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# DESIGN OF ROBUST PRECISION ATTITUDE AND HEADING REFERENCE SYSTEMS FOR MARINE VEHICLES

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***Abstract**—The paper deals with basic principles of the design of robust systems assigned for stabilization of navigation sensors of the precision attitude and heading reference systems operated on the marine vehicles. The possibilities of the mathematical description of coordinate disturbances for the marine vehicles are considered. The basic features of the robust parametric optimization and robust structural synthesis are described. Results of modeling in one of mode of marine vehicle operation are represented. The important problem of keeping accuracy in difficult conditions of real operation is solved. The obtained results can be useful for gyro-stabilized platforms used on moving vehicles of the wide class.*

**Index Terms**—Attitude and heading reference systems; parametrical optimization; structural synthesis; robust systems; sea irregularities.

## I. INTRODUCTION

Operation of precision attitude and heading reference systems is implemented in conditions of uncertainties caused by both inaccuracies of the mathematical description of the real system and influence of the parametrical and coordinate disturbance. Mainly, the systems designed for operation at marine vehicles are subjected to influence of the disturbances caused by sea irregular waves. The modern approach to these systems design is creation of the robust systems able to operate in conditions of both the parametrical structured and the external (coordinate) disturbances.

The precision attitude and heading reference systems of marine vehicles must carry out following functions:

1) preliminary stabilization of navigation sensors (accelerometers) in the mode of preliminary alignment in the horizontal plane (coarse levelling);

2) precision stabilization of navigation sensors (gyroscopes and accelerometers) in the mode of precision alignment in the horizontal plane (precision levelling);

3) initial alignment in the meridian plane;

4) joint stabilization and precision determination of heading in the mode of the gyroscopic compass.

The precision attitude and heading reference system can be developed based on the triaxial gimballed platform. Such a system includes two dynamically tuned gyros, three accelerometers and servo systems. The gyroscopes carry out functions of the vertical gyroscope and directional gyroscope respectively. Such a combination provides determination of the vehicle attitude and heading.

Design of attitude and heading reference system by means of such a scheme provides simulation of the true horizon plane. The system becomes undisturbed by external accelerations due to usage of the integral correction. This is important for operation of the system in conditions of sea irregular waves and the marine vehicle manoeuvring.

## II. REVIEW OF PREVIOUS RESEARCHES

Importance of development of the high precision gimballed heading and attitude reference systems is grounded in [1]. Basic principles of design of robust systems including the robust structural synthesis are represented in many textbooks and papers, for example, [2, 3]. Concepts of design of robust control systems of aircraft of the wide class based on the mixed $H_2/H_\infty$-approach, which simultaneously takes into consideration requirements given to accuracy and robustness of the designed system, are considered in [4]. Some approaches to design of the system for gyroscopic compass stabilization are given in [5]. Development of the appropriate design procedures of the robust optimization in the area of the high precision autonomous attitude and heading reference systems for marine vehicles stays a problem, which requires the further research.

## III. PROBLEM STATEMENT

The modern phase of marine vehicles development requires an improvement of systems of navigation and motion control. One of the most important problems is keeping high accuracy performances in conditions of parametrical and coordinate disturbances caused by the sea irregularities. To solve this problem it is possible using robust gyro-stabilized systems able to provide the stabilization of information and measuring devices in conditions of both internal parametrical an coordinate disturbances. It should be noted that nowadays usage of gimballed gyro-stabilized platforms is especially efficient for precision autonomous navigation systems operated on the marine moving vehicles.

One of the modern approaches to determination of criteria of the robust optimization is using the $H_\infty$-norm of the function of the complementary sensitivity of the closed loop system. To improve efficiency of the robust optimization is possible using the mixed $H_2/H_\infty$-optimization. It is known that the $H_2$-norm of the function of the sensitivity of the closed loop system characterizes its accuracy [8].

Design of robust systems for the gyroscopic stabilization of information and measuring devices is carried out in two directions including the modernization of operating and design of the perspective systems.

The modernization in conditions of uncertainty it is convenient to implement by means of the robust parametric optimization. The design of new perspective stabilization systems requires using the robust structural synthesis. The presented paper deals with approaches to solving both problems.

The goal of the research is representation of principles of the robust optimization of a system assigned for inertial stabilization of information and measuring devices operated on the marine moving vehicles.

## IV. MATHEMATICAL MODELS OF DISTURBED ATTITUDE AND HEADING REFERENCE SYSTEMS

The robust optimization of attitude and heading reference systems for the marine vehicles includes the following steps [6].

