THEORY AND METHODS OF SIGNAL PROCESSING

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CROSS-CORRELATION ANALYSIS OF SETS OF GENERALIZED BINARY BARKER SEQUENCES

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Abstract—*The effectiveness of many signal processing techniques and their practical value, which is particularly prominent in accuracy of detection and measurement, range and time resolution etc., depends on correlation properties of signals being processed. The article is focused on a study of correlation properties of generalized binary Barker sequences, namely cross-correlation between signal components in sets based on generalized binary Barker sequences. These sets of sequences provide low peak sidelobe level after their joint signal processing (multiplicative complementariness), but their cross-correlation characteristics can also have an impact upon the quality of radio and signal processing systems. There are 5 sets of generalized binary Barker sequences with different structures that were analyzed in the article. The presented results have established that signal components based on generalized binary Barker sequences are characterized by a relatively high level of cross-correlation, which can be up to a typical value 0.25 between different signal components in a set consisting of not more than 8 sequences. This fact restricts the application of generalized binary Barker sequences in some techniques (e.g., CDMA) due to the impossibility of separation of signal components or it requires enhancement of these techniques in order to take account of cross-correlation between signal components. Sets of generalized binary Barker sequences with the lowest maximum absolute values of cross-correlation were also identified and shown in the article.*

Index Terms—Generalized binary Barker sequences; correlation properties; cross-correlation; signal processing; signal detection; signal analysis.

I. INTRODUCTION

The effectiveness of many signal processing techniques and their practical value, which is particularly prominent in, e.g., detection characteristics, accuracy of detection and measurement, range or time resolution etc., depends on correlation properties of signals being processed.

Scientific literature and signal processing practice identify a few different kinds of correlations of sequences (signals) and their sets (systems), namely, aperiodic autocorrelation (e.g., [1]), periodic autocorrelation (e.g., [2]), aperiodic cross-correlation (e.g., [3]), and periodic cross-correlation (e.g., [4]) functions, the forms and features of which are important and critical for achieving different goals in the field of signal processing. It should be noted that above-indicated terms differ from some other commonly used ones in mathematics (e.g., “correlation”, “covariance” etc.), but frequently used with some variations in engineering.

The article is focused on a study of cross-correlation between signal components in sets of generalized binary Barker sequences [5], [6], which are characterized by a good equivalent (in the result of signal processing) aperiodic autocorrelation [7].

II. ANALYSIS OF PUBLICATIONS

A system of deterministic and regular mathematical models, which allow synthesizing of binary sequences with structural features like ones of well-known binary Barker sequences, were suggested in [5]. Because of their properties, sequences, which synthesized by means of these mathematical models (i.e. generation rules), may be taken for a kind of generalization of binary Barker sequences. Generation rules and structure features of generalized binary Barker sequences, which the article deals with, were described in [6]. Their autocorrelation functions, complete mathematical expressions for them, and analysis of their features (e.g., regularity, existence of periodic components) were presented in [8].

It was shown in [7] that generalized binary Barker sequences allow forming sets of multiplicative complementary signal constructions, which provide after multiplication of results of matched filtering a signal with a narrow central main lobe (its value equals to a result of multiplication of all lengths of sequences in the set consisting of K ones), and a normalized maximum absolute sidelobe level equals to $1/N_K$, where N_K is the maximum length of sequence in a set of generalized binary Barker sequences.

There are also known other different sequences and their sets, recently proposed in the literature and which are valuable with their correlation properties. It is worth mentioning that among such sequences and relevant synthesis approaches and algorithms reference may be made to:

1) synthesis algorithm [9], which has a heuristic nature and allows finding binary sequences with a very low autocorrelations for length of sequence up to 225; such relatively high values of length can significantly improve the effectiveness of many signal processing techniques, taking into account the fact that in some recent reviews and publications (e.g., [10]) were indicated binary sequences with low (and the best at the same time) autocorrelations for length of sequence up to 105;

2) algorithm [11] for synthesizing sets of orthogonal polyphase codes [12], which also can be characterized by good autocorrelation, in particular for synthesizing separately taken sequences with low autocorrelation; a feature of this approach boils down to a polyphase structure of codes (sequences), which can be more difficult for their practical implementation than in the case of binary structure of sequences.

