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[1]**Yu. M. Kemenyash,**
[2]**A. N. Kushnir**

MATHEMATICAL MODEL OF DAMPING OF VIBRATIONS OF A LIQUID IN A TANK WITH N ELASTIC RIBS

[1,2]Educational & Research Institute of Information and Diagnostic Systems,
National Aviation University, Kyiv, Ukraine
E-mails: [1]lindysik999@gmail.com, [2]aresal94@gmail.com

***Abstract*—*The problem of joint oscillations of a liquid and rigid elastically suspended damping ribs in a tank having the form of a rectangular parallelepiped is considered. A system of equations is drawn from which it can be seen that for a certain choice of parameters one can obtain a significant gain in damping. The Simulink model is constructed, which allows obtaining characteristics about the processes occurring in the tank. The aim of the paper is to describe such a mathematical model of damping of fluid vibrations with n elastic ribs in tanks, which has sufficient reliability, accuracy, simplicity and efficiency.*

Index Terms—Damping; ribs; oscillation of liquid; tank.

I. INTRODUCTION

Researches of free fluctuations of liquid in capacities are presented rather widely. Interest in problems of dynamics of bodies with the cavities containing liquid has considerably amplified in connection with fast development of the rocket and space equipment. The reserve of liquid fuel which is available onboard rockets, satellites and spaceships in some cases can have significant effect on the movement of these aircraft. Similar tasks arise also in the theory of the movement of the plane and ship, transportation of liquids, as well as in other technical questions. Thus, problems of dynamics of bodies which have the cavities containing liquid represent undoubted applied value. These tasks have also basic, theoretical interest.

Influence of fluctuations of fuel on dynamics of the aircraft is defined, first, by a ratio of mass of liquid in partially filled tanks and the mass of the device and, secondly, the level of automation of traffic control of the device concerning the center of masses. For devices of the rocket scheme at which in not indignant movement the overload is directed along a longitudinal axis of the device extensive researches [1], [2] in which are conducted tasks about definition of mathematical models of the movement of liquid in partially filled tanks are solved.

For restriction of mobility of liquid enter special dampers into a design of tanks.

Study of damping of liquid oscillations in tanks of aircrafts is one of the most important problems of the dynamics of elastic structures with tank partially filled with water. Along with determining of frequencies spectrum of natural oscillations of structure and liquid in tanks, the study of damping and, in particular, oscillation decrements, is quite necessary for a proper choice of the parameters of control systems, because in the case of convergence of natural frequencies of the structure or fluid with a frequency bandwidth of control system, violation of the normal operation of the latter and the loss of stability and controllability of the aircraft is possible.

However, if the frequency response of an elastic structure with tanks do not meet the requirements, then it is common to try by artificial means to change them in the desired direction by using damping baffles of different shapes and sizes into the tank. Effective means of limiting the mobility of the fluid are dampers in the form of ring and radial walls. With a specific choice of parameters of elastic baffles, there can be a significant gain in the magnitude of damping developed by them, as well as in the weight ratio.

Two simplest laws of dissipative forces are of the greatest interest:

– dissipative forces proportional to the velocity – viscous damping;

– dissipative forces that bear the harmonic character – hysteretic damping – proportional to displacement.

Viscous damping is described by the equation of motion in the form:

$$m\frac{d^2y}{dt^2}+h\frac{dy}{dt}+ky=F\cos\omega t.$$

Coefficient of damping $\beta = h / m$ and the natural frequency $\sigma = \sqrt{k / m}$ completely determine the dynamic properties of the system (natural oscillations of the system). Hysteresis damping –
$m\frac{d^2 y}{dt^2} + \frac{h}{\omega}\frac{dy}{dt} + ky = F \sin \omega t.$

We consider the problem of forced oscillations of a liquid in a rectangular parallelepiped in the presence of a damping septum in the form of a rigid rectangular plate that is elastically suspended in a vertical plane passing through the middle of the cavity [1].

II. Problem Statement

Suppose that a solid body with the cavity in the form of a rectangular parallelepiped filled with incompressible liquid be in the field of mass forces. The mechanical system to which the problem reduces is shown in Fig. 1.

