

AUTOMATIC CONTROL SYSTEMS

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ATTITUDE DETERMINATION USING GLOBAL POSITIONING SYSTEM

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Abstract—The effect of measurements errors of distances to satellites on the accuracy of attitude determination is analyzed. Matrix form of analysis and the least squares criterion are used. Algorithm for determining the angular orientation of the satellite based on a comparison of measured and calculated difference distances from the antennas installed on the object to GPS-satellites is proposed. The task of estimating the accuracy of determining the angular orientation in the presence of errors in determining distances to satellites is analyzed. For small measurements errors their influence on the accuracy of attitude determination can be characterized by additional rotation matrix. It is shown that the task is simplified if the normalized vectors form orthogonal basis. Relations between measurement errors in the object coordinate system and reference coordinate system are obtained. It is shown that to reduce the influence of measurement errors is advisable to install the slave antennas away from the master antenna.

Index Terms—Global Positioning System; attitude; determination.

I. INTRODUCTION

Currently researches on the use of Global Positioning System (GPS) to determine the attitude determination of moving objects are performed [1], [2]. Consider the algorithm of angular orientation determination of the object based on the GPS information [3], taking as a basis the matrix method of analysis, which is used in the methods of attitude determination of moving objects [4].

II. PROBLEM STATEMENT

Algorithm for determining the angular orientation of the satellite based on a comparison of measured and calculated difference distances from the antennas installed on the object to GPS-satellites. This is explained in Fig. 1, which shows two antennas (master) and the i th antenna (number of these antenna equals m).

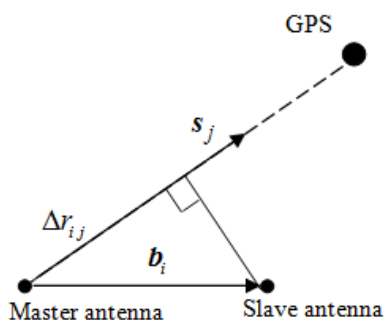


Fig. 1. Attitude determination

The direction to each antenna in connected with the object coordinate system (OCS) define the vector

\mathbf{b}_i . The direction to the j -GPS-satellite in the reference coordinate system (RCS) define the unit vector \mathbf{s}_j . Due to satellite rotation in space will be the difference $\Delta r_{i,j}$ in distance of shown antenna to the j th GPS-satellite. Comparing the measured and calculated difference of these distances, you can find a attitude matrix \mathbf{A} . From this matrix using formula

$$\psi = \arctg\left(\frac{A_{12}}{A_{11}}\right), \quad \theta = -\arcsin(A_{13}), \quad \varphi = \arctg\left(\frac{A_{23}}{A_{33}}\right), \quad (1)$$

you can find the angles of orientation.

The task is to design an algorithm determining the angles of orientation of the object based on the presence of GPS measurement errors distances to satellites.

III. PROBLEM SOLUTION

We have

$$\Delta r_{i,j} = \mathbf{b}_i^T \mathbf{h}_j, \quad (2)$$

where $\mathbf{h}_j = \mathbf{A} \mathbf{s}_j$.

The sense of the expression (2) is the distance $\Delta r_{i,j}$ can be seen as a projection of the vector \mathbf{b}_i on the direction of the unit vector \mathbf{s}_j . One must note that the vector \mathbf{b}_i set in OCS and the vector \mathbf{s}_j – in RCS. Therefore, in the expression (2) is written the vector \mathbf{s}_j in OCS, that is vector \mathbf{h}_j .

Rewrite formula (2) in matrix form

$$R = \begin{vmatrix} \mathbf{b}_1^T \mathbf{h}_1 & \mathbf{b}_1^T \mathbf{h}_2 \dots & \mathbf{b}_1^T \mathbf{h}_n \\ \mathbf{b}_2^T \mathbf{h}_1 & \mathbf{b}_2^T \mathbf{h}_2 \dots & \mathbf{b}_2^T \mathbf{h}_n \\ \dots & \dots & \dots \\ \mathbf{b}_m^T \mathbf{h}_1 & \mathbf{b}_m^T \mathbf{h}_2 \dots & \mathbf{b}_m^T \mathbf{h}_n \end{vmatrix} = \begin{vmatrix} \mathbf{b}_1^T \\ \mathbf{b}_2^T \\ \dots \\ \mathbf{b}_m^T \end{vmatrix} \begin{vmatrix} \mathbf{h}_1 & \mathbf{h}_2 \dots \mathbf{h}_n \end{vmatrix} \\ = \begin{vmatrix} \mathbf{b}_1 & \mathbf{b}_2 \dots \mathbf{b}_m \end{vmatrix}^T \begin{vmatrix} \mathbf{h}_1 & \mathbf{h}_2 \dots \mathbf{h}_n \end{vmatrix} = B^T H, \quad (3)$$

