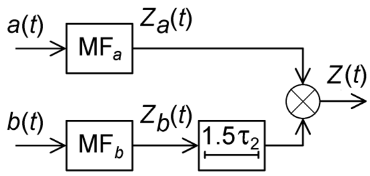

Fig. 5. Signal processing of results of matched filtering (MF_a and MF_b respectively) of signals $a(t)$ and $b(t)$

The signal $Z(t) = Z_a(t) Z_b(t - 1.5\tau_2)$ at the output of signal processing scheme (see Fig. 5) is shown in Fig. 6.

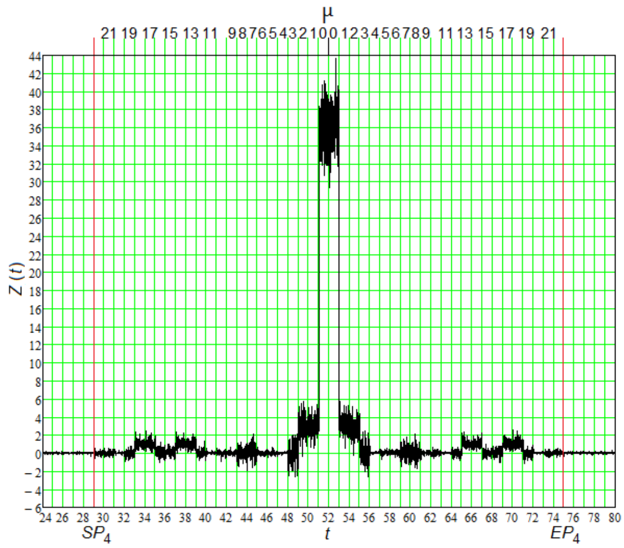

Fig. 6. Signal $Z(t)$ at the output of signal processing scheme and its structure at input SNR = 20 dB

The starting and the end time points of the time range, where components of output signal $Z(t)$ exist, are SP_4 = 29 μs and EP_2 = 75 μs. The structure of $Z(t)$ consists of the main lobe, which exists in the time range $t \in (51,\ 53)$ μs, and sidelobes, which exist in time ranges $t \in (29,\ 51)$ μs and $t \in (53,\ 75)$ μs. Taking into account the structure of input signals and signal processing scheme, the structure of $Z(t)$ contains 46 separately taken partial lobes, each of which has a duration 1 μs. These partial lobes are numbered in Fig. 6 by means of the index μ. The main lobe consists of 2 partial lobes with indices μ = 0, each group of sidelobes, i.e. at $t \in (29,\ 51)$ μs and at $t \in (53,\ 75)$ μs, consists of 22 partial lobes with indices $\mu = \overline{1,22}$. These explanations also mean that a maximum discretization interval of input and other signals must be $\Delta t = 1$ μs, or a minimum frequency of discretization must be $F_S = 1$ MHz, if digital signal processing is used for considered example.

It is also worth mentioning that the noise component in the signal $Z(t)$ is a non-stationary process, which is characterized by different values of variance of noise in each partial lobe.

A detailed analysis of these variances contains an evaluation of statistical characteristics of the result of multiplication of two independent random processes in each separately taken partial lobe.

The probability density function $p_Z(z)$ of a random variable $Z = Z_a Z_b$, where Z_a and Z_b are independent random variables, can be found using a mathematical expression (1) [11, p. 55].

$$p_Z(z) = \int_{-\infty}^{\infty} \frac{1}{|u|} p_{Za}\left(\frac{z}{u}\right) p_{Zb}(u)\, du, \qquad (1)$$

where $p_{Za}(z_a)$ and $p_{Zb}(z_b)$ are probability density functions of random variables Z_a and Z_b, respectively.