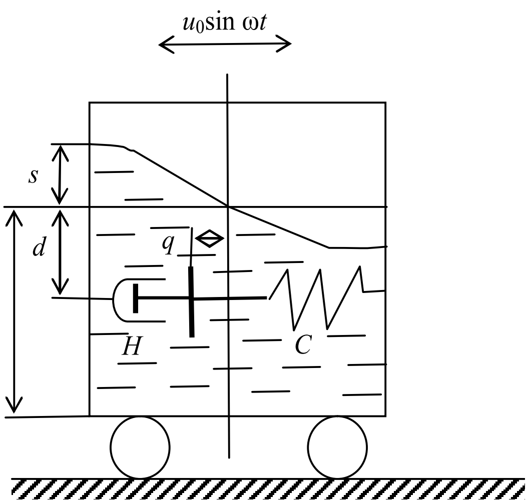

Fig. 1. The mechanical system

In the middle of a cavity at some distance from a free surface the rigid rectangular plate which width is much less than its length is suspended on springs. The ribs can perform translational movement relative to the walls of the cavity. We introduce the following notation: h is the cavity level, l_d is the cavity length, l_s is the width; ρ is the density of the liquid; j is the acceleration of the field of mass forces; l and b is the length and width of the ribs; d is the distance from the free surface of the liquid to the center of the ribs.

We will consider joint fluctuations of liquid and a ribs for a given translational motion of the body according to the harmonic law $u_0 \sin \omega t$.

III. Problem Solution

We form the equations of the movement of system. For the coordinate characterizing the oscillations of the liquid, we take the deviation of the free surface on the wall of the cavity s. We will designate the coordinate characterizing movement of a ribs concerning cavity walls through q. The equations of the movement can be written down in the following form [1]:

$$\begin{aligned} &\mu\left(\frac{d^2 s}{dt^2} + \beta_s \frac{ds}{dt} + \omega_s^2 s\right) - \alpha F = \lambda u_0 \omega^2 \sin \omega t, \\ &m^0\left(\frac{d^2 q}{dt^2} + \beta_q \frac{ds}{dt} + \omega_q^2 q\right) + F = m^0 u_0 \omega^2 \sin \omega t, \end{aligned} \quad (1)$$

where ω_s, β_s, μ and λ is the hydrodynamic coefficients corresponding to the cavity without the ribs; $\omega_q = \sqrt{\frac{c}{m^0}}$ is the natural frequency of oscillations of the ribs in the absence of liquid. The natural frequency has the form: $\omega_s^2 = \frac{\pi j}{l_d} \text{th} \frac{\pi h}{l_d}$.

Associated masses: $\mu = \frac{\rho l_d^2 l_s}{2\pi \, \text{th}(\pi h / l_d)}$; $\lambda = \frac{2\rho l_d^2 l_s}{\pi}$;

$\alpha_i = \frac{\text{ch}\left[\pi(h-d)/l_d\right]}{\text{sh}(h/l_d)}$; the coefficients β_s and β_q are determined experimentally; $\beta_q = H / m^0$ is the coefficient of damping, characterizing energy dissipation; m^0 is the ribs mass; α is the coefficient characterizing the decrease in the amplitude of fluid oscillations; F is the force acting on the bulkhead on the liquid side; q is the coordinate of the displacement of the septum relative to the wall of the cavity.

The hydrodynamic force can be represented in the following form:

$$F = -m\frac{d\vartheta}{dt} - \frac{4}{3\pi} c_b b \vartheta_0 \vartheta, ; \quad (2)$$

where m is the attached mass of the liquid; c_b is the coefficient of resistance of one rib, b is the rib height, ϑ is the flow velocity, l is the length of the septum. This dependence is valid only for the harmonic law of velocity variation [2].

The attached mass m and the resistance coefficient c_b in the general case are complex functions of a dimensionless parameter equivalent to the Strouhal number $S = 2\pi \upsilon_0 / b\omega$, also the depth of the drowning of the septum.

For the fixed provision of a ribs and rather small ranges of change of parameter S these functions with sufficient degree of accuracy can be approximated power-law dependences

$$c_b = K\left(\frac{2\pi\vartheta_0}{b\omega}\right)^{-n} = KS^{-n}. \qquad (3)$$

The coefficients K and n depend on the depth of the drowning of the ribs and are determined experimentally. For a large depth of drowning (the case of an infinite fluid) $K = 8.4$; $n = 1/3$.

