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COMPUTER-AIDED DESIGN OF FIXED WING UAV PATH TRACKING SYSTEM

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Abstract*—In this paper problem of smooth fixed-wing unmanned aerial vehicle path following is considered, when the reference path is the linear piecewise trajectory having points of discontinuity with abrupt changing of the heading angle. The ultimate goal is suppressing deflections of the state vector components, which are dangerous from the flight safety viewpoint, taking in consideration full mathematical model of the inner attitude control loop. The inner attitude control and outer guidance systems consist of the simplest commonly used elements like PD-controllers, washout filters and phase-lead compensators, which parameters have to be found by robust H_2 / H_∞-optimization based on genetic algorithms. In order to avoid over-parameterization of optimization procedure, we propose to solve this problem using its decomposition by two consecutive stages: finding optimal parameters of inner loop at the first stage, and then finding optimal parameters of outer loop at the second stage. We propose to use low pass filter for reference roll signal conditioning, in order to obtain unmanned aerial vehicle smooth pseudo-Dubins flight path. Simulation of unmanned aerial vehicle flight in calm and moderate turbulent atmosphere proves the efficiency of proposed design method.*

Index Terms—Attitude control; flight path control; $\mathrm{H}_2 / \mathrm{H}_\infty$-optimization; sensitivity function; complementary sensitivity function; genetic algorithms.

I. INTRODUCTION

Nowadays unmanned aerial vehicle (UAV) path tracking problems are one of the most important areas of the Unmanned Systems Technologies, and it attracts huge attention of researchers [1] – [5]. Description of all known path-tracking algorithms (PTA) given in [1], [3], and [4] represents two main types of these problems: straight line following and circle following (loitering). In both these cases, the dynamics of inner closed loop contour "autopilot+UAV" was either neglected [3] or the simplest approximation by the 2nd order linear system was assumed [1], [4]. If we consider the simplest single straight line following, this assumption can be quite acceptable. It could be also accepted in the loitering case, because the circle reference track (RT) is smooth and differentiable in any point. However, if the RT is linear piecewise trajectory consisting of several straight-line segments located with different heading angles, then in the transition points from one segment to another the discontinuities of RT arise. If the UAV linear attitude dynamic model is described with complete set of the state variables, including dynamics ailerons and rudder actuators, then these discontinuities cause huge transient deflections of the UAV state variables, which are incompatible with flight safety requirements, especially when the heading angles increments are significant (for instance, for closed rectangular RT these increments equal $\pm\ \pi/2$ rad). Example in [5] shows that deflection of sideslip β and roll φ angles in this case exceeds all acceptable values. Capability of smooth transition from one segment to another with acceptable values of all UAV state variables is called *dynamic feasibility* in [9]. This capability cannot be proved using simplified model of the UAV inner contour of attitude control, so achieving of dynamic feasibility is one of the goals of this paper.

There is another reason to use complete model of the UAV attitude control loop. As it is known any UAV flight is performed in the conditions of the turbulent atmosphere, which is simulated by Dryden model [1], [6]. Including the sideslip angle β and the yaw rate r in the UAV state space model is necessary condition for evaluation of the turbulent wind influence on the UAV guidance system [1], [6].

Finally, it is necessary to notice that in the majority of practical cases in the UAV inner control loop traditional PD and/or PID controllers are applied [1], [5], [6] and [7]. Note also that in a case of transition from one straight-line segment to another, for PID controller it is necessary to reset integrators after each switching to new segment [9]. That is why from practical point of view it is expedient to consider attitude control system with given structure consisting of PD-controllers, enhanced with the simplest 1st order phase-lead compensators [5] – [7] to improve dynamics of the inner loop. However, the parametric optimization proposed in this paper, based on results of [5], [7], [8], could be applied in more complicated cases of given control system structure.

II. UAV Mathematical Model, Control System Structure and Problem Statement

It is assumed that the path tracking process is considered in the horizontal plane, when the UAV altitude is constant. Therefore, we use the model of the UAV lateral motion dynamics, which is represented by following set of equations

$$\begin{cases} \dot{x} = Ax + B_u u + B_g g, \\ y = Cx, \end{cases} \quad x(0) = x_0, \qquad (1)$$

where $\mathbf{x} = [\beta,\ \varphi,\ p,\ r,\ \psi,\ e_d]^{\mathrm{T}}$, $\mathbf{u} = [\delta_a,\ \delta_r]^{\mathrm{T}}$, $g = [\beta_g,\ r_g]^{\mathrm{T}}$, $y = [\beta,\ \varphi,\ p,\ r,\ \psi,\ e_d,\ \dot{e}_d]^{\mathrm{T}}$ are the state, control, disturbance, and observation vectors respectively, β, φ, p, r, ψ, e_d are sideslip angle, roll angle, roll rate, yaw rate, yaw angle and cross track error respectively, δ_a, δ_r stand for deflections of ailerons and rudder; and, eventually, A, B_u, B_g, C stand for the state propagation, control input, disturbance input and observation matrices respectively.

