UDC 531/534:57+612.7 (045)
DOI: 10.18372/1990-5548.57.13248

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INVESTIGATION OF BLOOD FLOW ON THE INPUT OF AORTA

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***Abstract*—*The processes of blood flow in the aorta under the influence of swirling blood flow at the output from the left ventricle of the heart are studied. The problem of simulated swirling flow of blood in the left ventricle as a nonlinear boundary value problem in the form of a system of differential equations in partial derivatives with moving limits is formulated. Expressions for the field of blood flow velocity and pressure in the left ventricle are obtained. The flow of blood in the aorta under the influence of a swirling flow at the exit from the left ventricle is described by a system of nonlinear equations in partial derivatives. The solution of this boundary-value problem is sought using an iterative procedure based on using integral transformations for spatial variables and time.*

Index Terms—Aorta; vortex flows; swirling currents; Navier–Stokes equation; integral transformations; left ventricle.

I. INTRODUCTION

The nature of vortex currents in the left ventricle of the heart has been studied and studied by many leading scientists in the world. The left ventricle is seen as part of an elongated ellipsoid with a movable wall, the dynamics of which is induced from the outside. One of the most important properties of the blood that is observed in the left ventricle during diastole is the presence of vortex rings that curl through the jet phenomena coming from the mitral valve. The presence of vortex rings that develop during stasis of diastole is confirmed by numerous experimental studies on the basis of Doppler and magnetic resonance. During diastole, when the left ventricle is filled with a blood stream from the atrium, the ventricle expands, resulting in the area of the stomach moving in the opposite direction to the flow of blood.

II. PURPOSE OF THE RESEARCH

The purpose of the research is to develop a mathematical model of the vortex blood flow in the left ventricle, under the influence of which the flow of blood swirled at the entrance to the aorta and on this basis in the development and solution of the corresponding mathematical model of blood flow in the aorta.

III. OVERVIEW

It is believed [1] that the main dimensionless parameter for any viscous flow is the Reynolds number. The characteristic of the degree of rotation of the flow is the spin parameter. Experimental studies [2], usually use the integral spin parameter:

$$G = \frac{F_{mm}}{F_m R}. \tag{1}$$

The momentum flow in the axial direction, taking into account the contribution of the components of the turbulent shear stress [1].

$$F_{mm} = \int_\Sigma (\rho V_z V_\varphi + \rho \overline{V_z' V_\varphi'}) r d\Sigma.$$

The flow of traffic in axial direction, taking into account the contribution of turbulent normal stresses and pressures.

$$F_m = \int_\Sigma [\rho V_z^2 + \rho \overline{V_z^2} + (p - p_\infty)] d\Sigma.$$

Most of the twisty currents in the technical dictations are turbulent. Therefore, an effective Reynolds number is introduced and used to solve the complete Navier–Stokes differential equation system. A detailed review and analysis of such models is made in [4], and an example of a study of a swirling current in a vortex tube and jet is considered in [5].

A significant drawback of all these models, which are considered in the links, is the absence of external force, which prompts the emergence of twisted streams.

The Navier–Stokes equation and the continuity equation for an axially symmetric flow in cylindrical coordinates (r, θ, z) can be represented in the form [2].

$$\frac{1}{r}\frac{\partial r v_r}{\partial r} + \frac{\partial v_z}{\partial z} = 0. \tag{2}$$

$$\begin{aligned} &\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \\ &= -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{\partial^2 v_z}{\partial z^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right), \end{aligned} \tag{3}$$

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}$$
$$=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}-\frac{v_r}{r^2}\right), \quad (4)$$

Following the generally accepted method to exclude the variable p from these equations, we write them with respect to the function of the current ψ, the vorticity Ω and the azimuthal velocity V_φ:

$$\frac{\partial\Omega}{\partial t}+\frac{\partial}{\partial z}(V_z\Omega)+\frac{\partial}{\partial r}(V_r\Omega)$$
$$=\frac{1}{Re}\left[\frac{\partial^2\Omega}{\partial z^2}+\frac{\partial^2\Omega}{\partial r^2}+\frac{\partial(\Omega/r)}{\partial r}\right]+G^2\frac{1}{r}\frac{\partial (V_\varphi)^2}{\partial z}, \quad (5)$$

$$\frac{\partial V_\varphi}{\partial t}+\frac{\partial}{\partial z}(V_zV_\varphi)+\frac{\partial}{\partial r}(rV_rV_\varphi)+\frac{V_rV_\varphi}{r}$$
$$=\frac{1}{Re}\left[\frac{\partial^2 V_\varphi}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\varphi}{\partial r}\right)-\frac{V_\varphi}{r^2}\right], \quad (6)$$

$$\frac{1}{r}\frac{\partial^2\psi}{\partial z^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)=-\Omega, \quad (7)$$

where

$$V_r=-\frac{1}{r}\frac{\partial\psi}{\partial z},\quad V_z=\frac{1}{r}\frac{\partial\psi}{\partial r},\quad \Omega=\frac{\partial V_r}{\partial z}-\frac{\partial V_z}{\partial r}. \quad (8)$$