The adjoining mass of the fluid can be regarded as a constant for simplification and can be determined experimentally as an average value. This assumption is entirely possible, since the main interest is the small values of the parameter S, where the adjoined mass changes insignificantly.

Taking into account (3), the expression for the hydrodynamic force is written in the form

$$F = -m\frac{d\vartheta}{dt} - K_0\upsilon, \qquad (4)$$

where $K_0 = \dfrac{\rho K}{(S^n 3\pi)bl\upsilon_0}$.

Since the width of the septum is assumed to be small in comparison with its length, then in determining the hydrodynamic force for the velocity u it is possible to take the difference in the velocity of the displacement of the ribs and the oscillation of the liquid

$$\upsilon_n = \alpha\dot{s} - \dot{q}_n. \qquad (5)$$

Equation (1) describes the joint oscillation of a fluid and a single septum. Typically, several baffles are introduced into the tank to damp the oscillations. Then the system in which the n ribs are located takes the form:

$$\mu\left(\frac{d^2s}{dt^2} + \beta_s\frac{ds}{dt} + \omega_s^2 s\right) - \sum_{i=1}^{n}\alpha_n F_n = \lambda u_0\omega^2\sin\omega t,$$

$$m^0\left(\frac{d^2q_n}{dt^2} + \beta_q\frac{dq_n}{dt} + \omega_q^2 q_n\right) + F_n = m^0 u_0\omega^2\sin\omega t.$$

Considering the foregoing, let us write down the hydrodynamic force for n ribs $F_n = -m\alpha\ddot{s} - k_0\alpha\dot{s} + m\ddot{q}_n + k_0\dot{q}_n$.

Let's create a system of equations for two ribs located in the tank.

$$\mu\left(\frac{d^2s}{dt^2} + \beta_s\frac{ds}{dt} + \omega_s^2 s\right) - \alpha_1 F_1 - \alpha_2 F_2 = \lambda u_0\omega^2\sin\omega t,$$

$$m^0\left(\frac{d^2q_1}{dt^2} + \beta_q\frac{dq_1}{dt} + \omega_q^2 q_1\right) + F_1 = m^0 u_0\omega^2\sin\omega t,$$

$$m^0\left(\frac{d^2q_2}{dt^2} + \beta_q\frac{dq_2}{dt} + \omega_q^2 q_2\right) + F_2 = m^0 u_0\omega^2\sin\omega t, \qquad (6)$$

than

$$\alpha F_1 = -m\alpha\frac{d^2s}{dt^2} - \alpha K_0\frac{ds}{dt} + m\frac{d^2q_1}{dt^2} + K_0\frac{dq_1}{dt},$$
$$\alpha F_2 = -m\alpha\frac{d^2s}{dt^2} - \alpha K_0\frac{ds}{dt} + m\frac{d^2q_2}{dt^2} + K_0\frac{dq_2}{dt}. \qquad (7)$$

After substituting expression (7) into equation (6), and taking (5) into account, we obtain the following system of connected equations of fluid and ribs oscil:

$$\left(\mu + 2\alpha^2 m\right)\frac{d^2s}{dt^2} + \left(\mu\beta_s + 2\alpha K_0\right)\frac{ds}{dt} + \mu\omega_s^2 s - \alpha K_0\left(\frac{dq_1}{dt} + \frac{dq_2}{dt}\right) - \alpha m\left(\frac{d^2q_1}{dt^2} + \frac{d^2q_2}{dt^2}\right) = \lambda u_0\omega^2\sin\omega t,$$
$$\left(m^0 + m\right)\frac{d^2q_1}{dt^2} + \left(m^0\beta_q + K_0\right)\frac{dq_1}{dt} + m^0\omega_q^2 q_1 - \alpha K_0\frac{ds}{dt} - \alpha m\frac{d^2s}{dt^2} = m^0 u_0\omega^2\sin\omega t, \qquad (8)$$
$$\left(m^0 + m\right)\frac{d^2q_2}{dt^2} + \left(m^0\beta_q + K_0\right)\frac{dq_2}{dt} + m^0\omega_q^2 q_2 - \alpha K_0\frac{ds}{dt} - \alpha m\frac{d^2s}{dt^2} = m^0 u_0\omega^2\sin\omega t.$$

Since the initial conditions of this problem are not specified, we take them to be zero and find the solution of this system of equations describing the joint oscillations of the liquid and the ribs, as the Koshi problem with zero initial conditions and perturbations in the form of a harmonic oscillation with frequency ω and amplitude u_0.