Numerical values of these matrices entries for certain UAV benchmark model [11] are represented below. It is considered that UAV flies in a light turbulence. In this case disturbance vector g in (1) affecting the lateral motion of the aerial vehicle, is represented as the output of well-known Dryden forming filter [1], [6], having input signal as unit white noise g_w and output signals: the sideslip angle β_g, and the yaw rate r_g, so that $\vec{g} = [\beta_g,\ r_g]^{\mathrm{T}}$. The transfer functions of Dryden forming filter used in simulation to account external disturbances are well known and are given in [1], [6]. Block diagram of the closed-loop UAV guidance system is shown in the Fig. 1, where “Dryd.”, “UAV”, “PTCL” stand for Dryden filter, unmanned aerial vehicle, and path tracking control loop respectively. Block “UAV”, encompassed with inner loop feedback, “PTCL” and flight management system are forming outer path tracking loop, which is depicted in Fig. 1 with bold lines.

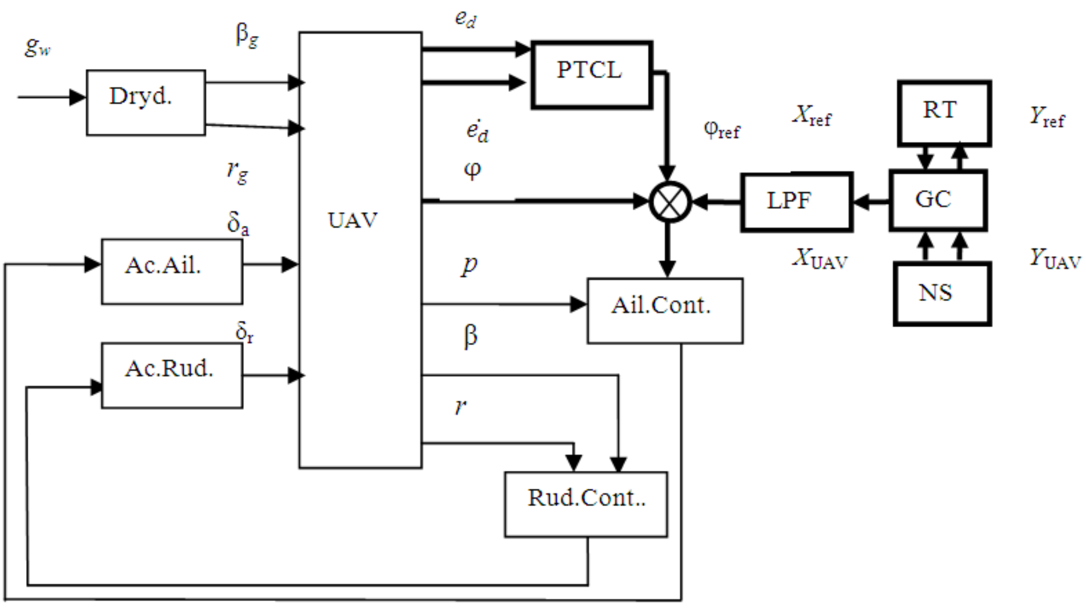


Fig. 1. Block diagram of the closed-loop UAV guidance system

Flight management system includes following blocks: Reference Track (RT), Navigation System (NS), Guidance Commander (GC), and Low Pass Filter (LPF). Comparing current UAV position in given navigation frame $(X_{\mathrm{UAV}}, Y_{\mathrm{UAV}})$ with corresponding points on RT $(X_{\mathrm{RT}}, Y_{\mathrm{RT}})$ (Fig. 2), the GC producing switching commands for transition from the current segment of the linear piecewise RT to the next one.

This switching generates abrupt increment of the roll reference value φ_{ref}, thus causing enormously large UAV attitude changing. In order to alleviate this problem, the Low Pass Filter (LPF) is applied.

Inner contour consists of aileron controller-“Ail.Cont.”, rudder controller –“Rud.Cont.”, aileron actuator – “Ac.Ail.”, and rudder actuator – “Ac. Rud.”; they are depicted with thin lines. These two closed loops create UAV Guidance and Control system. The PTCL represented in this paper is based on results obtained in [4]. As a control input for outer loop it uses a lateral linear acceleration, which is derived via current and predicted cross-track error [4]. The guidance system of UAV selects the reference point (pseudo-target) on desired path and then generates a lateral acceleration signal based on reference point using guidance control algorithm [4]. The reference point lies on the desired UAV reference track at a distance (L_d) ahead of the vehicle, as shown in Fig. 2. In accordance with [4] the lateral acceleration command is determined by following expression:

$$a_{y\text{comm}d} = 2\frac{V^2}{L_d}\left(\frac{e_d}{L_d} + \frac{\dot{e}_d}{V}\right), \qquad (2)$$