The flow is considered in the cylindrical region $D\{0\le z\le z_k,\ 0\le r\le r_k)\}$ bounded by the planes $z=0$, $z=z_k$ and the surface of rotation $r=r_k$. In the case of flow in an axisymmetric channel, the computational domain is bounded by a solid surface $r_k=1$, in the case of a free vortex, by the conditional boundary $r_k=\text{const}\gg 1$.

The main input data determining the flow development in the D region are given in the initial section $z=0$: $V_z=V_{z0}(r),\ V_\varphi=V_{\varphi 0}(r)$.

The radial velocity is generally assumed to be zero. The functions $V_{z0}(r)$, $V_{\varphi 0}(r)$ are taken either from experimental data or derived from theoretical considerations. The first possibility refers to the case when the initial field is formed using special devices; such flows are organized in pipes for technical applications. The second possibility arises in cases where the vortex flow with a twist is formed due to the natural development of the flow.

Thus, for $z=0$ there is some flow with a certain initial spin. It is required to determine the further structure of such an initial swirling flow in the D region. Ultimately, for the region under study, it is necessary to find the velocity field and construct a picture of the streamlines. Of particular interest here are the areas of return currents adjacent to the flow axis, which can be formed with certain combinations of the Reynolds number and the spin parameter. The formation and structure of such recirculation zones will be focused on.

In the output section, the boundary conditions can be set differently. It can be possible directly set the values of ψ, Ω and V_φ (hard boundary conditions) or assume that the derivatives of these variables on the coordinate z are zero (soft boundary conditions).

$$\frac{\partial\psi}{\partial z}=\frac{\partial\Omega}{\partial z}=\frac{\partial V_\varphi}{\partial z}=0,\quad 0\le r\le r_k,\quad z=z_k.$$

They have a weak effect on the structure of the flow upstream; therefore, when using them, one can confine oneself to a less extended region D along z.

The adhesion conditions are set on the side surface of the computational domain for swirling flow in an axisymmetric channel

$$\psi=\psi_1=\text{const},\ V_\varphi=0,\ \frac{\partial\psi}{\partial r}=0,\ 0\le z\le z_k,\ r=r_k,$$

The current function is determined up to a constant, so it is assumed that $\psi=0$ for $r=0$. Then on the flow axis it is had the following conditions of flow symmetry

$$\psi=0,\ V_\varphi=0,\ \Omega=0,\quad 0\le z\le z_k,\ r=0.$$

Initial conditions specified

$$\Omega=\Omega_0(r,z),\quad V_\varphi=V_{\varphi 0}(r,z),\ t=0,\ (r,z)\in D. \quad (9)$$

The general algorithm for solving the Navier – Stokes system (equations (5) – (7)) includes the following iterations [7]. For each time step, the Poisson equation is first solved for ψ, then the values of V_z, V_r are calculated using the formulas (8), then the equation (7) is solved for V_φ, after which the vorticity field from (6) is determined.

A significant feature of the movement of blood in the left ventricle is that the shape of the heart is a function of time, that is, it is mobile. Therefore, it is necessary to path from a cylindrical coordinate system to a mobile one, considering the shape (idealized) in the form of an elongated ellipsoid.

The degenerate ellipsoidal coordinates (α,β,φ) for an elongated ellipsoid of rotation are determined using the formulas

$$x=c\sin\beta\cos\varphi,\ y=c\sin\alpha\sin\beta\sin\varphi,\ z=c\alpha\cos\beta,$$

c is a scale factor, $0\le\alpha<\infty$, $0\le\beta\le\pi$, $-\pi\le\varphi\le\pi$.