where $B_{3 \times m} = [\mathbf{b}_1 \ \mathbf{b}_2 \dots \mathbf{b}_m]$;

$$H_{3 \times n} = \begin{vmatrix} \mathbf{h}_1 & \mathbf{h}_2 \dots \mathbf{h}_n \end{vmatrix} = \begin{vmatrix} A\mathbf{s}_1 & A\mathbf{s}_2 \dots A\mathbf{s}_n \end{vmatrix} = AS;$$

$$S = [\mathbf{s}_1 \ \mathbf{s}_2 \dots \mathbf{s}_n].$$

Apart from the measurement error, the matrix can be found directly from the expression (3), first multiplying the expression on the left by the matrix \mathbf{B} and then multiplying the resulting expression on the right by the matrix S^T

$$A = G^{-1} BRS^T N^{-1}. \quad (4)$$

where $G_{3 \times 3} = BB^T$; $N_{3 \times 3} = SS^T$.

In the presence of measurement errors is to use the least squares criterion [4]. Task orientation estimation using least squares criterion is to determine orthogonal matrix with determinant +1, which would minimize the loss function

$$g(A) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^m \|\Delta r_{ij} - \mathbf{b}_i^T A \mathbf{s}_j\|^2 \\ = \frac{1}{2} \text{tr} \left[(R - B^T H)(R - B^T H)^T \right]. \quad (5)$$

More convenient to use such loss function

$$g_1(A) = \frac{1}{2} \text{tr} \left\{ \left[G^{-1} B (R - B^T H) \right] \cdot \left[G^{-1} B (R - B^T H) \right]^T \right\}.$$

In this case

$$L = G^{-1} B (R - B^T H) = G^{-1} B B^T \left[(B B^T)^{-1} B R - H \right] \\ = G^{-1} B R - H = K - H,$$

where $K = G^{-1} B R$.

Write

$$g_1(A) = \frac{1}{2} \text{tr} \left[(K - H)(K - H)^T \right] \\ = \frac{1}{2} \text{tr} (K K^T - 2 H K^T + S^T S). \quad (6)$$

Thus, the problem is to minimize function

$$g_2(A) = \text{tr} (A S K^T). \quad (7)$$

Consider limitation $A^T A = I$ and take the function g_2 in the form

$$g_2 = \text{tr} (A Q) - \text{tr} \left(\frac{1}{2} \Lambda (A^T A - I) \right), \quad (8)$$

where Λ is the Lagrange multiplier matrix; $Q = S K^T$.

Similar to [4] will obtain

$$A = Q^T \left(\sqrt{Q Q^T} \right)^{-1}. \quad (9)$$

Thus, using formula (9) we can find the attitude matrix based on the information from GPS.

The matrix \mathbf{B} consists of non-normalized vectors. It is advisable to go to the normalized vectors

$\tilde{\mathbf{b}}_i = \frac{\mathbf{b}_i}{\rho_i}$, where $\rho_i = \|\mathbf{b}_i\|$. Then the matrix \mathbf{B} can be written

$$\mathbf{B} = \left[\rho_1 \tilde{\mathbf{b}}_1 \ \rho_2 \tilde{\mathbf{b}}_2 \dots \rho_m \tilde{\mathbf{b}}_m \right] = \tilde{B} M, \quad (10)$$

where $\tilde{B} = \left[\tilde{\mathbf{b}}_1 \ \tilde{\mathbf{b}}_2 \dots \tilde{\mathbf{b}}_m \right]$; $M = \text{diag}(\rho_1, \rho_2, \dots, \rho_m)$.