where V is a vehicle speed; L_d is the distance from the UAV to the pseudo target; e_d is a cross-track error, and $a_{y\text{comm}d}$ is the acceleration command signal. Note that value L_d is unknown from the very beginning. It is known that lateral acceleration is connected with roll angle as

$$a_y = g\,\varphi, \qquad (3)$$

where φ is the UAV roll angle, g is the gravity acceleration. Combining (2) and (3), the reference command for the roll angle control system will take the following form:

$$\varphi_{\text{ref}} = 2\frac{V^2}{g\,L_d}\left(\frac{e_d}{L_d} + \frac{\dot{e}_d}{V}\right), \qquad (4)$$

where φ_{ref} is the reference roll angle for the inner loop of the overall guidance system. Note that expression (4) is traditional PD control law for the PTCL. So the control problem at the 1st stage consists of the parametric optimization of the inner loop attitude control in order to obtain its robust suboptimal performance for following the command roll angle signal. In order to improve path-tracking algorithm it is proposed here to augment the control law (4) by heading error components [3]:

$$\varphi_{\text{ref}} = 2\frac{V^2}{g\,L_d}\left(\frac{e_d}{L_d} + \frac{\dot{e}_d}{V}\right) + \left(K_\psi \Delta\psi + K_{d\psi}\frac{d\Delta\psi}{dt}\right), \qquad (5)$$

where $\Delta\psi = \psi_{\text{tr}} - \psi_{\text{UAV}}$, and $\psi_{\text{tr}}, \psi_{\text{UAV}}$ stand for heading angle of the current reference track segment and current UAV heading angle respectively. It is necessary to notice that for obtaining desirable bandwidth of the closed-loop system it is useful to augment control law (5) with phase lead compensator [2], [6], and [7]:

$$W_{PL}(s) = \frac{K_n s + 1}{K_d s + 1}, \; K_n > K_d, \qquad (6)$$

which is connected in series with (5).

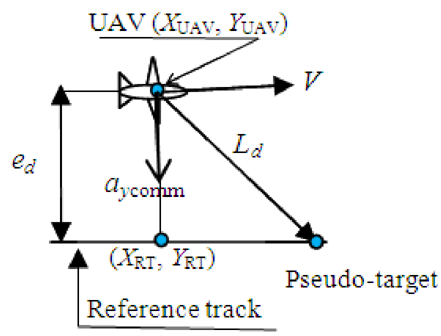


Fig. 2. Illustration of UAV guidance principle

The attitude control inner loop includes two controllers in ailerons $W_{ac}(s)$ and rudder $W_{rc}(s)$ channels, having following transfer functions [2], [6], and [7]:

$$W_{ac}(s) = \frac{K_{n1}s + 1}{K_{d1}s + 1}\cdot\left[K_\varphi, K_p\right]\cdot\left[\varphi(s), p(s)\right]^{\text{T}}, \qquad (7)$$

$$W_{rc}(s) = \left[(K_\beta + K_{d\beta}\frac{T_0 s}{T_0 s + 1}),\; \frac{T_1 s}{T_2 s + 1}\right]\cdot\left[\beta(s), r(s)\right]^{\text{T}}. \qquad (8)$$

In order to minimize the angular deflection in the transient processes arising due to transition from one straight-line segment to the next one it is necessary to find such values of parameter vector $\vec{K}_{AC}$ for attitude control:

$$\vec{K}_{AC} = [K_p, K_\varphi, K_{n1}, K_{d1}, K_\beta, K_{d\beta}, T_1, T_2], \qquad (9)$$

which is the solution of the following optimization problem:

$$\begin{aligned} &\text{Find: } \vec{K}^*_{AC} = \arg\min_{\vec{K}_{AC}} J_c(\vec{K}_{AC}), \\ &\text{Subject to: } \vec{K}_{AC} \in D_{st}, \end{aligned} \qquad (10)$$

where $J_c(\vec{K}_{AC})$ is some composite performance index, which will be defined below and D_{st} is the admissible domain in the adjustable parameter space, which is determined by the closed loop system stability conditions [5], [7], [8]. High dimension of the mathematical model of the closed-loop system and large amount of adjustable parameters, which are sought for, induces appearance of multi-extremal optimization problem. In order to alleviate this difficulty the genetic optimization algorithms are applied to solve optimization problem (10).

Now it is necessary to notice that dimension of problem, which includes dimension of UAV mathematical model (1) and dimensions of mathematical models of actuators and controllers (5) – (7), is very high. Also the dimension of adjustable parameters vector including parameters of inner loop control law (7) – (8) and outer loop control law (5) – (6) is also large. In order to avoid over-parameterization problem, to improve divergence of optimization procedure and to make it feasible for practical cases, it is proposed to decompose this problem by two stages: the 1st one is the robust optimization of the inner loop (attitude control) and the 2nd one is the robust optimization of the outer loop (path-tracking). The 1st stage of the optimization problem is designated to find parameters defined by expressions (8), (9). Both these stages use genetic algorithms.