We introduce the notation:

$$a_0 = \mu + 2\alpha^2 m,\ a_1 = \mu\beta_s + 2\alpha K_0,\ a_2 = \mu\omega_s^2,$$
$$b_0 = m^0 + m,\ b_1 = m^0\beta_q + K_0,\ b_2 = \omega_s^2,$$
$$g_1 = -\alpha m,\quad g_2 = -\alpha K_0,$$
$$f_1 = \beta_s\lambda\omega^3 u_0,\quad f_2 = m^0\beta_q\omega^3 u_0.$$

We apply the integral Laplace [3] transform to the system of equations (8). We obtain the following system of equations:

$$(a_0p^2 + a_1p + a_2)S(p) + (g_1p + g_2)pQ_1(p) + (g_1p + g_2)pQ_2(p) = \frac{f_1}{p^2 + \omega^2},$$

$$\left(b_0p^2 + b_1p + b_2\right)Q_1(p) + \left(g_1p + g_2\right)pS(p) = \frac{f_2}{p^2 + \omega^2}.$$

Solving this system with respect to $S(p)$ and $Q(p)$, we find

$$S(p)=\frac{1}{p^2+\omega^2}\frac{e_2p^2+e_1p+e_0}{p^4+c_1p^3+c_2p^2+c_3p+c_4},$$

$$Q_1(p)=\frac{1}{b_0p^2+b_1p+b_2}\left[\frac{f_1}{p^2+\omega^2}-(e_1p+e_2)pS(p)\right],$$

$$Q_2(p)=\frac{1}{b_0p^2+b_1p+b_2}\left[\frac{f_1}{p^2+\omega^2}-(e_1p+e_2)pS(p)\right].$$

Having executed the corresponding transformations and presenting the received expressions in the form of the sum of links of the second order, we receive the solution of a task in space of images in the following look:

$$S(p)=\sum_{k=1}^{3}\frac{s_{2k}+s_{2k+1}(p+\gamma_{sk})}{(p+\gamma_{sk})\pm\omega_{sk}^2}, \tag{9}$$

$$Q_1(p)=\sum_{k=1}^{4}\frac{q_{2k}+q_{2k+1}(p+\gamma_{sk})}{(p+\gamma_{sk})\pm\omega_{sk}^2}, \tag{10}$$

$$Q_2(p)=\sum_{k=1}^{4}\frac{q_{2k}+q_{2k+1}(p+\gamma_{sk})}{(p+\gamma_{sk})\pm\omega_{sk}^2}. \tag{11}$$

Expressions (9), (10), (11) in the origin space correspond to the following functions:

$$s(t)=\sum_{k=1}^{3}e^{-\gamma_{sk}}[\frac{s_{2k}}{\omega_{sk}}(\sin\omega_{sk}t+s_{2k+1}\cos\omega_{sk}t)]. \tag{12}$$

$$q_1(t)=\sum_{k=1}^{4}e^{-\gamma_{qk}}\left[\frac{q_{2k}}{\omega_{sk}}\left(\sin\omega_{sk}t+q_{2k+1}\cos\omega_{sk}t\right)\right], \tag{13}$$

$$q_2(t)=\sum_{k=1}^{4}e^{-\gamma_{qk}}\left[\frac{q_{2k}}{\omega_{sk}}\left(\sin\omega_{sk}t+q_{2k+1}\cos\omega_{sk}t\right)\right]. \tag{14}$$

The solution of this problem will be with the help of the model in Simulink Fig. 2.