Analysis of the influence of measurement errors will carry replacing the matrix \mathbf{R} on the matrix

$$R_v = R + D, \quad (11)$$

where matrix $\mathbf{D} = \begin{vmatrix} \Delta_{11} & \Delta_{12} \dots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} \dots & \Delta_{2n} \\ \dots & \dots & \dots \\ \Delta_{m1} & \Delta_{m2} \dots & \Delta_{mn} \end{vmatrix}$ is the matrix of measurement errors.

In this case the matrix \mathbf{K} can be written as

$$K = (B B^T)^{-1} B R = H + H_1, \quad (12)$$

where $H_1 = (\tilde{B}^T)^{-1} M_1 D = (\tilde{B}^T)^{-1} D_1$; $D_1 = M_1 D$;

$$M_1 = \text{diag}(\rho_1^{-1}, \rho_2^{-1}, \dots, \rho_m^{-1}).$$

From the expression for the matrix \mathbf{D}_1 we see that to reduce the influence of measurement errors advisable to increase modules of vectors \mathbf{b}_m , that is advisable to install the slave antennas away from the master antenna.

The task is simplified if we assume that the normalized vectors $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3$ form orthogonal basis. Then

$$S = N = I; \quad K = A + H_1; \quad Q = K^T;$$

$$Q Q^T = I + A^T H_1 + H_1^T A + H_1^T H_1.$$

For small errors

$$QQ^T \approx I + V, \quad (13) \quad \text{i.e.}$$

where $V = A^T H_1 + H_1^T A$.

As

$$\sqrt{QQ^T} = \sqrt{I+V} \approx I + \frac{1}{2}V; \quad \left(\sqrt{QQ^T}\right)^{-1} \approx I - \frac{1}{2}V$$

we get

$$\begin{aligned} A_r &= Q^T \left(\sqrt{QQ^T}\right)^{-1} = (A + H_1) \left(I - \frac{1}{2}V\right) \\ &= A + \frac{1}{2} \left(H_1 A^T - A H_1^T\right) A = (I + U) A = A_* A, \end{aligned} \quad (14) \quad \text{i.e.}$$

where $A_* = I + U$;

$$U = \frac{1}{2} \left(H_1 A^T - A H_1^T\right). \quad (15)$$

Explain the content of these expressions. Matrix **A** for small values of angles looks

$$A = \begin{vmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \varphi \\ \theta & -\varphi & 1 \end{vmatrix} = I + \begin{vmatrix} 0 & \psi & -\theta \\ -\psi & 0 & \varphi \\ \theta & -\varphi & 0 \end{vmatrix}.$$

That is, it is the sum of unit and small skew-symmetric matrix. Similar structure has matrix A_* . Therefore, to the error matrix **R** we can align the additional rotation, which is characterized by the matrix A_* . Write the matrix **U** in the form

$$U = \begin{vmatrix} 0 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 0 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 0 \end{vmatrix}, \quad (16)$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are errors of attitude determination in OCS.

Let us find the correspondence between the errors in OCS and RCS.

Represent the angles $\varepsilon_x, \varepsilon_y, \varepsilon_z, \Delta_\psi, \Delta_\theta, \Delta_\varphi$ in the form of vectors $\varepsilon_x, \varepsilon_y, \varepsilon_z, \Delta_\psi, \Delta_\theta, \Delta_\varphi$ (Fig. 2).

We have

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \Delta_\psi + \Delta_\theta + \Delta_\varphi. \quad (17)$$

Will project this expression on the axes allocated in Fig. 2 as dashed lines – it is the intersection of the planes, in which the Euler angles are specified.

A feature of these axes is that on each of them only one of the vectors $\Delta_\psi, \Delta_\theta, \Delta_\varphi$ is projected.