The 2nd stage of the design procedure for outer loop includes appropriate choice of parameters of the control law (5), (6) based on H_2/H_∞-optimization, and appropriate conditioning of the command signal φ_{ref} in order to obtain smooth transition from one linear segment of the reference track to another. The choice of parameters uses aforementioned optimization procedure (9), (10) for outer loop, considering inner loop with constant known parameters obtained at the 1st stage. So at the 2nd stage the role of controlled plant will play the closed inner loop, and finding parameters of PTCL (5), (6) will be done as in previous case.

III. Two-Stage Design of Robust UAV Path Following System

As it follows from the previous item, it is necessary to define composite performance index $J_c(\vec{K}_{AC})$ appearing in (8) for roll control inner loop. We have to notice that output vector $\mathbf{Y}$ for inner loop have the following form: $\mathbf{Y} = [\beta, \ \varphi, \ p, \ r]^T$ (other state variables appearing due to controllers (5), (6) and actuators are not measured), and matrices of the state space model (1) of inner loop $\mathbf{A}, \mathbf{B_u}, \mathbf{B_g}, \mathbf{C}, \mathbf{D}$ have corresponding dimensions. Following the procedure of the H_2/H_∞-optimization successfully applied in many similar cases [5], [7], and [8], we define performing index $J_c(\vec{K}_{AC})$ as follows [7], [8], and [10]:

$$J(\vec{K}_{AC}) = \left\| \begin{matrix} \lambda_1 S(j\omega,\cdot Q, \vec{K}_{AC}) \\ \lambda_2 T(j\omega,\cdot Q, \vec{K}_{AC}) \\ \lambda_3 K(j\omega,\cdot R, \vec{K}_{AC}) \end{matrix} \right\|_\infty + \lambda_4 \left\| W_{gz}(j\omega,\cdot Q, \vec{K}_{AC}) \right\|_2 + PF(\vec{K}_{AC}), \tag{11}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are weighting coefficients, $S(j\omega,\cdot Q, \vec{K}_{AC})$, $T(j\omega,\cdot Q, \vec{K}_{AC})$, $K(j\omega,\cdot R, \vec{K}_{AC})$ are sensitivity, complementary sensitivity and control sensitivity matrices respectively [10], $\|M\|_\infty, \|M\|_2$ stand for H_∞- and H_2-norms of corresponding matrices. The block matrix consisting of weighted matrices **S**, **T**, **K** represents "stacked approach" to the mixed sensitivity minimization [10]. Other notations in (9) are following: $W_{gz}(j\omega,\cdot Q, \vec{K}_{AC})$ stands for transfer matrix from disturbance $\vec{g}$ to weighted output $\mathbf{z} = \mathbf{Q} \cdot \mathbf{Y}$; $\mathbf{Q}, \mathbf{R}$ are weighting matrices for output and control vectors respectively. Eventually $PF(\vec{K})$ is the penalty function in order to keep all components of vector $\vec{K}_{AC}$ within the stability domain in the space of adjustable parameters (9) during each run of the optimization procedure (10). This penalty function restricts the location of the eigenvalues of the closed loop system state propagation matrix $\mathbf{A_{cl}}$ for given vector $\vec{K}_{AC}$ within a trapezoid in the left complex half-plane, defined by system stability margin, bandwidth, and system oscillativity [5], [7]. The penalty for violation these borders is determined by the following expression:

$$PF(d_m) = \begin{cases} 0, \text{ if } \ d_m \le d_{m1}, \\ \dfrac{P}{2}\left[1 + \cos\left(\dfrac{\pi(d_m - d_0)}{d_{m1} - d_0}\right)\right], \\ P, \text{ if } \ d_m \le d_o, \end{cases} \tag{12}$$

$$\text{if } d_o < d_m < d_{m1},$$

where $d_m = \min_i \left|\text{eig}_i(A_{cl}) - d_0\right|, \ i = 1, \dots, n$ is the minimal distance from ith eigenvalue to one of the borders, determined for set of all eigenvalues $\text{eig}_i(A_{cl})$, d_0 is the corresponding point on the given border, d_{m1} is the segment, where $PF(d_m) \ne 0$, P is a very large value (for example $P = 10^4 \ldots 10^6$) [5], [7]. This penalty function determines admissible domain in the adjustable parameter space D_{st}, appearing in (10).

Interactive procedure (10) based on genetic algorithm with performance index (11), (12) is running several times with different values of weighting matrices $\mathbf{Q}, \mathbf{R}$ and weighting coefficients $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ until designer will find the compromise between robustness and performance, which can be characterized by appropriate values of estimations of some state variables, which are critical from the flight safety viewpoint.