In contrast with indicated algorithms, a feature of considered in this article sequences is that they have binary structures and deterministic generation rules, but gives a good equivalent autocorrelation after signal processing (maximum absolute sidelobe level equals to $1/N_K$) only in case of using a set consisting of K binary sequences.

Despite the fact that the indicated algorithms and similar ones gives good results, the problem of synthesis of binary sequences with optimal (the lowest) autocorrelations, which is also known as the low autocorrelation binary sequence problem [9], [13], [14], is not still solved in general.

III. Problem Statement

In case of the use of generalized binary Barker sequences in different signal processing techniques and radio systems, the cross-correlation between signals, which based on considered sequences, in a set defines accuracy and features of transmission and division of these signals during their use. Moreover, a very high correlation might restrict the application of such signals due to the impossibility of separation of signal components in some techniques (e.g., CDMA) or requires a corresponding enhancement of these techniques [15], [16]. In this regard, the aim of the article is a study of the cross-correlation between signal components in sets of generalized binary Barker sequences.

IV. Sets of Generalized Binary Barker Sequences to be Analyzed

Sequences, which form analyzed in the article sets, may have different lengths ($N = 4k$, $N = 4k–1$, $N = 4k+1$, $k \in \mathbb{Z}_+$) and belong to different types and subtypes. They are synthesized by means of generation rules, which were proposed in [5], [6].

Generation rule for sequences $\mathbf{X} = \{x_i\}$, $i=\overline{1,N}$, $N = 4k$, $k \in \mathbb{Z}_+$, of type 1 (subtype A) is given in (1).

$$x_i = \begin{cases} -1, & i=1, \\ (-1)^{S^{(1)}}, & i = 2S^{(1)}+1, \\ (-1)^{S^{(2)}} x_{2S^{(2)}-1}, & i = 2S^{(2)}, \\ x_{2S^{(2)}}, & i = N+1-2S^{(2)}, \\ -x_{2S^{(2)}-1}, & i = N+2-2S^{(2)}, \\ S^{(1)}\big|_{k>1} = \overline{1,\left(\frac{N}{4}-1\right)}, & \\ S^{(2)} = \overline{1,\frac{N}{4}}, & \\ N=4k,\ k\in\mathbb{Z}_+. & \end{cases} \tag{1}$$

Generation rule (1) contains the hyperparameter $\mathbf{S} = \{S^{(1)}, S^{(2)}\}$ to determine indices and values of sequence elements.

The case of sequences of type 1 (subtype B) is not considered in the article due to the fact that a sequence of type 1 (subtype B) has an inverted second part in comparison with a sequence of type 1 (subtype A) at the same length [5], [6] (and the first parts are the same). It causes the same cross-correlation in a set of signals, which based only on sequences of type 1 and subtype A (R), and in a set of signals, which based only on sequences of type 1 and subtype B (R^*):

$$R^*_{mn} = \frac{1}{T}\int_0^T x^*_m(t)x^*_n(t)dt = \frac{1}{T}\left[\int_0^{T/2} x^*_m(t)x^*_n(t)dt + \int_{T/2}^{T} x^*_m(t)x^*_n(t)dt\right] = \frac{1}{T}\left[\int_0^{T/2} x_m(t)x_n(t)dt\right.$$

$$\left. + \int_{T/2}^{T}\big(-x_m(t)\big)\big(-x_n(t)\big)dt\right] = \frac{1}{T}\left[\int_0^{T/2} x_m(t)x_n(t)dt + \int_{T/2}^{T} x_m(t)x_n(t)dt\right] = \frac{1}{T}\int_0^T x_m(t)x_n(t)dt = R_{mn},$$

where $x_j(t)=\sum_{i=1}^{N_j} x_i \left\{\theta\left[t-(i-1)\frac{T}{N_j}\right]-\theta\left[t-i\frac{T}{N_j}\right]\right\}$ and $x_j^*(t)=\sum_{i=1}^{N_j} x_i^* \left\{\theta\left[t-(i-1)\frac{T}{N_j}\right]-\theta\left[t-i\frac{T}{N_j}\right]\right\}$ are jth signals in a sets, which are based on the sequences of type 1 (subtype A) $\mathbf{X}$ = $\{x_i\}$ or on the sequences of type 1 (subtype B) $\mathbf{X}^*$ = $\{x_i^*\}$ respectively; T is a duration of signals in a set and N_j is a length of sequence $\mathbf{X}$ or $\mathbf{X}^*$, which forms a jth signal in a set; $\theta(t)$ is the Heaviside step function; $j=\overline{1,K}$ $m=\overline{1,K}$, $n=\overline{1,K}$; K is the number of signals in a set.