1) Statement of problem of the optimal robust synthesis.

2) Creation of the full mathematical description of the system, which takes into consideration all nonlinearities inherent to real systems in the full measure.

3) Creation of the linearized mathematical model in the state space.

4) Analysis of requirements given to systems of the considered type and forming the appropriate objective and penalty functions.

5) Creation of the way to simulate the coordinate disturbances taking into consideration features of motion of the vehicle, on which the designed system is operated.

6) The choice of an optimization method.

7) Development of an algorithm of robust system synthesis.

8) Modeling and analysis of obtained results.

The mixed $H_2/H_\infty$-approach is the most efficient In the case of the parametric optimization, it is convenient to use the mixed $H_2/H_\infty$-approach. One of the modern approaches to the structural robust optimization is $H_\infty$-synthesis.

Implementation of both the robust parametric optimization and the robust structural synthesis can be realized by means of automated means of the optimal design such as MATLAB software, which includes toolboxes assigned for the design of robust systems.

It should be noted that the design of robust control systems is carried out in two stages. Firstly, the parametric or structural synthesis of a robust system by means of the linearized models in the state space is carried out. Secondly, checking a synthesized system is implemented. The goal of this checking is to analyze requirements given to the designed system. In this case, it is necessary to use the full mathematical models, which take into consideration nonlinearities inherent to real systems and also models of disturbances typical for conditions of real operation of the researched system (sea irregularities). If the technical requirements given to systems are not satisfied, the optimization procedure must be repeated after changing initial conditions and/or weighting coefficients as components of the optimization criterion.

Systems of the researched type can operate in some modes such as coarse levelling, precision levelling, and gyroscopic compass. These modes differ in types of sensors and control features.

Basis peculiarities of nonlinear models of the gyro-stabilized system can be researched on the example of a model in the mode of the coarse levelling. The basic function of the system in this mode is the stabilization of the platform based on information of accelerometers. The full mathematical model in this mode includes the model of a platform including dynamics and kinematics equations and also accelerometer models [6]:

– equations of platform dynamics –

$$\begin{aligned}\dot{\omega}_{xp} &= [-(J_z - J_y)\omega_{yp}\omega_{z\text{ï}} - (f_x + k_7)\omega_{xp} \\ &\qquad - k_1\delta_{xp} - k_5(-\delta_{xp} + k_3\beta) + M_0 \operatorname{sign}\omega_{x0}] / J_x, \\ \dot{\omega}_{yp} &= [-(J_x - J_z)\omega_{xp}\omega_{zp} - (f_y + k_8)\omega_{yp} - k_2\delta_{yp} \\ &\qquad - k_6(-\delta_{yp} + k_4\alpha) + M_0 \operatorname{sign}\omega_{y0}] / J_y, \\ \dot{\omega}_{zp} &= [-(J_y - J_x)\omega_{xp}\omega_{yp} - (f_y + k_9)\omega_{zp} \\ &\qquad + M_0 \operatorname{sign}\omega_{z0}] / J_z;\end{aligned} \tag{1}$$

– equations of accelerometers:

$$\begin{aligned}\dot{\delta}_{xp} &= (-\delta_{xp} + k_4\beta) / T_{\mathrm{A}}, \\ \dot{\delta}_{yp} &= (-\delta_{yp} + k_5\alpha) / T_{\mathrm{A}};\end{aligned} \tag{2}$$

– equations of platform kinematics:

$$\begin{aligned}
&\dot{\alpha} = (\omega_{xp}\sin\gamma + \omega_{yp}\cos\gamma)/\cos\beta,\\
&\dot{\gamma} = \omega_{zp} + \mathrm{tg}\beta(\omega_{xp}\sin\gamma + \omega_{yp}\cos\gamma),\\
&\dot{\beta} = \omega_{xp}\cos\gamma - \omega_{yp}\sin\gamma,
\end{aligned} \tag{3}$$

where $\omega_{xp}, \omega_{yp}, \omega_{zp}$ are angular rates of the platform onto its proper axes; $J_x, J_y, J_z$ are inertia moments of the platform with navigation sensors; $f_x, f_y, f_z$ are moments of the viscous friction; Ì $_0$ is the moment of the dry friction; $\omega_{x0}, \omega_{y0}, \omega_{z0}$ are projections of the external angular rate; $\delta_{xp}, \delta_{yp}$ are signals of accelerometers; $T_A$ is the time constant of the pendulum accelerometer; $\alpha, \beta, \gamma$ are angles of platform rotations; $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9$ are coefficients of control laws.

