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ACCURACY ESTIMATION OF FLIGHT TEST RESULTS ON THE BASIS OF APPROXIMATING THE CORRELATION FUNCTION

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Abstract— *The solution of the problem is considered, as a result of which estimates of statistical characteristics after processing the results of flight tests by the double averaging method, both in time and in the ensemble of realizations, are obtained. The approximation of the experimental curve of the correlation function is carried out by the analytical expression and the least squares method, the approximation parameters are determined. On the basis of the obtained approximation parameters, estimations of variance of statistical characteristics are determined, which are used subsequently to estimate the probability of finding trajectory parameters in the admissible region.*

Index Terms—Accuracy estimation; correlation function; probabilistic accuracy characteristics; double averaging; adaptive discretization; data processing technique; on-board control systems.

I. INTRODUCTION

The main point of the certification processes improvement of the on-board automatic control systems is the development of the most effective methods of processing data obtained as a result of flight tests and statistical modeling.

The theoretical basis of the certification processes for such systems is the methods of compliance of the tested aviation equipment with tactical and technical requirements. The basis for developing recommendations to improve the certification process is the methods of mathematical statistics in determining the accuracy characteristics of the on-board control systems for automatic approach and landing with the help of the respective algorithms.

In the article, an attempt is made to consider the possibility of using in the software one of the directions of statistical analysis-correlation analysis, which will increase the reliability of statistical conclusions.

II. PROBLEM STATEMENT

The construction of such algorithms can be based on information about the necessary parameters obtained from the definition of the higher moments of the random process distribution. Such information is provided by the correlation function, the estimation of which can be obtained on the basis of approximation of the experimental function by standard dependences.

The estimates of the mathematical expectation and the correlation function, obtained in the discrete processing of a random process, have the form:

$$m_x^* = \frac{1}{N}\sum_{j=1}^{N} x_j, \quad K_x^*(\tau) = \frac{1}{N-i}\sum_{j=1}^{N-i}(x_j - m_x^*)(x_{j-i} - m_x^*),$$

where $\tau = i\Delta t$, Δt is the interval between adjacent ordinates of the correlation function (the step of reading). For $\tau = 0$ the variance estimate is obtained.

The following expression is used to estimate the normalized correlation function: $\rho_x^*(\tau) = \dfrac{K_x^*(\tau)}{K_x^*(0)}$.

The estimates m_x^* and $K_x^*(\tau)$ with unknown mathematical expectation have an offset, in particular, the estimation of the correlation function has an offset whose maximum value is attained for $\tau = 0$ and is equal to the variance $\sigma_x^2(m^*)$ (the variance of mathematical expectation estimate). As is known, estimates of the correlation function (and, hence, of the normalized correlation function) are asymptotically unbiased. The effect of displacement is manifested with a small number of realizations and with small their duration for the use of the double averaging algorithm (time averaging and ensemble averaging) [1].

III. PROBLEM SOLUTION

The accuracy of estimates of the probability characteristics determines the confidence probability γ (reliability of estimation) in estimating the probability of finding the approach and landing parameters (vector $\vec{\mathbf{X}}$) in the acceptable area D.

$$\Pr\left\{\vec{\mathbf{X}} \in D\right\} \geq P_{\text{req}}.$$

It is known [2], [3], [1] that the variance of the estimate of the mathematical expectation is determined by the relation (for the case $M(m_x^*) = m_x$):

$$\sigma^2(m_x^*) = \frac{1}{N^2}\sum_{k,j}^{N} M\left[x^0(k\Delta t)x^0(j\Delta t)\right] = \frac{1}{N^2}\sum_{k,j}^{N} K_x^*[(k-j)] = \frac{1}{N^2}\sum_{p=-N}^{N}(N-p)K_x^*(p\Delta t)$$

$$= \frac{1}{N}\left[K_x^*(0) + 2\sum_{p=0}^{N-1}\left(1-\frac{p}{N}\right)K_x^*(p\Delta t)\right],$$

where x^0 is the centered value of the researched parameter, $p = k - j, N = \frac{T}{\Delta t}$ is the number of discrete samples in the observation interval T of the parameter realization.