Coordinate surfaces: extruded ellipsoids of rotation $\alpha = \text{const}$, [3].

$$h_1 = h_2 = c\sqrt{^2\alpha + \sin^2\beta}, \quad h_3 = h_\varphi = c\alpha\sin\beta.$$

$$\Delta u = \frac{1}{c^2\left(^2\alpha + \sin^2\beta\right)}\left[\frac{1}{\alpha}\frac{\partial}{\partial\alpha}\left(\alpha\frac{\partial u}{\partial\alpha}\right)\right.$$
$$\left.+\frac{1}{\sin\beta}\frac{\partial}{\partial\beta}\left(\sin\beta\frac{\partial u}{\partial\beta}\right)+\left(\frac{1}{^2\alpha}+\frac{1}{\sin^2\beta}\right)\frac{\partial^2 u}{\partial\varphi^2}\right]. \quad (10)$$

After solving the boundary value problem (5) – (7) taking into account (10) over the found velocity field, the pressure distribution in the flow can be determined from the following Poisson equation:

$$\frac{\partial^2 p}{\partial z^2}+\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}$$
$$=2\left(-\frac{\partial V_z}{\partial r}\frac{\partial V_r}{\partial z}+\frac{\partial V_r}{\partial r}\frac{\partial V_z}{\partial z}+G^2\frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial r}\right).$$

The boundary conditions for it are the Neumann conditions, which are obtained from the equations of the normal component of the impulse and in dimensionless form have the form:

$$\frac{\partial p}{\partial z}=-\frac{1}{Re}\left(\frac{\partial\Omega}{\partial r}+\frac{\Omega}{r}\right), \ 0\le r\le 1, \ z=0, z=z_k,$$

$$\frac{\partial p}{\partial r}=\frac{1}{Re}\frac{\partial\Omega}{\partial z}+\frac{V_\varphi^2}{r}+\frac{V_r^2}{r}, \ 0\le z\le z_k, \ r=1,$$

$$\frac{\partial p}{\partial r}=0, \ 0\le z\le z_k, \ r=0.$$

The metric coefficients of the moving coordinate system are

$$h_\mu=\alpha h_\eta, \ h_\theta=\delta(\alpha\mu)\sin\eta, \ h_\eta=\delta\sqrt{2(\alpha\mu)-\cos 2\eta},$$

where the time dependence is omitted for brevity. This coordinate system describes a moving object in physical space; therefore, a fixed point in (μ, η, θ) – space has a physical velocity c, whose components can be written in the general case as

$$c_\mu=\dot{\delta}\frac{\delta\alpha}{h_\mu}(\alpha\mu)(\alpha\mu)+\mu h_\mu\frac{\dot{\alpha}}{\alpha},$$

$$c_\eta=\dot{\delta}\frac{\delta}{h_\eta}\sin\eta\cos\eta, \quad c_\theta=0,$$

where the point denotes the time derivative. In this case, the expression for c_μ is simplified, since $\dot{\alpha}=0$.

The diameter of $D_0=D(t=0)$ at the beginning of the diastole filling phase is selected as the reference length scale. The time scale of T is the heart rate period. Thus, we have the Stokes number $\beta = D_0^2/\mu T$, ν is the kinematic viscosity of the fluid.

Here the diastolic phase is analyzed, therefore it has a dimensionless duration of approximately 0.5.

The system is excited by the arrival of a discharge with a given temporary law; A simple analytical form was chosen, which reproduces the rapid acceleration and deceleration of the flow pulsation inside the chambers of the heart and the main arterial vessels. It is represented by a dimensionless function.

$$\mathcal{L}(t)=A(St)t^2e^{-ft}, \quad (11)$$

$f=20$ is the characteristic deceleration frequency, giving a peak time of $t_p=0.1$.

The $A(St)$ function, which scales the total bit, depends on the Strouhal number $St=D_0/(UT)$; the scale of U is the speed at the input section, $\eta=\pi/2$, which corresponds to the maximum value of the discharge $\mathcal{L}_p=\mathcal{L}(t_p)$ averaged over the area actually occupied jet. The following velocity profile v_η is assigned to the input:

$$v_\eta(\mu,\theta)=C(t)\exp\left[-\left(\frac{(r\cos\theta-\varepsilon)^2+(r\sin\theta)^2}{\sigma^2}\right)^4\right] \quad (12)$$
for $\eta=\pi/2$,

where ε is the eccentricity of the profile, σ controls the relationship between the incoming jet and the diameter $D(t)$, C is the normalization factor for matching with (11). From the formula (12) the velocity scale is $U=4\mathcal{L}_p/(\pi(\sigma D_0)^2)$.