The mean value $m_Z(\mu)$ and the variance $\sigma_Z^2(\mu)$ of the signal $Z(t)$ in some separately taken partial lobe μ depend on mean values $m_{Za}(\mu)$, $m_{Zb}(\mu)$, and variances $\sigma_{Za}^2(\mu)$, $\sigma_{Zb}^2(\mu)$ of signals $Z_a(t)$ and $Z_b(t-1.5\tau_2)$, respectively, in this separately taken partial lobe [12, pp. 225–227]:

$$\begin{aligned} m_Z(\mu) &= m_{Za}(\mu) m_{Zb}(\mu), \\ \sigma_Z^2(\mu) &= \sigma_{Za}^2(\mu)\sigma_{Zb}^2(\mu) + m_{Za}^2(\mu)\sigma_{Zb}^2(\mu) \\ &\quad + m_{Zb}^2(\mu)\sigma_{Za}^2(\mu). \end{aligned} \qquad (2)$$

The probability density functions of signals $Z_a(t)$ and $Z_b(t-1.5\tau_2)$ in a separately taken partial lobe with index μ can be formalized by expressions (3) and (4), respectively.

$$p_{Za}(z_a \mid \mu) = \frac{1}{\sigma_{Za}(\mu)\sqrt{2\pi}} \exp\left\{-\frac{\left[z_a - m_{Za}(\mu)\right]^2}{2\sigma_{Za}^2(\mu)}\right\} = \frac{1}{\sigma_\eta \sqrt{2\pi N_1}} \exp\left\{-\frac{\left[z_a - m_{Za}(\mu)\right]^2}{2N_1\sigma_\eta^2}\right\}. \qquad (3)$$

$$p_{Zb}(z_b \mid \mu) = \frac{1}{\sigma_{Zb}(\mu)\sqrt{2\pi}} \exp\left\{-\frac{\left[z_b - m_{Zb}(\mu)\right]^2}{2\sigma_{Zb}^2(\mu)}\right\} = \frac{1}{\sigma_\eta \sqrt{2\pi N_2}} \exp\left\{-\frac{\left[z_b - m_{Zb}(\mu)\right]^2}{2N_2\sigma_\eta^2}\right\}. \qquad (4)$$

The values of $m_Z(\mu)$ and $\sigma_Z^2(\mu)$ are evaluated for each separately taken partial lobe in the Table I.

The values m_Z and σ_Z^2 for output components of $Z(t)$, which exist outside of the time range $t \in (SP_4, EP_4)$ are also indicated in the Table I and marked as values for a separately taken partial lobes "*NS*" ("no input signal"). These formally structured lobes also have a duration 1 μs, but formed only by input noise when components of input signals $a(t)$ and $b(t)$ do not exist.

It is worth mentioning that the output signal $Z(t)$ are characterized in some separately taken partial lobes by the same probability density $p_Z(z)$, m_Z and σ_Z^2:

1) sidelobes at $\mu = 1$ and $\mu = 2$ are statistically the same;

2) sidelobes at $\mu = 4$, $\mu = 11$, $\mu = 20$, and all lobes "*NS*" are statistically the same;

3) sidelobes at $\mu = 5$, $\mu = 6$, $\mu = 9$, and $\mu = 10$ are statistically the same;

4) sidelobes at $\mu = 7$ and $\mu = 8$ are statistically the same;

5) sidelobes at $\mu = 12$, $\mu = 15$, $\mu = 16$, and $\mu = 19$ are statistically the same;

6) sidelobes at $\mu = 13$, $\mu = 14$, $\mu = 17$, and $\mu = 18$ are statistically the same;

7) sidelobes at $\mu = 21$ and $\mu = 22$ are statistically the same.

Due to this fact, some sidelobes can be grouped during the statistical noise immunity analysis in relevant statistical models.