Generation rule for sequences $\mathbf{X}$ = $\{x_i\}$, $i=\overline{1,N}$, $N = 4k–1$, $k \in \mathbb{Z}_+$, of type 2 is given in (2).

$$x_i=\begin{cases}-1, & i=1,\\ (-1)^{S^{(1)}}, & i=2S^{(1)}+1,\\ (-1)^{S^{(2)}}x_{2S^{(2)}-1}, & i=2S^{(2)},\\ x_{2S^{(1)}}, & i=N+1-2S^{(1)},\\ -x_{2S^{(1)}+1}, & i=N-2S^{(1)},\\ -x_1, & i=N,\\ S^{(1)}\Big|_{k>1}=\overline{1,\left(\frac{N+1}{4}-1\right)}, & \\ S^{(2)}=\overline{1,(N+1)/4}, & \\ N=4k-1,\ k\in\mathbb{Z}_+. & \end{cases}\tag{2}$$

Generation rule (2) contains the hyperparameter $\mathbf{S}$ = $\{S^{(1)}, S^{(2)}\}$ to determine indices and values of sequence elements, as well as the rule (1).

Generation rule for sequences $\mathbf{X}$ = $\{x_i\}$, $i=\overline{1,N}$, $N = 4k+1$, $k \in \mathbb{Z}_+$, of type 3 for the subtype A is presented in (3), and for the subtype B is presented in (4). Generation rules (3) and (4) contain the parameter S to determine indices and values of sequence elements, analogous to the hyperparameter $\mathbf{S}$ in generation rules (1) and (2).

$$x_i=\begin{cases}-1,\ i=1,\ 2S+1,\ (N+1)/2,\ (N+3)/2,\ N,\\ -x_{2S-1},\ i=2S,\\ 1,\quad i=(N-1)/2,\\ -x_{2S},\quad i=N+1-2S,\\ x_{2S+1},\ i=N-2S,\\ S\big|_{k>1}=\overline{1,(N-5)/4},\\ N=4k+1,\ k\in\mathbb{Z}_+.\end{cases}\tag{3}$$

$$x_i=\begin{cases}-1,\quad i=1,\ 2S+1,\ (N-1)/2,\ N,\\ -x_{2S-1},\ i=2S,\\ 1,\quad i=(N+1)/2,\ (N+3)/2,\\ -x_{2S},\ i=N+1-2S,\\ x_{2S+1},\ i=N-2S,\\ S\big|_{k>1}=\overline{1,(N-5)/4},\\ N=4k+1,\ k\in\mathbb{Z}_+.\end{cases}\tag{4}$$

Structures of sets of generalized binary Barker sequences to be analyzed in the article are as follows.

1) Set 1 consists of K_{max} = 8 signals with the same duration T, which have a mathematical model (5).

$$x_j(t)=\sum_{i=1}^{N_j} x_i \left\{\theta\left[t-(i-1)\frac{T}{N_j}\right]-\theta\left[t-i\frac{T}{N_j}\right]\right\},\tag{5}$$

where $j=\overline{1,8}$ and $\mathbf{X}$ = $\{x_i\}$ are sequences of type 1 (subtype A) with lengths: N_1 = 4; N_2 = 16; N_3 = 32; N_4 = 64; N_5 = 128; N_6 = 256; N_7 = 512; N_8 = 1024.

2) Set 2 consists of K_{max} = 8 signals with the same duration T, which have a mathematical model (5), where $\mathbf{X}$ = $\{x_i\}$ for j = 1 is a sequence of type 2 with the length N_1 = 3, and $\mathbf{X}$ = $\{x_i\}$ for $j=\overline{2,8}$ are sequences of type 1 (subtype A) with lengths: N_2 = 12; N_3 = 24; N_4 = 48; N_5 = 96; N_6 = 192; N_7 = 384; N_8 = 768.