The real values for the parameters $\sigma\simeq(0.6-0.7)$ and $\varepsilon\simeq(0.1-0.14)$ cm; in this paper, we use the fixed value $\sigma=0.6$, therefore $C=209/St$.

A realistic flow profile $Q(t)$ for an ideal early filling period can be represented as

$$Q=At^2e^{-ft}, \quad (13)$$

where f is the scale frequency deceleration; A is the scale of the total volume input. The specification of the law of discharge corresponds to the change in time of the volume $V=\pi^2/6D^2H$ and gives the ratio between the diameter and height of the derivatives

$$Q=\frac{\pi^2}{6}D^2H\left(\frac{2}{D}\frac{dD}{dt}+\frac{1}{H}\frac{dH}{dt}\right).$$

The system was analyzed by changing the eccentricity of the ε inlet profile (13) in the range from 0.02 to 0.125. The Stokes number β was considered in the interval between 64 and 144, the Strouhal number was first set to $St = 0.072$, and then reduced to 0.05.

The case of $\beta = 144$, $\varepsilon = 124$ is discussed as a reference; the results are then compared with the results obtained with different parameter values.

To determine the moving boundaries in [5], a system of differential equations for $D(t)$ and $H(t)$ is proposed:

$$\frac{dD}{dt} = \frac{6Q}{\pi} \frac{8H^2 - D^2}{20DH^3 - 2HD^3}, \; t \in (0, T),$$

$$\frac{dH}{dt} = \frac{H}{D}\frac{dD}{dt}\frac{4H^2}{8H^2 - D^2}, \; t \in (0, T),$$

with initial conditions $D(0) = D_0$, $H(0) = H_0$.

IV. Setting Research Tasks

The considered mathematical models of fluid motion in the left ventricle (2) – (4) and the corresponding equations for vorticity (5) – (7), in our opinion, does not correspond to the actual state of affairs. It means the following. Fluid motion and, respectively, equations for vorticity are considered as non-stationary equations (6) – (7), that is, vorticity is a function of spatial coordinates and time, and Poisson's equation (5) – stationary. It is logical instead of (5) to write a nonstationary equation

$$\frac{\partial \psi}{\partial t} = \frac{1}{r}\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \Omega.$$

The study of vortex flows in the heart in the moving coordinate system leads to the need to consider the following system of nonlinear equations with respect to flow velocity and vorticity. First of all, we write down the expressions for the Laplacian and the divergence of the flow (the convective component in the moving coordinate system) taking into account the moving wall of the heart and equation:

$$\nabla v u = \frac{\partial v_r u}{\partial r} + \frac{\partial v_z u}{\partial z} + \frac{\partial v_\theta u}{\partial \theta}$$
$$= \frac{1}{\alpha h_\mu}\left[\left(\frac{\partial v_\mu}{\partial \mu} + \alpha \frac{\partial v_\eta}{\partial \eta} + \frac{\alpha h_\mu}{h_\theta}\frac{\partial v_\theta}{\partial \theta}\right) u + v_\mu \frac{\partial u}{\partial \mu} + \alpha v_\eta \frac{\partial u}{\partial \eta} + \frac{\alpha h_\mu}{h_\theta} v_\theta \frac{\partial u}{\partial \theta}\right].$$

Here, the symbol u in accordance with the equations (5) – (7) is denoted Ω, V_φ or ψ. The system of equations (5) – (7) can now be written in this form:

$$\frac{\partial \psi}{\partial t} = \Delta \psi + \Omega, \qquad (14)$$

$$\frac{\partial \Omega}{\partial t} + \nabla v \Omega = \frac{1}{\mathrm{Re}}\Delta\Omega + G^2 \frac{1}{r}\frac{\partial v_\theta^2}{\partial z}, \qquad (15)$$

$$\frac{\partial v_\theta}{\partial t} + \nabla v v_\theta = \frac{1}{\mathrm{Re}}\left[\Delta v_\theta - \frac{v_\theta}{r}\right], \qquad (16)$$

Formulas (8) take the form:

$$v_\mu = -\frac{1}{h_\mu}\frac{\partial \psi}{\partial \mu}, \qquad v_\eta = \frac{1}{h_\eta}\frac{\partial \psi}{\partial \eta}, \qquad (17)$$

$$\Omega = \frac{1}{\alpha h_\eta}\left(\frac{\partial v_\eta}{\partial \mu} - \alpha \frac{\partial v_\mu}{\partial \eta}\right) + \frac{\alpha}{\delta}\frac{1}{h_\eta^3}\left[\cos(2\eta) v_\eta - \alpha(2\alpha\mu) v_\mu\right].$$

V. Problem Solution

Since the reduced equations describing the swirling flows in the left ventricle represent a system of nonlinear equations for fluid flow velocity, vorticity and current functions, as well as wall motion, the real way to solve such a system of equations, in our opinion, lies in using approximate numerical-analytical methods, since the use of difference schemes for solving nonlinear differential equations seems attractive only at the stage of writing the corresponding difference schemes. Practical implementation of them is associated with significant difficulties both algorithmic and computational aspects.