To develop a mathematical model in the state space it is necessary to carry out linearization of the kinematics equations taking into consideration smallness of the angles of platform rotation. Moreover, it is possible to neglect a difference between the platform axial moments. Taking into account (1) – (3), the linearized model of the gyro-stabilized system in the state space becomes

$$A = \begin{bmatrix}
-\dfrac{f_x + k_7}{J_x} & 0 & 0 & -\dfrac{k_1 - k_5/T_A}{J_x} & 0 & -\dfrac{k_3 k_5/T_A}{J_x} & 0 & 0\\
0 & -\dfrac{f_x + k_8}{J_y} & 0 & 0 & -\dfrac{k_2 - k_6/T_A}{J_y} & 0 & -\dfrac{k_4 k_6/T_A}{J_x} & 0\\
0 & 0 & -\dfrac{f_x + k_9}{J_z} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\dfrac{1}{T} & 0 & \dfrac{k_3}{T} & 0 & 0\\
0 & 0 & 0 & 0 & -\dfrac{1}{T_A} & 0 & \dfrac{k_4}{T_A} & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$B = \begin{bmatrix}
\dfrac{H_x}{J_x} & 0 & 0\\
0 & \dfrac{H_y}{J_y} & 0\\
0 & 0 & \dfrac{H_z}{J_z}\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0
\end{bmatrix}, \quad
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad
D = \begin{bmatrix}
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0
\end{bmatrix},$$

where $H_x, H_y, H_z$ are the kinematical moments of the platform.

For the mathematical description of disturbances caused by the sea irregularities, it is necessary to use relationships for the spectral density based on using spectra of Neumann, Derbyshire, and Bretschneider [8]. It should be noted that these spectra do not include low frequencies. At the same time, the bandwidth of the marine vehicles is located just in this area. Therefore, for the system of the researched type it is convenient to use spectra of Rachmanin and Firsov [9]. The transfer function of the forming filter obtained on the basis of the known expressions for the spectral density taking into consideration an angle of the slope can be represented in the following form

$$W_f(j\omega) = 2\sqrt{\frac{D_r\mu(\mu^2 + \lambda^2)}{\pi}}\,\frac{j\omega}{g} \cdot \frac{\sqrt{\mu^2 + \lambda^2}}{(j\omega)^2 + 2\lambda j\omega + \mu^2 + \lambda^2}, \tag{4}$$

here $D_r$ is the variance of the wave ordinates; $\mu, \lambda$ are parameters of the sea irregularities; $g$ is the gravity acceleration.

Design of the system for stabilizing navigation sensors requires solving two basic problems. Firstly, it is necessary to create control laws of navigation contours. Secondly, it is necessary to create controllers operating in stabilization contours.

Taking into consideration the great experience of design of similar navigation systems, the first problem can be solved by means of the robust parametrical optimization. In this case, the structure of control laws is believed to be known. Therefore, carrying out optimization allows determining coefficients of control laws. The second problem foresees creation of the robust controllers in the stabilization contours by means of the robust structural synthesis.

## V. PARAMETRIC ROBUST OPTIMIZATION

The parametrical optimization it is convenient to implement using the combined criterion on the basis of $H_2$, $H_\infty$-norms of the functions of sensitivity, which take into consideration performance and robustness of the system [3]. Influence of every component of the combined quality criterion can be regulated by means of the weighting coefficients depending on features of the system