As it was stated before at the 2nd stage we have to find vector of unknown parameters for PTCL (5), (6) $\vec{K}_{PT}$:

$$\vec{K}_{PT} = [K_r, K_\psi, K_{e_d}, K_{de_d}, K_n, K_d], \qquad (13)$$

where K_{e_d}, K_{de_d} is the proportional and differential gains of control law (4). In this case the output vector of state space model of outer loop will have the following form:

$$\mathbf{Y_{PT}} = \left[r, \quad \psi, \quad e_d, \quad \dot{e}_d\right]^{\mathrm{T}}. \qquad (14)$$

Considering the inner loop with numerical values of adjustable parameters, which were found at the 1st stage, as new controlled plant, we can apply previously described $\mathrm{H}_2 / \mathrm{H}_\infty$- optimization procedure in order to find unknown parameters for $\vec{K}_{PT}$ (13).

At this stage it is necessary also to find parameters of the low-pass filter (LPF, see Fig. 1), which is used for command signal conditioning. It is inevitable from the point of view of the RT dynamic feasibility. In order to avoid unacceptable deflections of all state variables in the moments of transition from current straight-line leg of RT to the next one, it is necessary to determine the LPF transfer function. It must satisfy two following conditions: the velocity and acceleration at the beginning of transient process must be equal to zero, and the bandwidth of filter must be less than the bandwidth of the outer contour with PTCL (5), (6). In accordance with initial value theorem of Laplace transform the 3th order LPF will be proper for this goal, because the first 2 derivatives (velocity, acceleration) of output signal equal to zero at the beginning of transient process. Therefore, LPF transfer function looks as follows:

$$W_{LPF}(s) = \frac{1}{s^3 + m_1 s^2 + m_2 s + 1}. \qquad (15)$$

As far as LPF is applied to the reference track (RT) generation, it makes UAV track smoother, filling the transition points of RT with circle-like fillets and making it similar to the Dubins flight path [1]. This LPF was used in simulation; however, the Butterworth filter might be also successful alternative. Speaking about the bandwidths, it is necessary to note that on the one hand, the LPF bandwidth must be less than outer loop bandwidth, but on the other hand, both of them might be wide enough in order to provide minor time of switching from one segment to the next one. The reasonable trade-off between these two requirements can be achieved by the choice of transfer functions coefficients of LPF (15) and phase-lead compensator PLC (6).

IV. CASE STUDY

To demonstrate the efficiency of the proposed approach, we use a lateral channel of the UAV Aerosonde [11] as a case study. The cruise flight with true airspeed equals V_t = 26.0 m/s at the altitude 200 m is simulated. The linear state space model (1) is represented by matrices $[\mathbf{A}, \mathbf{B}, \mathbf{C}]$:

$$\mathbf{A} = \begin{bmatrix} -0.72 & 1.07 & -25.98 & 9.81 & 0 & 0 \\ -4.74 & -23.31 & 11.22 & 0 & 0 & 0 \\ 0.77 & -3.02 & -1.17 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 26.0 & 0 & 0 & 0 & 26.0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -1.6 & 4.08 \\ -140.33 & 2.52 \\ -5.53 & -25.78 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 26 & 0 & 0 & 0 & 26 & 0 \end{bmatrix}. \qquad (16)$$

The transfer function of the Dryden filter as well as its parameters for a case of a moderate turbulence at low altitude are given in [1, p.56]. The inner loop $\mathrm{H}_2 / \mathrm{H}_\infty$- optimization (10) – (12) was performed with output vector $\mathbf{Y} = [\beta, \quad \varphi, \quad p, \quad r]^{\mathrm{T}}$ and corresponding matrices of state space model $\mathbf{A_{inn}}, \mathbf{B_{inn}}, \mathbf{C_{inn}}$ produced from (16). Following weight matrices $\mathbf{Q}, \mathbf{R}$ in (11) at the final run of the genetic optimization procedure were used:

$$R = 0.01 \cdot \mathrm{eye}(2), \quad Q = \mathrm{diag}[0.4\ 0.7\ 0.9\ 0.4].$$

After running this procedure, the following numerical values of the adjustable parameters vector $\vec{K}_{AC}$ (9) were produced:

$$\vec{K}_{AC} = [3.05, 7.51, 0.67, 2.82, 1.15, \\ 2.01, 0.8, 0.16]. \quad (17)$$

At the 2nd stage of the design procedure we define the dynamic feasibility conditions, which must be imposed on the maximal magnitudes of angles and cross-track error in transient processes, as follows:

$$|\beta_{\max}| < 10°, |\varphi_{\max}| < 45°, |\delta a| < 10°, \\ |\delta r| < 6°, |e_{d\max}| < 20 \text{ m}. \quad (18)$$

Satisfying inequalities (18), which define the UAV flight safety is the ultimate goal of entire procedure of path tracking system design.