The construction of iterative schemes for numerical-analytical modeling consists of several stages, the first of which deals with the linear approximation of the corresponding boundary value problem. In this case, we begin with the search for a solution to the equation for the v_θ – equation (17). We write the equations (14) – (16) in the form

$$\frac{\partial v_\theta}{\partial t} = \frac{1}{\mathrm{Re}}\left[\Delta v_\theta - \frac{v_\theta}{r}\right] - N_{v_\theta}, \qquad (18)$$

$$\frac{\partial \Omega}{\partial t} = \frac{1}{\mathrm{Re}}\Delta\Omega + G^2 \frac{1}{r}\frac{\partial v_\theta^2}{\partial z} - N_\Omega, \qquad (19)$$

$$\frac{\partial \psi}{\partial t} = \Delta\psi + \Omega, \qquad N_{v_\theta} = \nabla v v_\theta, \quad N_\Omega = \nabla v \Omega.$$

Boundary and Initial conditions on the bottom of rigion afte Laplace transform ve be as follows:

$$G(p)=\frac{a}{p+a^2}\frac{1}{p+\alpha_{n,m}}\frac{1}{1-e^{-\pi/ap}}.$$

$$g(t)=\frac{1}{12\pi}\left\{b_0+b_1e^{-\alpha_{n,m}t}+b_2\cos at+b_3/a\sin at\right\}.$$

Note that the statements of the authors of numerous publications related to solving the problem under study, that the equation is solved first (19) (in the stationary case, this Poisson equation) is doubtful, since it contains the unknown in the right-hand side the function $\Omega(\mu,\eta,t)$.

The general scheme for the numerical-analytical solution of nonlinear equations of mathematical physics is given in [8], [9].

The equation (18) can be represented in the following form.

$$\frac{\partial v_\theta^{(0)}}{\partial t}=\frac{1}{Re}\left[\Delta v_\theta^{(0)}-\frac{v_\theta^{(0)}}{r}\right]-N_{v_\theta^{(0)}}. \quad (20)$$

In the linear approach we have a parabolic equation. The use of integral transformations in the η, θ and μ variables gives a solution

$$v_\theta^{(0)}=\sum_{m=1}^{M}\sum_{n=0}^{N}e^{-a^2\nu_{nm}^2t}Z_{nm}\left((\alpha\mu)\right)Y_{nm}(\cos\eta). \quad (21)$$

In this solution, Z_{nm} and Y_{nm} are the associated Legendre polynomials.

$$P_n^{(m)}(\cos(\eta)=\sin^m\eta\frac{d^m}{d\cos\eta^m}P_n(\cos\eta),$$

$$P_n(x)=\frac{1}{2^n n!}\frac{d}{dx^n}[(x^2-1)^n],$$

$$(n=0,1,2,\ldots).$$

The resulting expression for the azimuthal flow velocity is used to find the solution in the linear approximation of the equation (18). After that we find the solution of the equation (19).

V. Simulation of Aortic Flow Dynamics

An equation system, which describes the distribution of fluid in the aorta, is conveniently presented in cylindrical coordinates. For the assumption of the existence of the coefficients of turbulent exchange, the stationary three-dimensional equations have the form [6]:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r u_r)+\frac{1}{r}\frac{\partial}{\partial\varphi}(\rho r u_\varphi u_r)+\frac{\partial}{\partial z}(\rho r u_z u_r)=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{ef}\frac{\partial u_r}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{\mu_{ef}}{r}\frac{\partial u_r}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\mu_{ef}\frac{\partial u_r}{\partial z}\right)+S_{u_r} \quad (22)$$

$$S_{u_r}=\frac{\rho u_\varphi^2}{r}-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{ef}\frac{\partial u_r^2}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\mu_{ef}\frac{r\partial(u_\varphi/r)}{\partial r}\right)-2\frac{\mu_{ef}}{r}\frac{\partial u_\varphi}{r\partial\varphi}+\frac{\partial}{\partial z}\left(\mu_{ef}\frac{\partial u_z}{\partial r}\right),$$