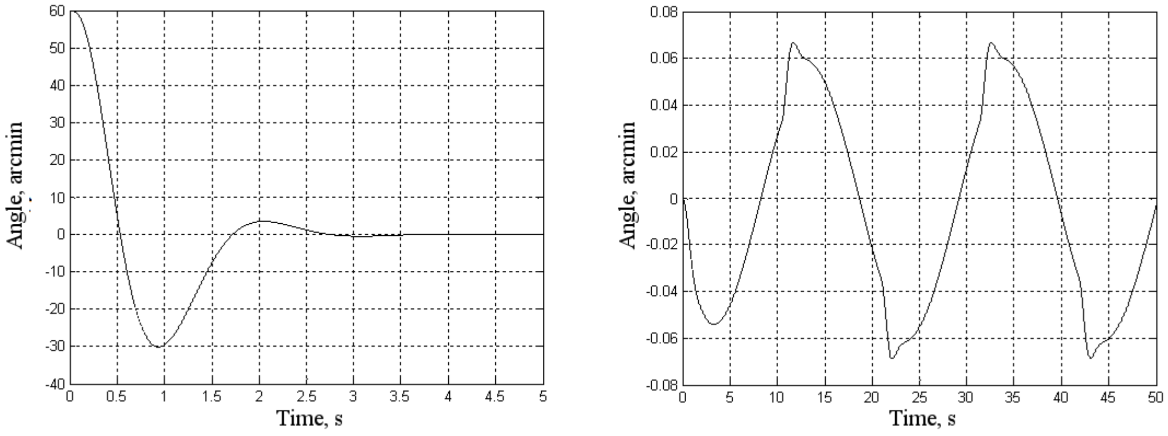
$$\begin{aligned} J_{H_2/H_\infty} &= \lambda_2^{\text{nom d}} \| \Phi_s(K, j\omega) \|_2^{\text{nom d}} + \\ &\lambda_2^{\text{nom s}} \| \Phi_s(K, j\omega) \|_2^{\text{nom s}} + \\ &\lambda_\infty^{\text{nom}} \| \Phi_c(K, j\omega) \|_\infty^{\text{nom}} + PF, \end{aligned} \qquad (5)$$

where $\| \cdot \|_2^{\text{nom d}}$, $\| \cdot \|_2^{\text{nom s}}$, are $H_2$-norms of the sensitivity functions of the nominal and disturbed systems, $\| \cdot \|_\infty^{\text{nom}}$ is $H_\infty$-norm of the complementary sensitivity function; $\lambda_2^{\text{nom d}}, \lambda_2^{\text{nom s}}, \lambda_\infty^{\text{nom}}$ are the weighting coefficients of the appropriate norms; $PF$ is the penalty function, which provides carrying out conditions of system stability during optimization process; $K$ is the vector of coefficients of control laws. In this case, the problem statement of the robust optimization becomes

$$K^* = \arg \inf_{K \in D} J_{H_2/H_\infty}(K),$$

where the optimization criterion $J_{H_2/H_\infty}$ is determined by the expression (5).

Results of modeling of the designed system in the mode of coarse levelling are represented in Fig. 1. Disturbances have been given by means of the expression (4). As result of the robust parametrical optimization, the vector of coefficients of the optimal control laws has been obtained $k_1 = k_2 = 5$, $k_3 = k_4 = 4$, $k_5 = k_6 = 0.2$, $k_7 = k_8 = k_9 = 1.5$.



a b

Fig. 1. Transient processes of the angle of pitch in the mode of coarese levelling for nominal (a) and optimized (b) systems

## VI. STRUCTURAL ROBUST SYNTHESIS

The robust structural synthesis is based on solving two Riccati equations, checking some conditions and minimization of $H_\infty$-norm of the system's function of the mixed sensitivity. The researched system includes the plant and controller [10]. The modern approach to the robust structural synthesis foresees providing desired frequency characteristics of the designed system by means of forming of the augmented plant using the weighting transfer functions. The optimization criterion represents the $H_\infty$-norm of the mixed function of the sensitivity [1], [2]

$$J_{H_\infty} = \left\| \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \right\|_\infty ,$$

where $W_1, W_2, W_3$ are the weighting transfer functions; $S, R, T$ are functions of sensitivity by reference and control and the complementary sensitivity function.

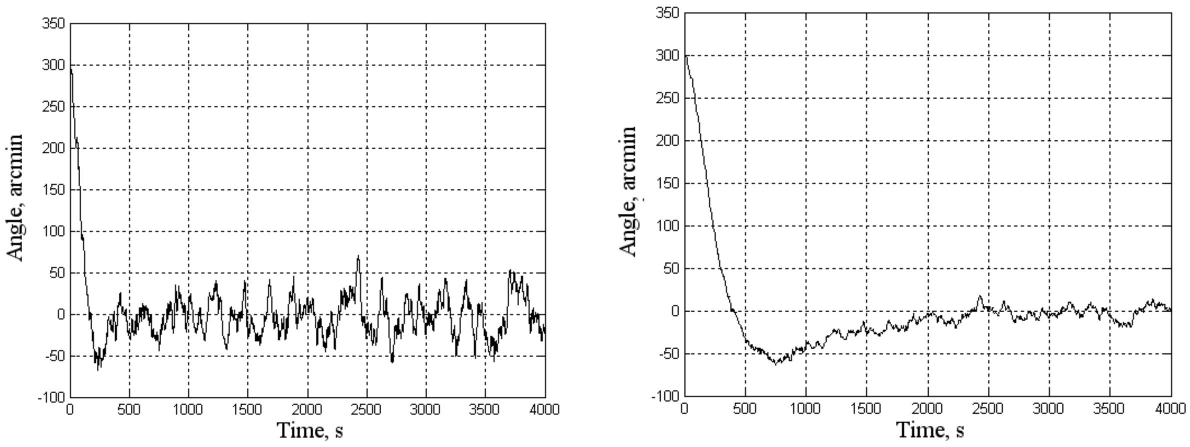
Results of modeling of the robust synthesized system are represented in Fig. 2.