The structure of the guidance command system (GC) is described in [5] and is not given here for the sake of brevity. The output vector for the path-tracking loop looks as follows: $\mathbf{Y_{PT}} = [r, \psi, e_d, \dot{e}_d]^{\mathrm{T}}$. The matrices of the state space model of outer loop $\mathbf{A_{out}}, \mathbf{B_{out}}, \mathbf{C_{out}}$ are not given here due to their high dimension. Using parameters (17) as known for the inner loop, we can perform $\mathrm{H_2} / \mathrm{H_\infty}$-optimization (10) – (12) for outer loop. The result of this procedure is represented by following numerical values of vector $\vec{K}_{PT}$ components (13):

$$\vec{K}_{PT} = [0.3,\ 0.15,\ 0.035, \\ 0.175,\ 0.75,\ 0.02]. \quad (19)$$

Now it is possible to choose parameters of LPF transfer function (15) using Bode plot of outer loop contour (see Fig. 1: from input φ_{ref}, to output φ). Choosing $m_1 = 5,\ m_2 = 7$ in (15), we obtain relation between the bandwidth of system and bandwidth of LPF represented in Fig. 3, where system and LPF Bode plots are presented with black solid and grey dashed lines respectively. It coud be seen that LPF supresses high frequencies, thus producing smooth response of system.

The simulation of path tracking system behavior with parameters (17), (19) was performed for calm and moderate turbulent atmosphere using Simulink package. The implementation of Guidance Commander GC for path tracking loop (see Fig. 1) in Simulink is given in [5]. Initial conditions of translational motion in the inertial frame (in meters) equal [–100, 200] (Fig. 4a).

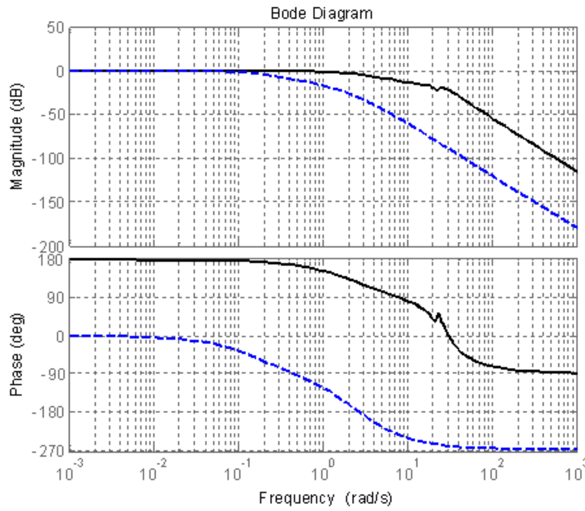
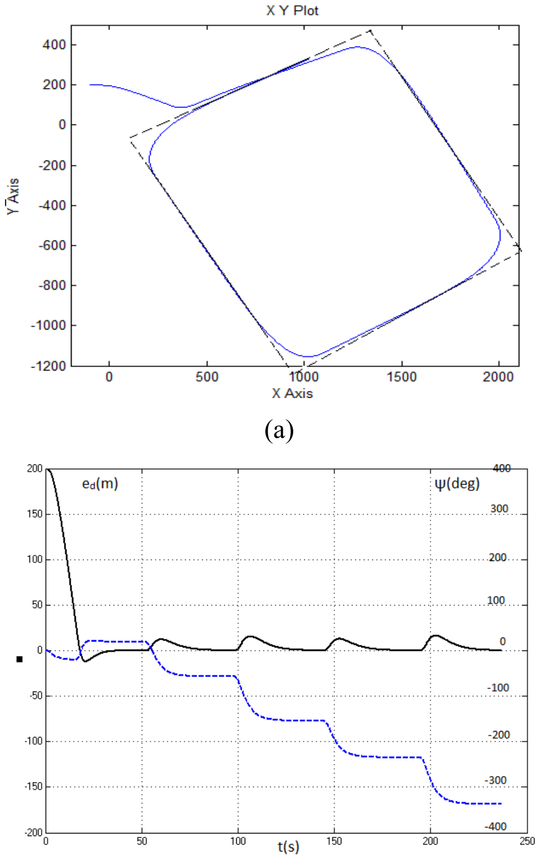


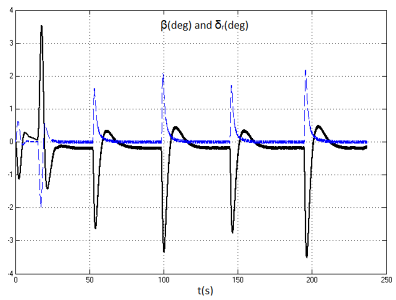
Fig. 3. Bode plots of path tracking system (solid, black) and LPF (dashed, grey)