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r u_\varphi)+\frac{1}{r}\frac{\partial}{\partial\varphi}(\rho r u_\varphi u_\varphi)+\frac{\partial}{\partial z}(\rho r u_z u_\varphi)=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{ef}\frac{\partial u_\varphi}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{\mu_{ef}}{r}\frac{\partial u_\varphi}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\mu_{ef}\frac{\partial u_\varphi}{\partial z}\right)+S_{u_\varphi}, \quad (23)$$

$$S_{u_\varphi}=\frac{\rho u_r u_\varphi}{r}-\frac{1}{r}\frac{\partial p}{\partial\varphi}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu_{ef}\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)\right]+\frac{\mu_{ef}}{r}\left(r\frac{\partial(u_\varphi/r)}{\partial r}\right)+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}+\frac{1}{r}\frac{\partial}{\partial\varphi}\left[\mu_{ef}\left(\frac{\partial u_\varphi}{\partial r\varphi}+\frac{2u_r}{r}\right)\right]+\frac{\partial}{\partial z}\left(\mu_{ef}\frac{\partial u_z}{r\partial\varphi}\right),$$

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r u_z)+\frac{1}{r}\frac{\partial}{\partial\varphi}(\rho r u_\varphi u_z)+\frac{\partial}{\partial z}(\rho r u_z u_z)=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{ef}\frac{\partial u_z}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{\mu_{ef}}{r}\frac{\partial u_z}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\mu_{ef}\frac{\partial u_z}{\partial z}\right)+S_{u_z}, \quad (24)$$

$$S_{u_z}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{ef}\frac{\partial u_r}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\mu_{ef}\frac{\partial u_\varphi}{\partial z}\right)+\frac{\partial}{\partial z}\left(\mu_{ef}\frac{\partial u_z}{\partial z}\right).$$

Simulation of twisted streams will be performed according to the iterative scheme [7]. For this we present the equation of the components of the speed of the fluid in the form

$$\frac{\partial\Phi}{\partial t}+\mu_{ef}\left(\frac{1}{r^3}\frac{\partial\Phi}{\partial\varphi}-\frac{1}{r}\frac{\partial\Phi}{\partial r}\right)$$
$$=\mu_{ef}\left[\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{\partial^2\Phi}{\partial z^2}\right]+N_\Phi, \quad (25)$$

$$N_\Phi=\rho\left(\frac{1}{r}u_r u_\Phi+u_r\frac{\partial\Phi}{\partial r}+\Phi\frac{\partial u_r}{\partial r}\right.$$
$$\left.+\frac{1}{r}\frac{\partial(u_\varphi\Phi}{\partial\varphi}+\frac{1}{r}u_\varphi\Phi+u_z\Phi\right)+S_\Phi.$$

Since the Navier–Stokes equation contains a term relative to pressure, one needs to add another equation for closing this equation system. This equation is an equation with respect to the flow temperature

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \qquad (26)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=\mu_T\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\partial \tilde{p}}{\partial x}-\frac{2}{3}\rho\frac{\partial K}{\partial x},$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=\mu_T\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\partial \tilde{p}}{\partial x}-\frac{2}{3}\rho\frac{\partial K}{\partial x},$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}\right)=\mu_T\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\partial \tilde{p}}{\partial z}-(\rho-\rho_\infty)g-\frac{2}{3}\rho\frac{\partial K}{\partial z},$$

$$c_p\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=\lambda_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right),$$

$$p=\rho RT.$$

Then $\frac{\partial p}{\partial x}=\rho R\frac{\partial T}{\partial x},\ \frac{\partial p}{\partial z}=\rho R\frac{\partial T}{\partial z}.$

Given the equation of continuity of the flow (25) we obtain the following system of equations:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=\mu_T\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)-\rho R\frac{\partial T}{\partial x}-\frac{2}{3}\rho\frac{\partial K}{\partial x},$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}\right)=\mu_T\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-\rho R\frac{\partial T}{\partial z}-(\rho-\rho_\infty)g-\frac{2}{3}\rho\frac{\partial K}{\partial z},$$

$$c_p\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=\lambda_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right).$$