In the projection on axis OX_1 we get

$$\Delta_\varphi \cos \theta = \varepsilon_x \cos \theta + \sin \theta (\varepsilon_z \cos \varphi + \varepsilon_y \sin \varphi),$$

$$\Delta_\varphi = \varepsilon_x + \operatorname{tg} \theta (\varepsilon_z \cos \varphi + \varepsilon_y \sin \varphi). \quad (18)$$

In the projection on axis OY_2 we get

$$\Delta_\theta = \varepsilon_y \cos \varphi - \varepsilon_z \sin \varphi, \quad (19)$$

In the projection on axis OZ_2 we get

$$\Delta_\psi \cos \theta = \varepsilon_z \cos \varphi + \varepsilon_y \sin \varphi,$$

$$\Delta_\psi = \frac{1}{\cos \theta} (\varepsilon_z \cos \varphi + \varepsilon_y \sin \varphi). \quad (20)$$

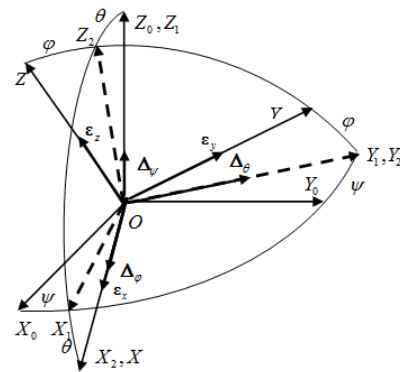


Fig. 2. Errors as vectors

Perform numerical estimation of attitude determination accuracy. We assume that measurement errors are random with a normal distribution with a maximum value of $\Delta_{ij\max} = 0.3$ m. Taking into account that the mean-square value is about $\sigma = \Delta_{\max} / 3$, matrix errors will be set as $D = 0.1 \cdot \operatorname{randn}(3,3)$. Value after simulation is

$$D = \begin{bmatrix} -0.0691 & 0.0826 & -0.0132 \\ 0.0449 & 0.0536 & -0.0147 \\ 0.0101 & 0.0898 & 0.1008 \end{bmatrix}.$$

Assume: $\psi = 30^\circ; \theta = 20^\circ; \varphi = 10^\circ;$

$$\mathbf{b}_1 = [4 \ 2 \ 0]^T; \quad \mathbf{b}_2 = [1 \ 4 \ 0,5]^T; \quad \mathbf{b}_3 = [0,2 \ 0 \ 3]^T;$$

$$\mathbf{s}_1 = [1 \ 0 \ 0]^T; \quad \mathbf{s}_2 = [0 \ 1 \ 0]^T; \quad \mathbf{s}_3 = [0 \ 0 \ 1]^T.$$

The numerical values of the vectors $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ are given in meters.

By using formula (1) the error angles (in degrees) following

$$\Delta_\psi = 0.3285; \quad \Delta_\theta = 0.3691; \quad \Delta_\varphi = -0.7148.$$

Here in the matrix **A** errors of measurement were taken into consideration.

Matrix \mathbf{U} calculated by the formula

$$\mathbf{U} = \mathbf{A}_r \mathbf{A}^{-1} - \mathbf{I}, \quad (21)$$

looks like

$$\mathbf{U} = \begin{bmatrix} -0.0020 & 0.2391 & -0.4172 \\ -0.2451 & -0.0065 & -0.8272 \\ 0.4137 & 0.8289 & -0.0075 \end{bmatrix}.$$

We see that this matrix is close to the skew-symmetric matrix. For further calculations instead of matrix \mathbf{U} appropriate to take skew-symmetric matrix

$$\mathbf{U}_1 = \frac{1}{2}(\mathbf{U} - \mathbf{U}^T) = \begin{bmatrix} 0 & 0.2421 & -0.4155 \\ -0.2421 & 0 & -0.8281 \\ 0.4155 & 0.8281 & 0 \end{bmatrix}.$$

Errors of the angles determination using the formulas (18) – (20) and matrix \mathbf{U}_1 are

$$\Delta\psi = 0.3305; \quad \Delta\theta = 0.3671; \quad \Delta\varphi = -0.7150.$$

Calculated by the formula (15) matrix \mathbf{U} equals

$$\mathbf{U}_2 = \begin{bmatrix} 0 & 0.2367 & -0.4278 \\ -0.2367 & 0 & -0.8450 \\ 0.4278 & 0.8450 & 0 \end{bmatrix}.$$

Errors of the angles determination using the formulas (18) – (20) and matrix \mathbf{U}_2 are

$$\Delta\psi = 0.3271; \quad \Delta\theta = 0.3802; \quad \Delta\varphi = -0.7331.$$