Structure of the synthesized controller is represented in the state space by the quadruple of matrices:

$$\mathbf{A} = \begin{bmatrix} 0.1871 & -0.1805 & 0.2126 & 0.372 & -0.0496 & 0.295 & -0.384 \\ -0.29 & 0.196 & 0.433 & 0.168 & 0.078 & -0.065 & -0.56 \\ 1.865 & -1.004 & -0.613 & -0.201 & -0.591 & -1.547 & 0.029 \\ -0.126 & 0.329 & 0.019 & 0.067 & 0.888 & 0.698 & -0.153 \\ -9.389 & 3.538 & -0.405 & -0.678 & 4.466 & 8.172 & -1.434 \\ 2.805 & -1.213 & -0.02 & 0.527 & -1.43 & -1.649 & 0.581 \\ -4.814 & 1.71 & 0.142 & -0.464 & 2.332 & 4.278 & -0.434 \end{bmatrix},$$

$$\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} 0.115 & -3.957 & 26.01 & -6.504 & -132.2 & 35.04 & -67.35 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0.0234 & -0.012 & 0.0036 & 0.0025 & -0.012 & -0.0219 & 0.0061 \end{bmatrix}, \quad \mathbf{D} = [-0.375].$$



a b

Fig. 2. Results of modeling of the robust controller: the transient process by the pitch (a); the dynamic error (b)

The presented modeling results prove the comparatively high speed of operation and stabilization accuracy of the designed system and also acceptable dynamic error.

## VII. CONCLUSIONS

The basic principles of the robust parametrical optimization and robust structural synthesis of the precision attitude and heading reference systems operated on the marine vehicles are represented. The basic types of mathematical models necessary for design of robust systems are characterized. Efficiency of suggested approaches is proved by modeling results. Results of researches can be spread on attitude and heading reference systems operated on moving vehicles of the different types.

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**О. А. Сущенко. Синтез робастних високоточних систем визначення просторового положення для морських рухомих об’єктів**

У статті розглянуто основні принципи синтезу робастних систем, призначених для стабілізації навігаційних датчиків, експлуатованих на морських рухомих об’єктах. Розглянуто можливості математичного опису координатних збурень морських рухомих об’єктів. Описано основні особливості робастної параметричної оптимізації і робастного структурного синтезу. Представлено результати моделювання в одному з режимів функціонування морського рухомого об’єкта. Вирішено важливу проблему збереження точності в складних умовах реальної експлуатації. Отримані результати можуть бути корисними для гіростабілізованих платформ, що використовуються для рухомих об’єктів широкого класу.

**Ключові слова**: система визначення просторової орієнтації; параметрична оптимізація; структурний синтез; робастні системи; морське хвилювання.

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Напрям наукової діяльності: системи стабілізації інформаційно-вимірювальних пристроїв, експлуатованих на рухомих об’єктах широкого класу.
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**О. А. Сущенко. Синтез робастных высокоточных систем определения пространственного положения для морских подвижных объектов**

В статье рассмотрены основные принципы синтеза робастных систем, предназначенных для стабилизации навигационных датчиков, эксплуатируемых на морских подвижных объектах. Рассмотрены возможности математического описания координатных возмущений морских подвижных объектов. Описаны основные особенности робастной параметрической оптимизации и робастного структурного синтеза. Представлены результаты моделирования в одном из режимов функционирования морского подвижного объекта. Решена важная проблема сохранения точности в сложных условиях реальной эксплуатации. Полученные результаты могут быть полезными для гиростабилизированных платформ, используемых для подвижных объектов широкого класса.

**Ключевые слова**: система определения пространственной ориентации; параметрическая оптимизация, структурный синтез; робастные системы; морское волнение.

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