If we take $\Delta t = \frac{\tau_c}{n}$, $n = 1, 2, ...,$ (τ_c is the interval of correlation) then we can estimate upper bound of the obtained expression neglecting by the values of $K_x^*\left(p\frac{\tau_c}{n}\right)$, whose argument is greater than τ_c ($K_x^* \approx 0$). In this case we can write:

$$\sum_{p=1}^{N-1}\left(1-\frac{p}{N}\right)K_x^*\left(p\frac{\tau_c}{n}\right) < \sum_{p=1}^{N-1}\left|K_x^*\left(p\frac{\tau_c}{n}\right)\right| < \sum_{p=1}^{n}K_x^*(0).$$

Since

$$\sum_{p=1}^{n}K_x^*(0). = nK_x^*(0) = K_x^*(0)\frac{\tau_c}{\Delta t},$$

then upper bound can be represented in the form.

$$\bar{\sigma}^2(m_x^*) = \frac{K_x^*(0)}{N} + \frac{2}{N}K_x^*(0)\frac{\tau_c}{\Delta t}$$

$$= K_x^*(0)\frac{\Delta t}{T} + \frac{2\Delta t}{T}K_x^*(0)\frac{\tau_c}{\Delta t} = K_x^*(0)\left\{\frac{\Delta t + 2\tau_c}{T}\right\}.$$

The variance of the estimation of the correlation function is given by the general expression:

$$\sigma^2\left[K_x^*(j\Delta t)\right] = \frac{1}{N}\sum_{p=-(N-1)}^{N-1}\left(1-\frac{p}{N}\right)$$

$$\cdot\left\{K_x^2(i\Delta t) + K_x^*[(i+j)\Delta t]K_x^*[(i-j)\Delta t]\right\}.$$

The variance of the estimation of the correlation function includes a fourth-order probability moment, the calculation of which is a difficult problem. For normal random processes, we can use the relation [2]:

$$\sigma^2\left[K_x^*(j\Delta t)\right] = 2K_x^2(0)\frac{\Delta t + 2\tau_c}{T}.$$

Dividing the left and right expressions by $K_x^2(0)$, we obtain the maximum relative mean square error:

$$\sigma\left[K_x^*(j\Delta t)\right] = \sqrt{\frac{2\Delta t + 4\tau_c}{T}} = \sqrt{\frac{4\tau_c}{T}\left(1+\frac{1}{2n}\right)}.$$

In article [3], the possibility of using in the calculations, instead of the unknown function $K_x^*(j\Delta t)$, an estimate of it, found as a result of processing realizations of a random process, is justified.

The estimation of the correlation function of any researched parameter (for example, the lateral deviation of the aircraft from the equiangular line or from the axis of the runway) is calculated on the basis of processing a stationary parts in the test flights, each of which contains 5–10 realizations. The problem of estimating its accuracy is substantially simplified if this function is approximated by the respective analytical expression [4], [5].

As an example, Fig. 1 (curve 2) shows a graph of the correlation function estimation of the process of changing the angle of heel during automatic landing of the aircraft in the area adjacent to the point of touchdown.

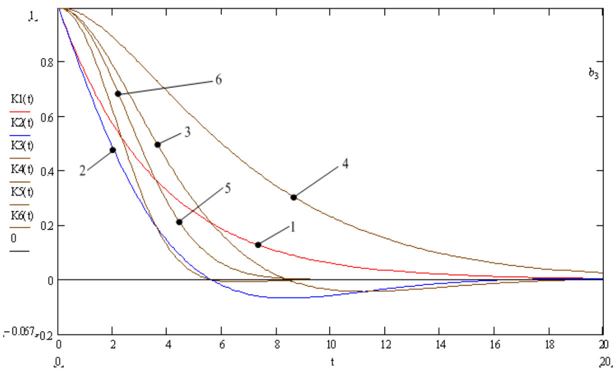

Fig. 1. Normalized correlation function of the process of changing the angle of heel