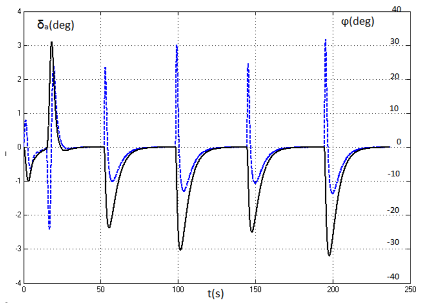
(a)

(b)

Fig. 4. Results of path tracking system simulation in calm atmosphere: (a) flight trajectory, where the grey solid line represents actual UAV trajectory and black dashed line shows reference track; (b) cross-track error e_d in m (black line) and UAV heading angle ψ in deg (grey line); (c) sideslip angle β (black line) and rudder deflections δr (grey line), both in deg; (d) roll angle φ (black line) and aileron deflections δa (grey line), both in deg



(c)



(d)

Fig. 4. Ending. (See also p. 134)

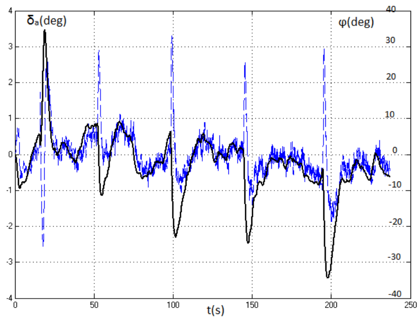
From this Figure, it can be seen that the maximal deflections of state variables produced by switching from current path segment to the next one are following:

$$|\beta_{\max}| < 4°, |\varphi_{\max}| < 34°, |\delta a| < 4°, |\delta r| < 3°, \quad |e_{d\max}| < 17 \text{ m}. \quad (20)$$

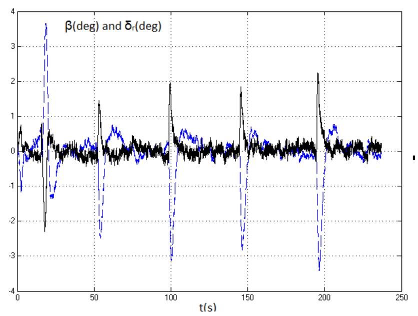
Comparing (20) and (18) it is possible to conclude that for the calm atmosphere the goal of design procedure is achieved.

As we can see from Fig. 4a, including low pass filter in the command switching for transition of current segment of reference track to the next one leads to the circle–like fillets between these segments, thus creating Dubins-like UAV path. In Figure 5 results of path following process simulation for UAV flight in moderate turbulent atmosphere are represented.

The flight path obtained here has the slightest distinctions from Fig. 4a, and that is why it is impossible to notice difference between them in the scales of Figs 4a and b.



(a)



(b)

Fig. 5. Results of simulation in turbulent atmosphere: (a) roll angle φ (black line) and aileron deflections δa (grey line), both in deg, (b) sideslip angle β (black line) and rudder deflections δr (grey line), both in deg.

The same we can say about track error e_d and the heading angle ψ. So corresponding results are not shown in Fig. 5. Only plots of $\varphi, \delta a, \beta$ and δr are worthy to be shown here. This Figure shows that requirements (18) are also satisfied in a case of turbulent atmosphere.

VII. Conclusions

Based on the results of the work, the following conclusions can be drawn.

1) UAV smooth path following the linear piecewise reference track requires taking in account complete mathematical model of the closed-loop attitude control system with known structure incorporated in the path following control system.

2) In order to avoid the over-parameterization problem in performing of robust H_2 / H_∞-optimization procedure for entire system, it is proposed to decompose this procedure by two sequential stages consisting of optimization of the inner loop (attitude control) and then optimization of the outer loop (path following control).

3) In order to achieve dynamic feasibility of path tracking system in the switching processes for transition from current linear segment to the next one we propose to include low pass filter in the switching algorithm for reference signals conditioning. The bandwidth and parameters of this filter can be determined after finding of all parameters of two contours path following system.

4) Efficiency of proposed design procedure was proved by simulation of this system in conditions of calm and light turbulent atmosphere.

References

[1] Randal W. Beard and Timothy W. McLain, Small unmanned aircraft: theory and practice. Princeton University Press, 2012.

[2] G. H. Elkaim, F. A. Pradipta Lie, and D. Gebre-Egziabher, Principles of Guidance, Navigation, and Control of UAVs, in "Handbook of Unmanned Aerial Vehicles". Springer Netherlands, 2015, pp. 347–380.

[3] P. Sujit, S. Saripally, and J. B. Sousa, "Unmanned Aerial Vehicle Path Following," *IEEE Control Systems Magazine,* vol. 34, no.1, February, 2014, pp. 42–59.

[4] S. Park, J. Deyst, and J. P. How, "Performance and Lyapunov Stability of a Nonlinear Path-following Guidance Method," *J. Guidance, Control, Dyn.*, vol. 30, no. 6, pp. 1718–1728, 2007.