Taking into account (1), (3), and (4), the probability density function of signal $Z(t)$ in a separately taken partial lobe with index μ can be formalized by expression:

$$p_Z(z \mid \mu) = \frac{1}{2\pi\sigma_{Za}(\mu)\sigma_{Zb}(\mu)} \times \int_0^{\infty} \frac{1}{u} \left\{ \exp\left[-\frac{\left(\frac{z}{u} - m_{Za}(\mu)\right)^2}{2\sigma_{Za}^2(\mu)} - \frac{\left(u - m_{Zb}(\mu)\right)^2}{2\sigma_{Zb}^2(\mu)}\right] + \exp\left[-\frac{\left(-\frac{z}{u} - m_{Za}(\mu)\right)^2}{2\sigma_{Za}^2(\mu)} - \frac{\left(-u - m_{Zb}(\mu)\right)^2}{2\sigma_{Zb}^2(\mu)}\right] \right\} du$$

$$= \frac{1}{2\pi\sigma_\eta^2\sqrt{N_1 N_2}} \int_0^\infty \frac{1}{u} \left\{ \exp\left[-\frac{\left(\frac{z}{u} - m_{Za}(\mu)\right)^2}{2N_1\sigma_\eta^2} - \frac{\left(u - m_{Zb}(\mu)\right)^2}{2N_2\sigma_\eta^2} \right] + \exp\left[-\frac{\left(-\frac{z}{u} - m_{Za}(\mu)\right)^2}{2N_1\sigma_\eta^2} - \frac{\left(-u - m_{Zb}(\mu)\right)^2}{2N_2\sigma_\eta^2} \right] \right\} du.$$

TABLE I. RESULTS OF STATISTICAL ANALYSIS OF OUTPUT SIGNAL FOR CONSIDERED EXAMPLE OF SIGNAL PROCESSING

μ	$m_{Za}(\mu)$	$\sigma_{Za}^2(\mu)$	$m_{Zb}(\mu)$	$\sigma_{Zb}^2(\mu)$	$m_Z(\mu)$	$\sigma_Z^2(\mu)$
0	$N_1 = 3$	$N_1\sigma_\eta^2$	$N_2 = 12$	$N_2\sigma_\eta^2$	$N_1N_2 = 36$	$N_1N_2\sigma_\eta^2(\sigma_\eta^2 + N_1 + N_2)$
1	$N_1 = 3$	$N_1\sigma_\eta^2$	1	$N_2\sigma_\eta^2$	3	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + N_1N_2 + 1)$
2	$N_1 = 3$	$N_1\sigma_\eta^2$	1	$N_2\sigma_\eta^2$	3	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + N_1N_2 + 1)$
3	$N_1 = 3$	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_1N_2\sigma_\eta^2(\sigma_\eta^2 + N_1)$
4	0	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_1N_2\sigma_\eta^4$
5	0	$N_1\sigma_\eta^2$	1	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 1)$
6	0	$N_1\sigma_\eta^2$	1	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 1)$
7	0	$N_1\sigma_\eta^2$	-4	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 16)$
8	0	$N_1\sigma_\eta^2$	-4	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 16)$
9	0	$N_1\sigma_\eta^2$	1	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 1)$
10	0	$N_1\sigma_\eta^2$	1	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 1)$
11	0	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_1N_2\sigma_\eta^4$
12	-1	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_2\sigma_\eta^2(N_1\sigma_\eta^2 + 1)$
13	-1	$N_1\sigma_\eta^2$	-1	$N_2\sigma_\eta^2$	1	$\sigma_\eta^2(N_1N_2\sigma_\eta^2 + N_1 + N_2)$
14	-1	$N_1\sigma_\eta^2$	-1	$N_2\sigma_\eta^2$	1	$\sigma_\eta^2(N_1N_2\sigma_\eta^2 + N_1 + N_2)$
15	-1	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_2\sigma_\eta^2(N_1\sigma_\eta^2 + 1)$
16	-1	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_2\sigma_\eta^2(N_1\sigma_\eta^2 + 1)$
17	-1	$N_1\sigma_\eta^2$	-1	$N_2\sigma_\eta^2$	1	$\sigma_\eta^2(N_1N_2\sigma_\eta^2 + N_1 + N_2)$
18	-1	$N_1\sigma_\eta^2$	-1	$N_2\sigma_\eta^2$	1	$\sigma_\eta^2(N_1N_2\sigma_\eta^2 + N_1 + N_2)$
19	-1	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_2\sigma_\eta^2(N_1\sigma_\eta^2 + 1)$
20	0	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_1N_2\sigma_\eta^4$
21	0	$N_1\sigma_\eta^2$	-1	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 1)$
22	0	$N_1\sigma_\eta^2$	-1	$N_2\sigma_\eta^2$	0	$N_1\sigma_\eta^2(N_2\sigma_\eta^2 + 1)$
NS	0	$N_1\sigma_\eta^2$	0	$N_2\sigma_\eta^2$	0	$N_1N_2\sigma_\eta^4$