Now consider the case without imposing restrictions on vectors \mathbf{s}_1 , \mathbf{s}_2 , \mathbf{s}_3 orientation. Assume

$$\mathbf{s}_1 = \frac{[1 \ 0 \ 1]^T}{\text{norm}([1 \ 0 \ 1]^T)}; \quad \mathbf{s}_2 = \frac{[0 \ 1 \ 2]^T}{\text{norm}([0 \ 1 \ 2]^T)}; \\ \mathbf{s}_3 = \frac{[0 \ 0 \ 1]^T}{\text{norm}([0 \ 0 \ 1]^T)}.$$

Errors of the angles determination using the formula (1) are

$$\Delta\psi = 0.7872; \quad \Delta\theta = -0.0936; \quad \Delta\varphi = 0.0917.$$

Matrix \mathbf{U} calculated by the formula (21) equals

$$\mathbf{U}_1 = \begin{bmatrix} 0 & 0.7449 & -0.0369 \\ -0.7449 & 0 & -0.1769 \\ 0.0369 & 0.1769 & 0 \end{bmatrix}$$

Errors of the angles determination using the formulas (18) – (20) and matrix \mathbf{U}_1 are

$$\Delta\psi = 0.7875; \quad \Delta\theta = -0.0930; \quad \Delta\varphi = 0.0924.$$

In this case we cannot use the formula (15), because the vectors \mathbf{s}_1 , \mathbf{s}_2 , \mathbf{s}_3 are not orthogonal. Although the matrix

$$\mathbf{U} = \begin{bmatrix} -0.0049 & 0.7448 & -0.0381 \\ -0.7449 & -0.0051 & -0.1767 \\ 0.0358 & 0.1771 & -0.0003 \end{bmatrix},$$

calculated by the formula (21), close to skew-symmetric matrix.

To analyze the influence of the location of antennas on the accuracy of the algorithm assume

$$\mathbf{b}_1 = k[4 \ 2 \ 0]^T; \quad \mathbf{b}_2 = k[1 \ 4 \ 0.5]^T; \quad \mathbf{b}_3 = k[0.2 \ 0 \ 3]^T$$

where k is the parameter.

The results of calculation of errors $\Delta\psi$ (line 1), $\Delta\theta$ (line 2), $\Delta\varphi$ (line 3) are presented in Fig. 3.

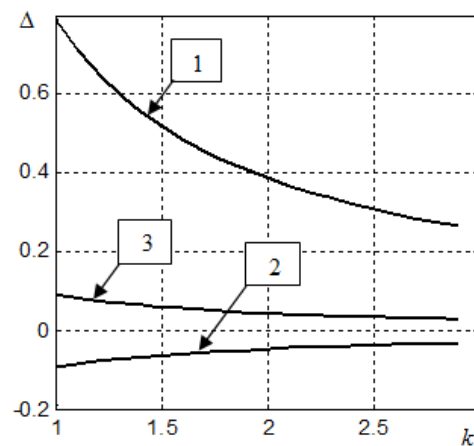


Fig. 3. Errors of attitude determination

We see that it is advisable to install slave antennas away from the master antenna.

IV. CONCLUSION

Matrix form and the least squares criterion are an effective means of attitude determination analysis using GPS. For small measurements errors their influence on the accuracy of attitude determination can be characterized by additional rotation matrix.

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Л. М. Рижков, О. В. Прохорчук. Визначення орієнтації з використанням системи глобального позиціонування

Аналізується точність визначення орієнтації об'єкта за наявності помилок вимірювання відстаней до супутників. Використовується матрична форми аналізу та метод найменших квадратів. Показано, що для малих помилок вимірювань їх вплив на точність визначення орієнтації може характеризуватися додатковою матрицею повороту.

Ключові слова: система глобального позиціонування; орієнтація; визначення.

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Л. М. Рижков, А. В. Прохорчук. Определение ориентации с использованием системы глобального позиционирования

Анализируется точность определения ориентации объекта при наличии ошибок измерения расстояний до спутников. Используется матричная форма анализа и метод наименьших квадратов. Показано, что для малых погрешностей измерений их влияние на точность определения ориентации может характеризоваться дополнительной матрицей поворота.

Ключевые слова: система глобального позиционирования; ориентация; определение.

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