[5] A. A. Tunik and M. M. Komnatska, "Robust Optimization of the UAV Path Following Guidance Systems," *IEEE 4th International Conference on Methods and Systems of Navigation and Motion Control (MSNMC)*. October, 18–20, 2016, pp. 25–30.

[6] Prentice Hall International, 1990, 593p.

[7] A. A. Tunik, H. Ryu, and H.-C. Lee, "Parametric Optimization Procedure for Robust Flight Control System Design," *International Journal of Aeronautical and Space Sciences*. vol. 2, no. 2, pp. 95–107, November 2001.

[8] E. Schoemig and M. Sznaier, "Mixed H_2/H_∞ Control of Multimodel Plants," *Journal of Guidance, Control and Dynamics*. no. 3, pp. 525– 31, May–June 1995.

[9] G. M. Hoffman, S. L. Waslander, and C. J. Tomlin, "Quadrotor Helicopter Trajectory Tracking Control," *Proc. of the AIAA Guidance, Navigation and Control Conf. and Exhibition*, 18-21 August 2008, Hawaii, Honolulu, pp.1–14.

[10] S. Skogestad ad I. Postlethwaite, Multivariable Feedback Control. Analysis and Design, John Wiley & Sons, 1997, 559 p.

[11] AEROSIM BLOCKSET. Version 1.2 User's Guide Unmanned Dynamics, LLC No.8 Fourth St. Hood River, OR 97031 (503) 329-3126 http://www.u-dynamics.com

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А. А. Тунік, М. О. Лаванова. Комп'ютерне проектування системи траєкторного керування БПЛА з фіксованими крилами

В статті розглянуто задачу плавного відслідковування еталонної траєкторії для безпілотного літального апарата з фіксованими крилами, коли еталонна траєкторія є кусково-лінійною, що має точки розриву з різкою зміною курсового кута. Кінцевою ціллю є придушення відхилень компонентів вектора стану , які є небезпечними з точки зору безпеки польоту, приймаючи до уваги повну математичну модель внутрішнього контуру кутової стабілізації. Внутрішня система кутової стабілізації та зовнішня система траєкторного керування складається з найпростіших елементів, що часто використовуються, таких як ПД-регулятори, згладжуючі фільтри та фазовипереджаючі ланки, параметри котрих були знайдені з допомогою процедури H_2/H_∞- оптимізації, що базується на генетичних алгоритмах. Щоб запобігти надмірної параметризації процедури оптимізації, запропоновано вирішити цю проблему, використовуючи її розклад на два послідовних етапи: пошук оптимальних параметрів внутрішнього контуру на першому етапі, а потім пошук оптимальних параметрів зовнішнього контуру на другому етапі. Запропоновано використати фільтр низьких частот для перетворення еталонного сигналу крену для того, щоб отримати гладку псевдо-Дубінсовську траєкторію польоту безпілотного літального апарата. Моделювання польоту безпілотного літального апарата в спокійній та помірно турбулентній атмосфері доводить ефективність запропонованого методу проектування.

Ключові слова: керування кутовим положенням; траєкторне керування; H_2/H_∞ - оптимізація; функція чутливості, комплементарна функція чутливості, генетичні алгоритми.

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А. А. Туник, М. О. Лаванова. Компьютерное проектирование системы траекторного управления БПЛА с фиксированными крыльями

В статье рассмотрена задача плавного отслеживания эталонной траектории для беспилотного летательного аппарата с фиксированными крыльями, когда эталонная траектория является кусочно-линейной, имеющей точки разрыва с резким изменением курсового угла. Конечной целью является подавление отклонений компонентов вектора состояния, которые являются опасными с точки зрения безопасности полета, принимая во внимание полную математическую модель внутреннего контура угловой стабилизации. Внутренняя система угловой стабилизации и внешняя система траекторного управления состоят из простейших часто используемых элементов, таких как ПД-регуляторы, сглаживающие фильтры и фазоопережающие звенья, параметры которых были найдены с помощью процедуры H_2/H_∞ - оптимизации, основанной на генетических алгоритмах. Чтобы избежать чрезмерной параметризации процедуры оптимизации, предложено решить эту проблему, используя ее разложение на два последовательных этапа: поиск оптимальных параметров внутреннего контура на первом этапе, а затем поиск оптимальных параметров внешнего контура на втором этапе. Предлагается использовать фильтр низких частот для преобразования эталонного сигнала крена, с тем, чтобы получить гладкую псевдо-Дубинсовскую траекторию полета беспилотного летательного аппарата. Моделирование полета беспилотного летательного аппарата в спокойной и умеренной турбулентной атмосфере доказывает эффективность предлагаемого метода проектирования.
Ключевые слова: управление угловым положением; траекторное управление; H_2/H_∞ - оптимизация; функция чувствительности; комплементарная функция чувствительности; генетические алгоритмы.

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