The integral in the expression for $p_Z(z\,|\,\mu)$ cannot be evaluated and represented analytically in general case, therefore $p_Z(z\,|\,\mu)$ can be represented as tabulated function for each set of parameters $\{m_{Za}(\mu),\ m_{Zb}(\mu),\ \sigma_\eta^2\}$. Results of evaluation of $p_Z(z\,|\,\mu)$ at $\sigma_\eta^2 = 0.2$ (input $SNR = 7$ dB) for some separately taken partial lobes from the Table I are shown in Fig. 7.

Analysis of probability density functions for output signal $p_Z(z\,|\,\mu)$ at $\mu = 0$, $\mu = 7$ and $\mu = 13$ (Fig. 7) shows that $p_Z(z\,|\,\mu)$ can be characterized by very different forms at stationary input noise.

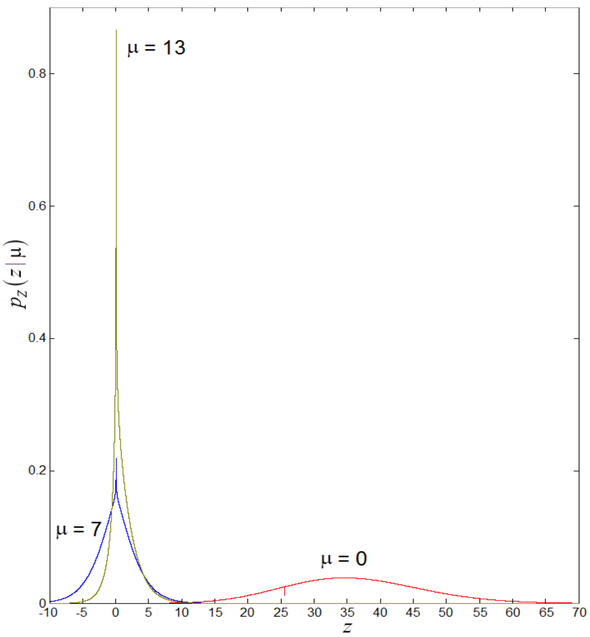

Fig. 7. Results of evaluation of $p_Z(z\,|\,\mu)$ at $\sigma_\eta^2 = 0.2$ (input *SNR* = 7 dB) for some partial lobes

V. CONCLUSIONS

The multiplication of results of matched filtering of multiplicative complementary binary Barker sequences forms a nonstationary output noise in the case of stationary input noise. This fact might significantly affect the noise immunity, detection and other characteristics in signal processing system.

The value of variance of this noise is larger in the main lobe of output signal in comparison with the values of variance in its sidelobes (see the Table I).

The structure of output signal in the case of signal processing of multiplicative complementary binary Barker sequences can be represented by some number of separately taken partial lobes (46 in the considered example), each of which is characterized by constant mean value and variance of signal.

The probability density function of values of output signal can be represented by the probability density function of two independent random variables. This probability density function cannot be evaluated analytically in general case, and therefore it can be used for further noise immunity analysis as tabulated function for each set of parameters within each separately taken partial lobe.