The solution of the linear part of this system, taking into account the initial and boundary conditions, is obtained as: ($\zeta=z-h_2,\ \zeta\in[0,L_z]$, $x\in[l_{x-},L_{x+}]$):

$$u^{(0)}(x,\zeta,t)=\sum_{m_u,l_u}Z_u\left(\alpha_{m_u},\zeta\right)X_u\left(\beta_{l_u},x\right)\overline{\overline{u}}_n e^{-\gamma^u_{m_u},l^t_u t},$$

$$Z_u(\alpha_{mu},\zeta)=\frac{1}{\|Z_u\|}\left(\sin\alpha^u_{m_u}\zeta-\frac{\alpha^u_{m_u}}{\nu}\cos\alpha^u_{m_u}\zeta\right),$$

$$X_u(x)=\frac{1}{\|X_u\|}\sin\beta^u_{l_u}x,$$

$$w^{(0)}(x,\zeta,t)=\sum_{m_w,l_w}Z_w(\alpha_{m_w},\zeta)X_w\left(\beta_{l_w},x\right)\cdot\left(-\overline{\overline{G}}_{m_w,l_w}\right)\left[1-e^{-(\gamma^w_{m_w},l^t_w)t}\right],$$

$$Z_w(\alpha_{m_w},\zeta)=\frac{1}{\|Z_w\|}\cos\alpha^w_{m_w}\zeta,$$

$$X_w\left(\beta_{l_w},x\right)=\frac{1}{\|X_w\|}\cos\beta^w_{l_w}x,$$

$$T^{(0)}(x,\zeta,t)=\sum_{m_T,l_T}Z_T(\alpha_{m_T},\zeta)X_T\left(\beta_{l_T},x\right)\cdot\left[T0^{(0)}_{m_T,l_T}+T1^{(0)}_{m_T,l_T}e^{-(\gamma^T_{m_T},l^t_T)t}\right],$$

$$Z_T(\alpha_{m_T},\zeta)=\frac{1}{\|Z_T\|}\cos\alpha^T_{m_T}\zeta,$$

$$X_T(\beta_{l_T}x)=\frac{1}{\|X_T\|}\cos\beta^T_{l_T}x.$$

In the first approximation we get

$$u^{(1)}(x,\zeta,t)=\sum_{m_u,l_u}Z_u(\alpha^u_{m_u},\zeta)X_T(\beta_{l_u},x)\cdot\left[U1^0_{m_u,l_u}+e^{\sigma^u_{m_u,l_u}t}[U1^1_{m_u,l_u}\varphi1^u_{m_u,l_u}(t)+U2^2_{m_u,l_u}\varphi2^u_{m_u,l_u}(t)]\right]. \qquad (27)$$

$$w^{(1)}(x,\zeta,t)=\sum_{m_w,l_{wu}}Z_w(\alpha^w_{m_w},\zeta)X_T(\beta_{l_w},x)\cdot\left[W1^0_{m_w,l_w}e^{\sigma^w_{m_w,l_w}t}[W1^1_{m_w,l_w}\varphi1^w_{m_w,l_w}(t)+W2^2_{m_w,l_w}\varphi2^w_{m_w,l_w}(t)]\right], \qquad (28)$$

where $\sigma^u_{m,l}$ are self-values of ( ) with conditions ( ).

Further approximations are performed according to a similar scheme. Obviously, the application of the simplification algorithm leads to errors in the solutions of the corresponding boundary value problems. But these errors can be offset by additional iterations, using relatively simple expressions of the form (27) – (28). These iterations do not lead to additional complications of a computational nature, since they are realized by similar algorithms, which enables to automate the process of finding approximate solutions of the formulated boundary value problem.

At the next iteration, using the found vorticity expressions in the system of equations (18) and (19), we proceed to the consideration of the convective components.

Continue these iterations until the required accuracy of the solution is achieved. The results of modeling the components of the speed of the fluid in the root of the aorta are presented on Figs 1–3.

After the initial development of the flow at the entrance to the aorta there is a tendency to spin it with the further formation of vortices.