The estimation was obtained from six test flights (each flight contained 10 realizations) and approximated by the standard expression [1]:

$$\rho_\gamma^*(\tau) = e^{-\alpha|\tau|}\cos\beta\tau = e^{-0,28|\tau|}\cos 0,28\tau.$$

In article [6], the correlation interval is recommended to be taken from the condition: $\tau_0 = \left\{\tau : \left|\rho_x^* \le 0,3\right|\right\}$. For the considered example the quantity τ_0 at the level $\rho_\gamma^*(\tau) = 0,3$ is 2.9 s.

According to [3], [5], analytical expressions of typical normalized correlation functions are (Fig. 1):

$$\rho_1^*(\tau) = e^{-\alpha|\tau|};\ \rho_2^*(\tau) = e^{-\alpha|\tau|}\cos\beta|\tau|;$$

$$\rho_3^*(\tau) = e^{-\alpha|\tau|}\left(\cos\beta|\tau| + \frac{\alpha}{\beta}\sin\beta|\tau|\right);$$

$$\rho_4^*(\tau) = e^{-\alpha|\tau|}(1+\alpha|\tau|);$$

$$\rho_5^*(\tau) = e^{-\alpha^2\tau^2};\ \rho_6^*(\tau) = e^{-\alpha^2\tau^2}\cos\beta|\tau|.$$

The parameters of the correlation function α and β are calculated with the help of least-squares method.

For typical correlation functions, analytical expressions for the variances of estimates of the expectation and the correlation function are equaled to:

$$\sigma^2(m_x^*) = \frac{b_m}{T} = \frac{\sigma_x^2 b_m^*}{\alpha T},\ \sigma^2\left[K_x^*(\tau)\right] = \frac{b_c(\tau)}{T} = \frac{\sigma_x^4 b_c^*(0)}{\alpha T},$$

where

$$b_m^* = 2\int_0^\infty \rho_x(\tau)d\tau,\quad b_c^*(0) = 4\alpha\int_0^\infty \rho_x^2(\tau)d\tau.$$

In Table I, for typical correlation functions, the ratios for b_b^* and $b_c^*(0)$ are given for $T >> \tau_c$.

TABLE I COEFFICIENTS FOR FINDING ESTIMATION OF $\sigma^2(m_x^*)$ AND $\sigma^2\left[K_x^*(\tau)\right]$

ρ / b	ρ1	ρ2	ρ3	ρ4	ρ5	ρ6
b_m^*	2	$\frac{2}{1+\bar{\beta}^2}$	$\frac{4}{1+\bar{\beta}^2}$	4	$\sqrt{\pi}$	$\sqrt{\frac{\pi}{2}}\left(1+e^{-\bar{\beta}^2/4}\right)$
$b_c^*(0)$	2	$\frac{2+\bar{\beta}^2}{1+\bar{\beta}^2}$	$\frac{5+\bar{\beta}^2}{1+\bar{\beta}^2}$	4	$\sqrt{2\pi}$	$\sqrt{\frac{\pi}{2}}\left(1+e^{-\bar{\beta}^2/2}\right)$

The above dependences and relations allow to determine the variances of the corresponding estimates approximately (with a relative error of up to several percents).

We also note that in determining the estimation of the correlation function in rhythm with experiment (successive arrival of realizations during the test flight), one can refuse from a uniform change in the signal delay, which leads to redundancy of the information, since the frequency of the calculated points remains constant both for large and small changes in $\rho_x^*(\tau)$.

The delay step is chosen in inverse proportion to the rate of change of the function $\rho_x^*(\tau)$. For example, for *i*th step the following expression may be used:

$$\Delta\tau_i = \frac{k}{|\rho'(\tau)|} \approx \frac{k}{\left|\frac{\Delta\rho_{i-1}^*(\tau)}{\Delta\tau_{i-1}}\right|},$$

where k is a coefficient, determined from a priori considerations [3].