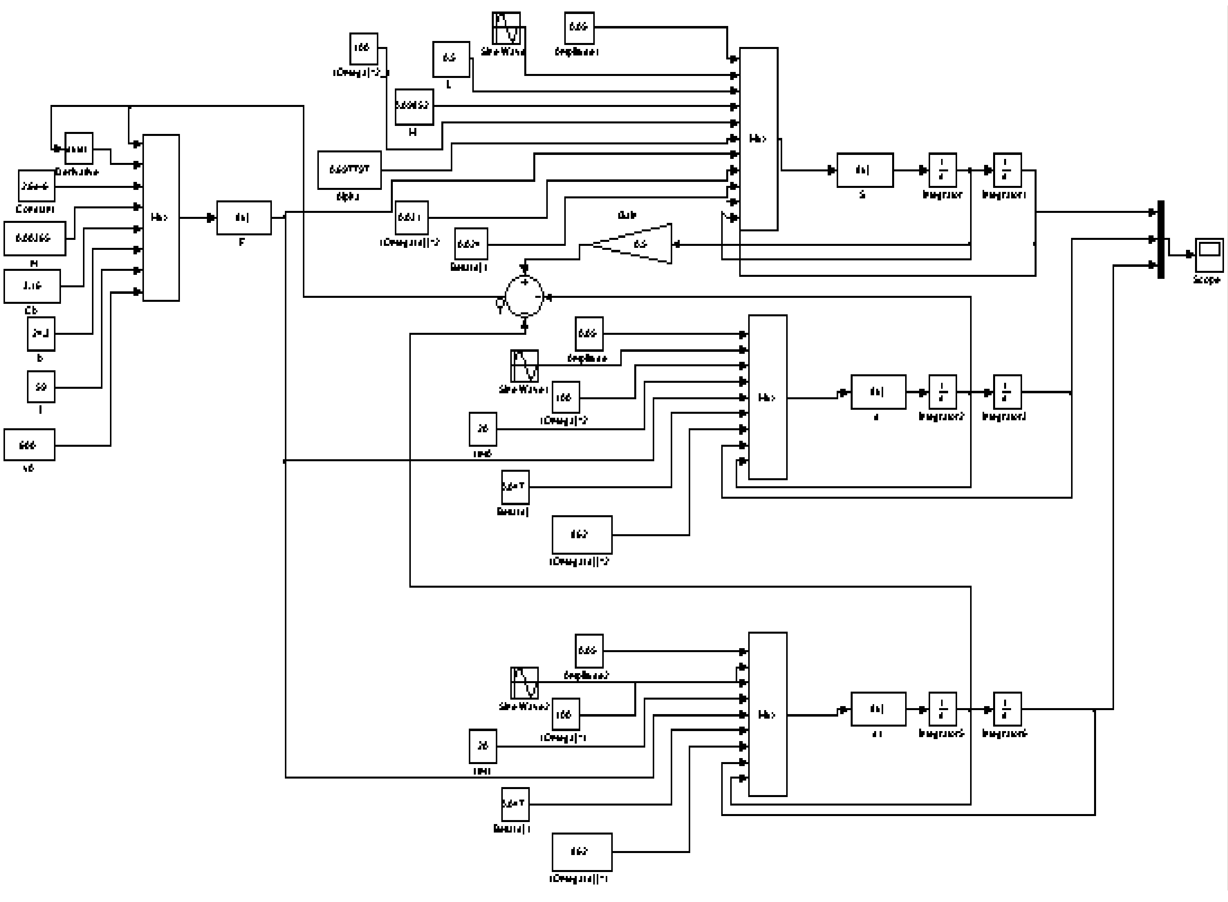

Fig. 2. Model of tank and ribs

IV. RESULTS

Analyzing the received result it is possible to tell that the task about oscillations of liquid in cavities it is possible to reduce accounting of the effects caused by elasticity of the damping baffles to determination of coefficients of damping and natural frequencies Fig 3. The minimum vibration amplitude of liquid is watched with the relative amplitude of relocation of a cavity 0.05. Its value is 3.2 times less than amplitude for a rigid baffle. Therefore, in case of a certain choice of parameters it is possible to receive a scoring in damping.