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О. Г. Голубничий. Статистичний аналіз вихідного сигналу у системі обробки сигналів на основі мультиплікативно комплементарних узагальнених бінарних послідовностей Баркера

Математичні вирази, які є регулярними детермінованими правилами синтезу узагальнених бінарних послідовностей Баркера, розглянуто в науковій літературі. Послідовності, які можна синтезувати в рамках цих математичних виразів, узагальнюють структурні особливості відомих бінарних послідовностей Баркера. Системи мультиплікативно комплементарних узагальнених бінарних послідовностей Баркера дозволяють отримати мале еквівалентне абсолютне значення максимального рівня бічних пелюсток сигналу після сумісної обробки сигналів, який дорівнює $1/N_{max}$, де N_{max} – максимальна довжина послідовності в системі послідовностей. Однією з особливостей розглянутих послідовностей є те, що множення результатів узгодженої фільтрації компонентів сигналу формує нестаціонарний шум на виході системи обробки сигналів у випадку дії на її вході стаціонарного шуму. Цей факт впливає на завадостійкість, характеристики виявлення та інші характеристики, пов'язані з системою обробки сигналів. Метою статті є статистичний аналіз вихідного сигналу в системі обробки сигналів на основі мультиплікативно комплементарних узагальнених бінарних послідовностей Баркера. Результати аналізу показують, що значення дисперсії шуму на виході системи обробки сигналів більше в головній пелюстці вихідного сигналу у порівнянні із значеннями дисперсії у його бічних пелюстках. Структура сигналу на виході системи обробки сигналів у випадку обробки узагальнених бінарних послідовностей Баркера може бути представлена певною кількістю виокремлених частинних пелюсток, кожна з яких характеризується сталими значеннями математичного сподівання та дисперсії сигналу. У статті також представлено та проаналізовано статистичні характеристики (щільності розподілу ймовірностей, математичні сподівання та дисперсії) вихідного сигналу з використанням прикладу обробки сигналів.

Ключові слова: узагальнені бінарні послідовності Баркера; завадостійкість; статистичний аналіз; аналіз сигналів; обробка сигналів; виявлення сигналів.

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А. Г. Голубничий. Статистический анализ выходного сигнала в системе обработки сигналов на основе мультипликативно комплементарных обобщённых бинарных последовательностей Баркера

Математические выражения, являющиеся регулярными детерминированными правилами синтеза обобщенных бинарных последовательностей Баркера, рассмотрены в научной литературе. Последовательности, которые можно синтезировать в рамках этих математических выражений, обобщают структурные особенности известных бинарных последовательностей Баркера. Системы мультипликативно комплементарных обобщенных бинарных последовательностей Баркера позволяют получить малое эквивалентное абсолютное значение максимального уровня боковых лепестков сигнала после совместной обработки сигналов, равное $1/N_{max}$, где N_{max} – максимальная длина последовательности в системе последовательностей. Одной из особенностей рассматриваемых последовательностей является то, что умножение результатов согласованной фильтрации компонентов сигнала формирует нестационарный шум на выходе системы обработки сигналов в случае воздействия на ее входе стационарного шума. Этот факт влияет на помехоустойчивость, характеристики обнаружения и другие характеристики, связанные с системой обработки сигналов. Целью статьи является статистический анализ выходного сигнала в системе обработки сигналов на основе мультипликативно комплементарных обобщенных бинарных последовательностей Баркера. Результаты анализа показывают, что значение дисперсии шума на выходе системы обработки сигналов больше в главном лепестке выходного сигнала по сравнению со значениями дисперсии в его боковых лепестках. Структура сигнала на выходе системы обработки сигналов в случае обработки обобщенных бинарных последовательностей Баркера может быть представлена определенным количеством выделенных частных лепестков, каждый из которых характеризуется постоянными значениями математического ожидания и дисперсии сигнала. В статье также представлены и проанализированы статистические характеристики (плотности распределения вероятностей, математические ожидания и дисперсии) выходного сигнала с использованием примера обработки сигналов.

Ключевые слова: обобщённые бинарные последовательности Баркера; помехоустойчивость; статистический анализ; анализ сигналов; обработка сигналов; обнаружение сигналов.

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