3) Set 3 consists of K_{max} = 8 signals with the same duration T, which have a mathematical model (5), where $\mathbf{X}$ = $\{x_i\}$ for j = 1 is a sequence of type 2 with the length N_1 = 7, and $\mathbf{X}$ = $\{x_i\}$ for $j=\overline{2,8}$ are sequences of type 1 (subtype A) with lengths: N_2 = 28; N_3 = 56; N_4 = 112; N_5 = 224; N_6 = 448; N_7 = 896; N_8 = 1792.

4) Set 4 consists of K_{max} = 8 signals with the same duration T, which have a mathematical model (5), where $\mathbf{X}$ = $\{x_i\}$ for j = 1 is a sequence of type 3 (subtype A) with the length N_1 = 5, and $\mathbf{X}$ = $\{x_i\}$ for $j=\overline{2,8}$ are sequences of type 1 (subtype A) with lengths: N_2 = 20; N_3 = 40; N_4 = 80; N_5 = 160; N_6 = 320; N_7 = 640; N_8 = 1280.

5) Set 5 is similar to set 4, but the sequence of subtype B (type 3) is used for the signal component j = 1.

V. Results of Cross-Correlation Analysis of Considered Sets of Generalized Binary Barker Sequences

The values of cross-correlation $R(J)_{mn} = = \frac{1}{T}\int_0^T x_m(t)x_n(t)dt$ between signals $x_m(t)$ and $x_n(t)$, $m=\overline{1,8}$, $n=\overline{1,8}$, which part of a set with a number J and are consistent with the model (5), are presented below for 5 above-considered sets.

1) For the set 1 of considered sequences:

$$\mathbf{R}(1)=\begin{pmatrix} 1 & 0 & -\frac{1}{8} & -\frac{3}{16} & -\frac{7}{32} & -\frac{15}{64} & -\frac{31}{128} & -\frac{63}{256} \\ 0 & 1 & \frac{1}{8} & \frac{3}{16} & \frac{5}{32} & \frac{9}{64} & \frac{17}{128} & \frac{33}{256} \\ -\frac{1}{8} & \frac{1}{8} & 1 & \frac{3}{16} & \frac{7}{32} & \frac{13}{64} & \frac{25}{128} & \frac{49}{256} \\ -\frac{3}{16} & \frac{3}{16} & \frac{3}{16} & 1 & \frac{7}{32} & \frac{15}{64} & \frac{29}{128} & \frac{57}{256} \\ -\frac{7}{32} & \frac{5}{32} & \frac{7}{32} & \frac{7}{32} & 1 & \frac{15}{64} & \frac{31}{128} & \frac{61}{256} \\ -\frac{15}{64} & \frac{9}{64} & \frac{13}{64} & \frac{15}{64} & \frac{15}{64} & 1 & \frac{31}{128} & \frac{63}{256} \\ -\frac{31}{128} & \frac{17}{128} & \frac{25}{128} & \frac{29}{128} & \frac{31}{128} & \frac{31}{128} & 1 & \frac{63}{256} \\ -\frac{63}{256} & \frac{33}{256} & \frac{49}{256} & \frac{57}{256} & \frac{61}{256} & \frac{63}{256} & \frac{63}{256} & 1 \end{pmatrix}$$

2) For the set 2 of considered sequences:

$$\mathbf{R}(2)=\begin{pmatrix} 1 & \frac{1}{6} & 0 & -\frac{1}{12} & -\frac{1}{8} & -\frac{7}{48} & -\frac{5}{32} & -\frac{31}{192} \\ \frac{1}{6} & 1 & \frac{1}{6} & \frac{1}{6} & \frac{1}{8} & \frac{5}{48} & \frac{3}{32} & \frac{17}{192} \\ 0 & \frac{1}{6} & 1 & \frac{1}{6} & \frac{5}{24} & \frac{3}{16} & \frac{17}{96} & \frac{11}{64} \\ -\frac{1}{12} & \frac{1}{6} & \frac{1}{6} & 1 & \frac{5}{24} & \frac{11}{48} & \frac{7}{32} & \frac{41}{192} \\ -\frac{1}{8} & \frac{1}{8} & \frac{5}{24} & \frac{5}{24} & 1 & \frac{11}{48} & \frac{23}{96} & \frac{15}{64} \\ -\frac{7}{48} & \frac{5}{48} & \frac{3}{16} & \frac{11}{48} & \frac{11}{48} & 1 & \frac{23}{96} & \frac{47}{192} \\ -\frac{5}{32} & \frac{3}{32} & \frac{17}{96} & \frac{7}{32} & \frac{23}{96} & \frac{23}{96} & 1 & \frac{47}{192} \\ -\frac{31}{192} & \frac{17}{192} & \frac{11}{64} & \frac{41}{192} & \frac{15}{64} & \frac{47}{192} & \frac{47}{192} & 1 \end{pmatrix}$$

3) For the set 3 of considered sequences:

$$\mathbf{R}(3)=\begin{pmatrix} 1 & \frac{1}{14} & 0 & -\frac{1}{28} & -\frac{3}{56} & -\frac{1}{16} & -\frac{15}{224} & -\frac{31}{448} \\ \frac{1}{14} & 1 & \frac{3}{14} & \frac{3}{14} & \frac{11}{56} & \frac{3}{16} & \frac{41}{224} & \frac{81}{448} \\ 0 & \frac{3}{14} & 1 & \frac{3}{14} & \frac{13}{56} & \frac{25}{112} & \frac{7}{32} & \frac{97}{448} \\ -\frac{1}{28} & \frac{3}{14} & \frac{3}{14} & 1 & \frac{13}{56} & \frac{27}{112} & \frac{53}{224} & \frac{15}{64} \\ -\frac{3}{56} & \frac{11}{56} & \frac{13}{56} & \frac{13}{56} & 1 & \frac{27}{112} & \frac{55}{224} & \frac{109}{448} \\ -\frac{1}{16} & \frac{3}{16} & \frac{25}{112} & \frac{27}{112} & \frac{27}{112} & 1 & \frac{55}{224} & \frac{111}{448} \\ -\frac{15}{224} & \frac{41}{224} & \frac{7}{32} & \frac{53}{224} & \frac{55}{224} & \frac{55}{224} & 1 & \frac{111}{448} \\ -\frac{31}{448} & \frac{81}{448} & \frac{97}{448} & \frac{15}{64} & \frac{109}{448} & \frac{111}{448} & \frac{111}{448} & 1 \end{pmatrix}$$

4) For the set 4 of considered sequences:

$$\mathbf{R}(4)=\begin{pmatrix} 1 & \frac{1}{10} & \frac{1}{5} & \frac{1}{4} & \frac{11}{40} & \frac{23}{80} & \frac{47}{160} & \frac{19}{64} \\ \frac{1}{10} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{7}{40} & \frac{13}{80} & \frac{5}{32} & \frac{49}{320} \\ \frac{1}{5} & \frac{1}{5} & 1 & \frac{1}{5} & \frac{9}{40} & \frac{17}{80} & \frac{33}{160} & \frac{13}{64} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{5} & 1 & \frac{9}{40} & \frac{19}{80} & \frac{37}{160} & \frac{73}{320} \\ \frac{11}{40} & \frac{7}{40} & \frac{9}{40} & \frac{9}{40} & 1 & \frac{19}{80} & \frac{39}{160} & \frac{77}{320} \\ \frac{23}{80} & \frac{13}{80} & \frac{17}{80} & \frac{19}{80} & \frac{19}{80} & 1 & \frac{39}{160} & \frac{79}{320} \\ \frac{47}{160} & \frac{5}{32} & \frac{33}{160} & \frac{37}{160} & \frac{39}{160} & \frac{39}{160} & 1 & \frac{79}{320} \\ \frac{19}{64} & \frac{49}{320} & \frac{13}{64} & \frac{73}{320} & \frac{77}{320} & \frac{79}{320} & \frac{79}{320} & 1 \end{pmatrix}$$

Signal processing of sets of generalized binary Barker sequences with a set number $J=\overline{1,5}$ gives a good equivalent autocorrelation (low peak sidelobe level in the output signal) after signal processing, which is described in detail in [7].