The value in the denominator is the averaged derivative of the function $\rho_x^*(\tau)$ on the interval $\Delta\tau_{i-1}$, Fig. 2. For a linear extrapolation of the function $\rho_x^*(\tau)$ between readings, this quantity is defined as $\tan\beta_{i-1}$. Obviously, such a delay variation can be considered as a quantization of the correlation function in terms of the level.

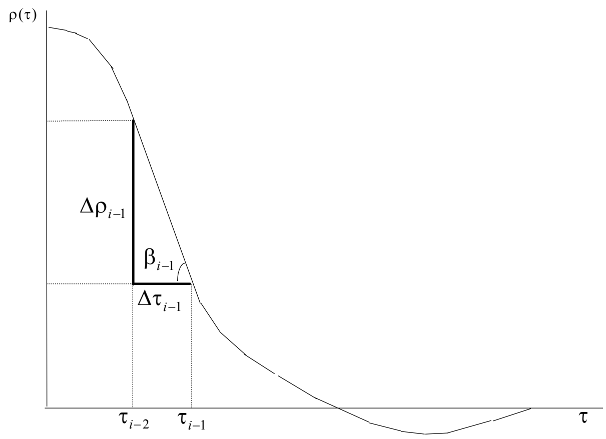

Fig. 2. Definition of delay step

When $\Delta\tau_i$ is chosen by according to such algorithm, the measurement frequency increases in the region of a significant change in the slope of the $\rho_x^*(\tau)$ curve. Thus, the adaptive discretization of the realizations allows to choose measurement steps in accordance with the change of the accumulated curve $\rho_x^*(\tau)$ and eliminate the redundancy of the calculated points of the correlation function.

IV. CONCLUSIONS

The results are aimed at developing more advanced methods for processing the results of flight tests to estimate the accuracy characteristics of the automatic approach and landing of the aircraft with the purpose of determining the compliance of on-board control systems with specified requirements.

This will make it possible to extract more information from the experimental data with the same volume of flight tests and significantly reduce the time of processing the test results while increasing the accuracy of the data obtained.

The development of effective methods of statistical processing parameters that characterize the automatic landing process can reduce the processing time, the cost of the experiment and increase its effectiveness. The creation of the corresponding software will allow realizing the operative analysis of the measurement information, processing data coming from several sensors.

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О. А. Зеленков, О. О. Бунчук, А. П. Голік. Оцінка точності результатів льотних випробувань на основі апроксимації кореляційної функції

Розглянуто розв'язання задачі, в результаті чого отримано оцінки статистичних характеристик після оброблення результатів льотних випробувань методом подвійного усереднення, як в часі, так і за ансамблем реалізацій. Здійснено апроксимацію експериментальної кривої кореляційної функції аналітичним виразом і методом найменших квадратів визначено параметри апроксимації. На підставі отриманих параметрів апроксимації визначено оцінки дисперсії статистичних характеристик, які використовуються в подальшому для оцінки ймовірності знаходження траєкторних параметрів у межах, дозволеної області.

Ключові слова: оцінка точності; кореляційна функція; ймовірнісні точнісні характеристики; подвійне усереднення; адаптивна дискретизація; методика обробки даних; бортова система керування.

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А. А. Зеленков, А. А. Бунчук, А. П. Голик. Оценка точности результатов летных испытаний на основе аппроксимации корреляционной функции

Рассмотрено решение задачи, в результате чего получены оценки статистических характеристик после обработки результатов летных испытаний методом двойного усреднения, как во времени, так и по ансамблю реализаций. Осуществлена аппроксимация экспериментальной кривой корреляционной функции аналитическим выражением и методом наименьших квадратов определены параметры аппроксимации. На основании полученных параметров аппроксимации определены оценки дисперсии статистических характеристик, которые используются в дальнейшем для оценки вероятности нахождения траекторных параметров в допустимой области.

Ключевые слова: оценка точности; корреляционная функция; вероятностные точностные характеристики; двойное усреднение; адаптивная дискретизация; методика обработки данных; бортовая система управления.

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