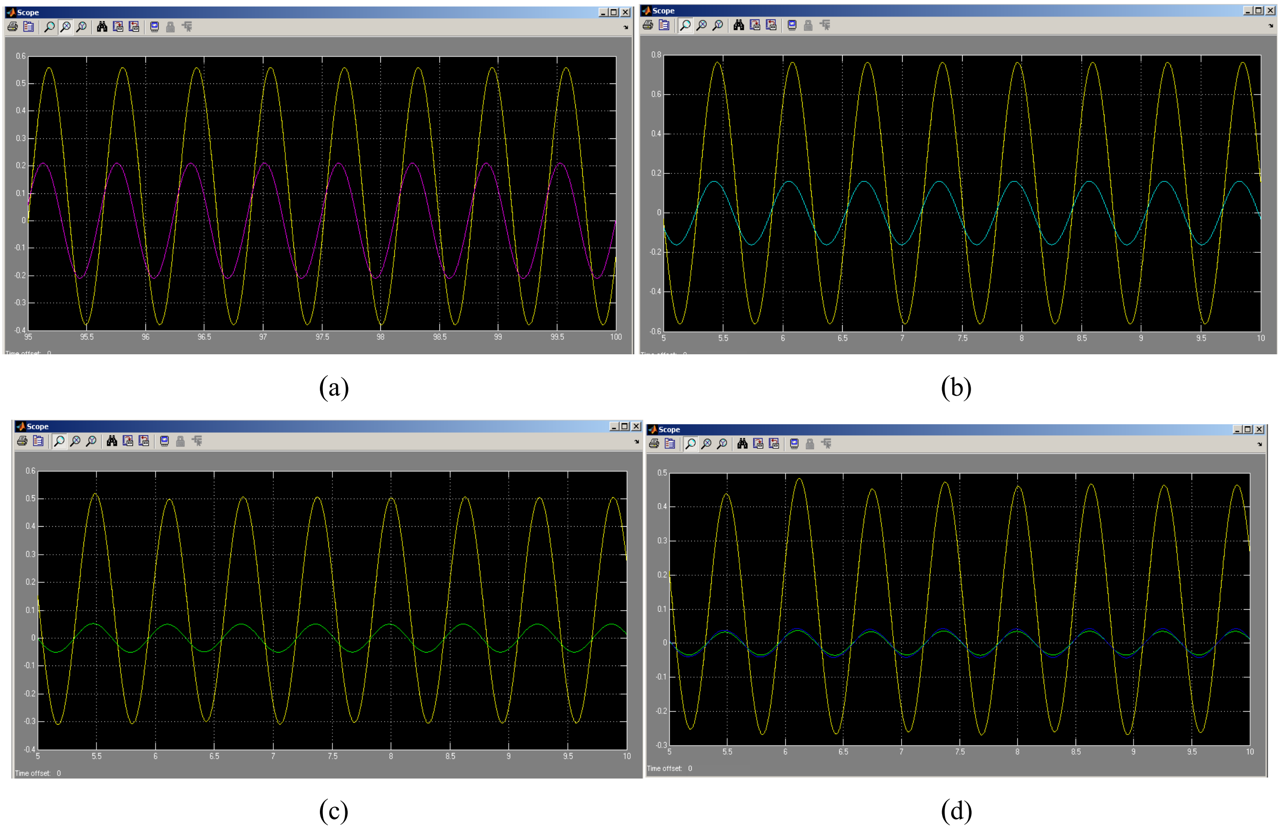

Fig. 3. Result of simulation of fluid oscillations in a tank with fins: (a) – one ribs; (b) – two ribs; (c) – three ribs; (d) – four ribs

V. CONCLUSIONS

In this paper we consider the problem of joint oscillations of a liquid and rigid elastically suspended damping partitions in a tank. It is shown that with a certain choice of system parameters, one can obtain a significant gain in damping. It is obvious that the quantity of ribs leads to the considerable damping effect.

The mathematical model has sufficient reliability, accuracy, simplicity and efficiency, and completely agrees with the conclusion of the paper [1].

Analyzing the results obtained, it can be seen that with the addition of ribs the coefficient α increases. And this, in turn, leads to a decrease in the amplitude of oscillation of the liquid in the tank.