Note that in [7] all sequences are presented in inverted form and starts from "1" in order to match representation form for polynomials and elements of GF(2^n), taking also into account that "-1" should be replaced with "0".

5) *For the set 5 of considered sequences:*

$$\mathbf{R}(5)=\begin{pmatrix} 1 & -\frac{1}{10} & 0 & \frac{1}{20} & \frac{3}{40} & \frac{7}{80} & \frac{3}{32} & \frac{31}{320} \\ -\frac{1}{10} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{7}{40} & \frac{13}{80} & \frac{5}{32} & \frac{49}{320} \\ 0 & \frac{1}{5} & 1 & \frac{1}{5} & \frac{9}{40} & \frac{17}{80} & \frac{33}{160} & \frac{13}{64} \\ \frac{1}{20} & \frac{1}{5} & \frac{1}{5} & 1 & \frac{9}{40} & \frac{19}{80} & \frac{37}{160} & \frac{73}{320} \\ \frac{3}{40} & \frac{7}{40} & \frac{9}{40} & \frac{9}{40} & 1 & \frac{19}{80} & \frac{39}{160} & \frac{77}{320} \\ \frac{7}{80} & \frac{13}{80} & \frac{17}{80} & \frac{19}{80} & \frac{19}{80} & 1 & \frac{39}{160} & \frac{79}{320} \\ \frac{3}{32} & \frac{5}{32} & \frac{33}{160} & \frac{37}{160} & \frac{39}{160} & \frac{39}{160} & 1 & \frac{79}{320} \\ \frac{31}{320} & \frac{49}{320} & \frac{13}{64} & \frac{73}{320} & \frac{77}{320} & \frac{79}{320} & \frac{79}{320} & 1 \end{pmatrix}$$

VI. Discussion of Results

The main point of the discussion boils down to determining of a set, which contains K $\left(2 \le K \le K_{\max}\right)$ sequences and has the lowest maximum absolute value of cross-correlation, i.e. $\min_J \max_{\substack{m,n \\ m\neq n,\ m,n\le K}} \left(|R(J)_{mn}|\right)$. This criterion is chosen due to the fact that a maximum cross-correlation in a set typically defines the greatest influence on accuracy of signal detection. Results of analysis of sets by means of this criterion are shown in the Table I, where for each J and K the value of $\max_{\substack{m,n \\ m\neq n,\ m,n\le K}} \left(|R(J)_{mn}|\right)$ is indicated (optimal set for each K is marked by using bold style for values).

TABLE I. Analysis of Sets Using the Criterion $\min_J \max_{\substack{m,n \\ m\neq n,\ m,n\le K}} \left(|R(J)_{mn}|\right)$

K \ J	1	2	3	4	5
2	**0**	1/6	1/14	1/10	1/10
3	**1/8**	1/6	3/14	1/5	1/5
4	3/16	**1/6**	3/14	1/4	1/5
5	7/32	**5/24**	13/56	11/40	9/40
6	15/64	**11/48**	27/112	23/80	19/80
7	31/128	**23/96**	55/224	47/160	39/160
8	63/256	**47/192**	111/448	19/64	79/320

VII. Conclusions

Despite the fact that sets of generalized binary Barker sequences provide low peak sidelobe level after their joint signal processing (multiplicative complementariness), they are characterized by a relatively high level of cross-correlation, which can be up to a typical value $|R(J)_{mn}| \approx 0.25$, $m \ne n$, for sets consisting of $K \le 8$ sequences. This fact restricts the application of generalized binary Barker sequences in some techniques (e.g., CDMA) due to the impossibility of separation of signal components or it requires enhancement of these techniques in order to take account of the cross-correlation.

The cross-correlation analysis has also shown (see the Table I) that the set 1, which consists of sequences of type 1 (subtype A) with lengths $N_1 = 4$, $N_2 = 16$, $N_3 = 32$, has the lowest maximum absolute values of cross-correlation at $K \le 3$; the set 2, which consists of sequence of type 2 with the length $N_1 = 3$ and sequences of type 1 (subtype A) with lengths $N_2 = 12$, $N_3 = 24$, $N_4 = 48$, $N_5 = 96$, $N_6 = 192$, $N_7 = 384$, $N_8 = 768$, has the lowest maximum absolute values of cross-correlation at $4 \le K \le 8$.