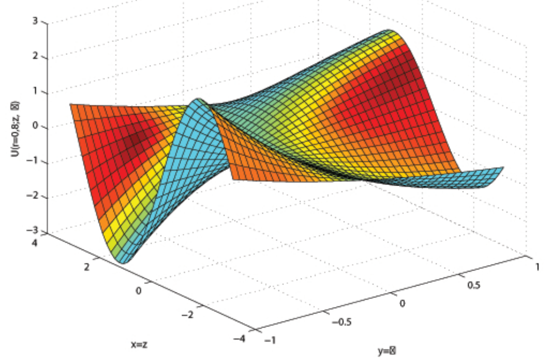

Fig. 1. Distribution of the longitudinal component of the speed of the liquid

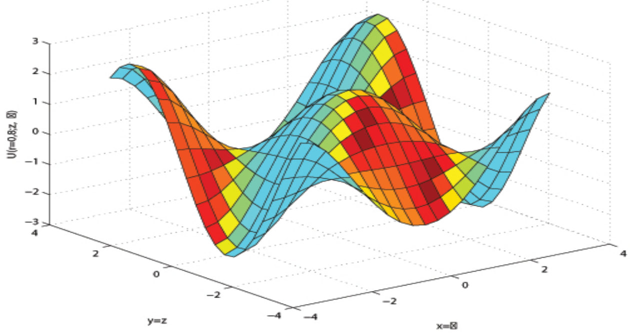

Fig. 2. Distribution of the azimuthal component of the fluid velocity

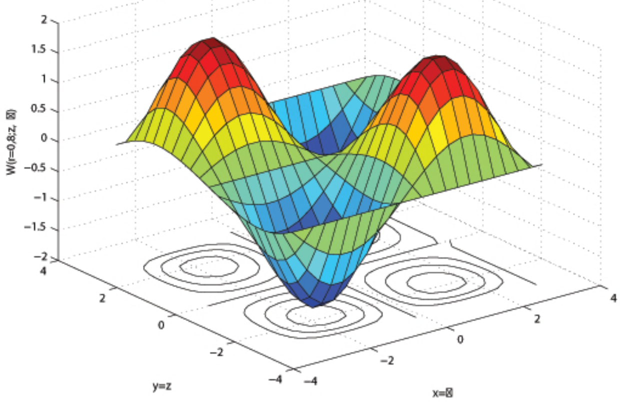

Fig. 3. Distribution of the radial component of the fluid velocity

VI. CONCLUSION

In this paper, on the basis of existing work related to the study of swirling flows in the heart, the problem of vortex flows in the left ventricle in a moving coordinate system was first formulated as a system of nonlinear differential equations in partial derivatives. To solve this system of equations, an iterative method has been proposed using integral transformations in finite limits along the corresponding coordinates.

Further studies are related to obtaining numerical-analytical solutions of this system of equations.

VII. DISCUSSION

The issues discussed in this paper attract the attention of numerous researchers all over the world. Despite the large number of works devoted to the formation of vortex flows in the heart and aorta and approaches to their study, there are currently no works that suggest approaches to constructive solutions to this complex issue.

The proposed work, too, does not pretend to be a final solution to this problem, but it is the first attempt to study the edema in the left ventricle of the heart and aorta, which, according to the author, should contribute to the diagnosis of heart disease and the development of recommendations for the treatment of these diseases.

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Received May 14, 2018.

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Є. А. Настенко. Дослідження потоку крові у гирлі аорти

Досліджуються процеси руху крові в аорті під впливом закручених потоків крові на виході із лівого шлуночка серця. Сформульовано задачу моделювання закручених потоків крові у лівому шлуночку як нелінійну крайову задачу у вигляді системи диференційних рівнянь у частинних похідних із рухомими межами. Отримано вирази для поля швидкості потоку крові та тиску у лівому шлуночку. Рух потоку крові в аорті під впливом закрученого потоку на виході із лівого шлуночка описується системою нелінійних рівнянь у частинних похідних. Розв'язок цієї крайової задачі відшукується за допомогою ітераційної процедури, що ґрунтується на використанні інтегральних перетворень за просторовими змінними та часу.

Ключові слова: аорта, вихрові потоки; закручені течії; рівняння Нав'є–Стокса; інтегральні перетворення; лівий шлуночок.

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Е. А. Настенко. Исследование потока крови в корне аорты

Исследуются процессы движения крови в аорте под влиянием закрученных потоков крови на выходе из левого желудочка сердца. Сформулирована задача моделирования закрученных потоков крови в левом желудочке как нелинейную краевую задачу в виде системы дифференциальных уравнений в частных производных с подвижными границами. Получены выражения для поля скорости потока крови и давления в левом желудочке. Движение потока крови в аорте под воздействием закрученного потока на выходе из левого желудочка описывается системой нелинейных уравнений в частных производных. Решение этой краевой задачи отыскивается с помощью итерационной процедуры, основанной на использовании интегральных преобразований по пространственным переменным и времени.

Ключевые слова: аорта; вихревые потоки; закрученные течения; уравнения Навье–Стокса; интегральные преобразования; левый желудочек.

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