From the result obtained, it can be seen that with an increase in the number of partitions, the mass increases, and at some stage of the increase, the efficiency decreases. Weight costs are very important for transportation. The problem arises of the optimal number of fines. It will not be a big deal having a Simulink model.

The task about joint fluctuations of liquid is considered and rigid it is elastic the suspended damping ribs in the tank having the form of a rectangular parallelepiped. It is shown that at a certain choice of parameters of system it is possible to receive a considerable prize in damping.

The purpose of work is the description of mathematical model of damping of fluctuations of liquid elastic ribs in tanks of a difficult design which has sufficient reliability, accuracy, simplicity and efficiency [4].

REFERENCES

[1] G. Mikishev and B. Rabinovich, *The dynamics of thin-walled structures with compartments containing liquid.* Moscow: Engineering, 1971, 532 p. (in Russian).

[2] J. W. Miles, “Ring damping of free surfaces Oscillations in a circular tank,” *J. Appl. Mech.*, vol. 25, no. 6, 1958.

[3] K. Kh. Zelensky, V. N. Ignatenko, and A. P. Kots, *Computer methods of applied mathematics*. Kyiv: Design-In, 1999, 352 p.

[4] V. Yatskivskiy and Yu. Kemenyash**.** “Damping of liquid oscillations in a right circular cylinder with radial ribs,” *Electronics and Control Systems*, no. 1(35), pp. 142–145, 2013.

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Kemenyash Yuriy. Assistant Professor.
Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine, (1996).
Research area: Identification of Complex Systems.
Publications: more than 30 papers.
E-mail: lindysik999@gmail.com

KushnirAlexey. Student.
Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine.
Research area: Identification of Complex Systems.
Publications: 3.
E-mail: aresal94@gmail.com

Ю. М. Кеменяш, О. М. Кушнір. Математична модель демпфування коливань рідини в ємності з *n* жорсткими ребрами

Розглянуто задачу про спільні коливаннях рідини і жорстких пружно підвішених демпфуючих перегородок в баку, що має форму прямокутного паралелепіпеда. Складено систему рівнянь з якої видно, що вибираючи певні параметри можна отримати значний виграш в демпфуванні. Побудовано Simulink модель, яка дозволяє отримувати характеристики, що описують процеси у баку. Метою роботи є опис такої математичної моделі демпфування коливань рідини з *n* пружними ребрами в баках, яка характеризується достатньою надійністю, точністю, простотою та ефективністю.
Ключові слова: демпфування; перегородка; коливання рідини; ємність.

Кеменяш Юрій Михайлович. Старший викладач.
Навчально-науковий інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна, (1996).
Напрям наукової діяльності: ідентифікація складних систем.
Кількість публікацій: більше 30 наукових робіт.
E-mail: lindysik999@gmail.com

Кушнір Олексій Миколайович.Студент.
Навчально-науковий інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.
Напрям наукової діяльності: ідентифікація складних систем.
Кількість публікацій: 3.
E-mail: aresal94@gmail.com

Ю. М. Кеменяш., А. Н. Кушнир. Математическая модель демпфирования колебаний жидкости в баке с *n* упругими ребрами

Рассмотрена задача о совместных колебаниях жидкости и жестких упруго подвешенных демпфирующих ребрах в баке, имеющем форму прямоугольного параллелепипеда. Составлена система уравнений из которой видно, что при определенном выборе параметров можно получить значительный выигрыш в демпфировании. Построена Simulink модель, позволяющая получать характеристики о происходящих процессах в баке. Целью работы является описание такой математической модели демпфирования колебаний жидкости с *n* упругими ребрами в баках, которая обладает достаточной надежностью, точностью, простотой и эффективностью.
Ключевые слова: демпфирование; перегородка; колебание жидкости; бак.

Кеменяш Юрий Михайлович. Старший преподаватель.
Учебно-научный институт информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.
Образование: Киевский авиационный унриверситет, Киев, Украина, (1996).
Направление научной деятельности: идентификация сложных систем.
Количество публикаций: более 30 научных работ.
E-mail: lindysik999@gmail.com

Кушнир Алексей Николаевич. Студент.
Учебно-научный институт информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.
Направление научной деятельности: идентификация сложных систем.
Количество публикаций: 3.
E-mail: aresal94@gmail.com