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О. Г. Голубничий. Аналіз взаємної кореляції у системах узагальнених бінарних послідовностей Баркера

Ефективність багатьох методів обробки сигналів та їх практична цінність, яка особливо проявляється у таких характеристиках як точність виявлення сигналів та вимірювання їх параметрів, роздільній здатності по дальності та часу тощо, залежить від кореляційних властивостей оброблюваних сигналів. Статтю присвячено дослідженню кореляційних властивостей узагальнених бінарних послідовностей Баркера, а саме значень коефіцієнта взаємної кореляції між сигнальними складовими у системі сигналів, яку побудовано на основі узагальнених бінарних послідовностей Баркера. Такі системи послідовностей забезпечують низький максимальний рівень бічних пелюсток сигналу після їх спільної обробки (мультиплікативна комплементарність), однак їх взаємнокореляційні характеристики також можуть впливати на якість функціонування радіотехнічних систем та систем обробки сигналів. У статті проаналізовано п’ять систем сигналів з різними структурами, які побудовані на основі узагальнених бінарних послідовностей Баркера. Представлені результати показали, що сигнальні складові на основі узагальнених бінарних послідовностей Баркера характеризуються відносно великим рівнем взаємної кореляції, який для систем сигналів, побудованих на основі не більше ніж восьми послідовностей, може бути охарактеризований типовим значенням коефіцієнта кореляції між різними сигнальними складовими, що дорівнює 0,25. Цей факт обмежує використання узагальнених бінарних послідовностей Баркера у деяких системах (наприклад, CDMA) через неможливість якісного розділення сигнальних складових, або потребує удосконалення таких систем з метою врахування взаємної кореляції, яка існує між сигнальними складовими. У статті також визначено та показано системи узагальнених бінарних послідовностей Баркера з найменшими максимальними абсолютними значеннями коефіцієнта взаємної кореляції.

Ключові слова: узагальнені бінарні послідовності Баркера; кореляційні властивості; взаємна кореляція; обробка сигналів; виявлення сигналів; аналіз сигналів.

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А. Г. Голубничий. Анализ взаимной корреляции в системах обобщённых бинарных последовательностей Баркера

Эффективность многих методов обработки сигналов и их практическая ценность, особенно проявляющаяся в таких характеристиках как точность обнаружения сигналов и измерения их параметров, разрешающей способности по дальности и времени и других, зависит от корреляционных свойств обрабатываемых сигналов. Статья посвящена исследованию корреляционных свойств обобщённых бинарных последовательностей Баркера, а именно значений коэффициента взаимной корреляции между сигнальными составляющими в системе сигналов, построенной на основе обобщённых бинарных последовательностей Баркера. Такие системы последовательностей обеспечивают низкий максимальный уровень боковых лепестков сигнала после их взаимной обработки (мультипликативная комплементарность), однако их взаимнокорреляционные характеристики также могут влиять на качество функционирования радиотехнических систем и систем обработки сигналов. В статье проанализированы пять систем сигналов с разными структурами, которые построены на основе обобщённых бинарных последовательностей Баркера. Представленные результаты показали, что сигнальные составляющие на основе обобщённых бинарных последовательностей Баркера характеризуются относительно высоким уровнем взаимной корреляции, который для систем сигналов, построенных на основе не более чем восьми последовательностей, может быть охарактеризован типовым значением коэффициента корреляции между разными сигнальными составляющими, равным 0,25. Этот факт ограничивает использование обобщённых бинарных последовательностей Баркера в некоторых системах (например, CDMA) из-за невозможности качественного разделения сигнальных составляющих, или требует усовершенствования таких систем с целью учёта взаимной корреляции, которая существует между сигнальными составляющими. В статье также определены и показаны системы обобщённых бинарных последовательностей Баркера с наименьшими максимальными абсолютными значениями коэффициента взаимной корреляции.
Ключевые слова: обобщённые бинарные последовательности Баркера; корреляционные свойства; взаимная корреляция; обработка сигналов; обнаружение сигналов; анализ сигналов.

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