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METHOD OF BROWN'S EXPONENTIAL FILTER ADAPTATION BY USING THE METHOD OF LEAST SQUARES

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Abstract**—***Proposed structure of the filtering algorithm gives an opportunity to avoid some of disadvantages of exponential smoothing. The main aim of proposed algorithm is to estimate filtration quality and get an ability to change smoothing factor during the work of the system. We used the method of least squares to estimate the difference between smoothed signal and signal that was built from filtered signal got by its approximation. This method might be used in the case if the trajectory of the tracking signal is not changing during the estimating process or it might be changed inconsiderably. This data processing algorithm can be used as filtering and forecasting system and integrated in systems with lags.*

Index Terms—Exponential smoothing; noise; forecast; tracking signal; low-pass filter; smoothing factor; least squares.

I. INTRODUCTION

The quality of the filtering data process is very important for systems under disturbance that can grow during the time of researching process. If system works with the same smoothing factor during the full period, the information got from measuring devices might be inaccurate. This is because filtering system with the constant smoothing factor will not work correctly if noise amplitude grows. To understand the reason of this effect it should be clear how filtering algorithm works.

Browns filter is based on the exponential smoothing. It is well known that exponential smoothing is a technique for smoothing time series data. The signal measured during some technological process can be represented as time series data as well. There are many reasons of the input or output signal disturbance.

This filtering method can be used in the systems with the noise which frequency is higher than tracking signal trajectory change. If tracking signal changes quickly and the difference between changes of the tracking signal trajectory and noise frequency is low this filter will not be able to smooth input signal because of the lag. This effect was widely described in our previous research [1].

II. PROBLEM STATEMENT

In my present research modified structure of Brown exponential smoothing algorithm was used. The full structure is shown on the Fig. 1 and was widely described in our previous research [1]. The reasonable disadvantage of this system is that it does not have the system of adaptation.

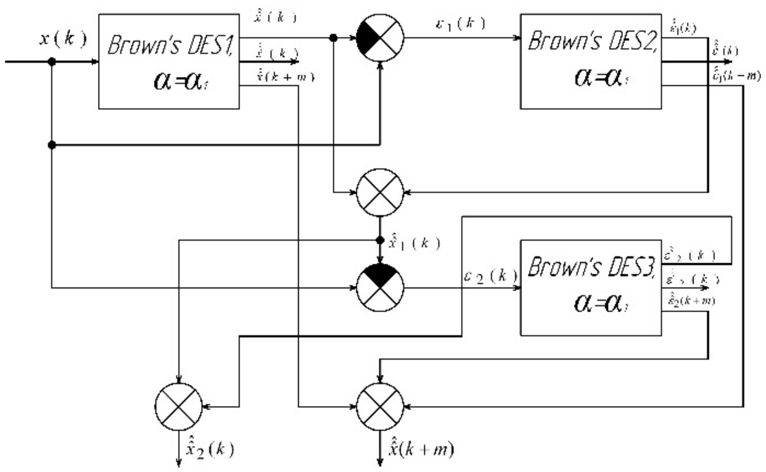

Fig. 1. Modified structure of the Brown exponential smoothing algorithm

According to the reasons mentioned above our main task was how to get information about the quality of filtration. One of the difficulties we face is that we cannot objectively determine if the filtering process goes well. Finding error between the input signal and the filtered signal does not give such information as well. Let us compare the results of the forecasting filtering process with different smoothing factors and graphs of differences between the input signal and the filtered signal.

Comparing the results of the filtering process which smoothing factor α = 0.1 (Fig. 2) and the results of the filtering process which smoothing factor α = 0.01 (Fig. 3) we can see the difference between filtration quality. As a tracking signal we have a line signal. The quality of the forecast and filtration in the case α = 0.1 is much worse than in the case α = 0.01 but we can make this conclusion only if we have values of the real tracking signal. The problem is that we do not have these values. Due to this fact we cannot estimate the quality of filtration and forecast.

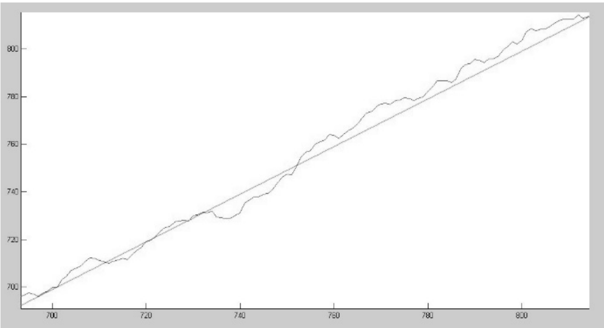

Fig. 2. Filtered forecasted signal with smoothing factor α = 0.1 and the tracking signal

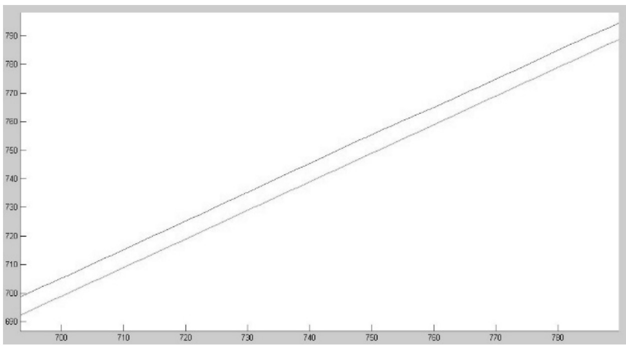

Fig. 3. Filtered forecasted signal with smoothing factor α = 0.01 and the tracking signal

To get some information about the results of the filtering process we can try to compare differences between input signal and filtered signal. This graphics are shown on the Figs 4 and 5.

According to this graphics we can see that the difference between received information from filtering systems with different smoothing factors is insignificant. There is one considerable difference between these two graphics. This is the waving part of the figure in the beginning of the filtering process that characterizes the setup of the filtering algorithm caused by lag. Such effect can be observed only in the filtering systems with the small value of the smoothing factor such as α = 0.01. Other parts of the graphics do not give any information about filtration quality.

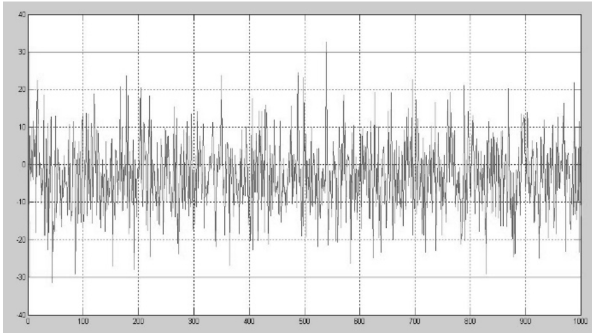

Fig. 4. Difference between input signal and filtered signal with smoothing factor α = 0.1

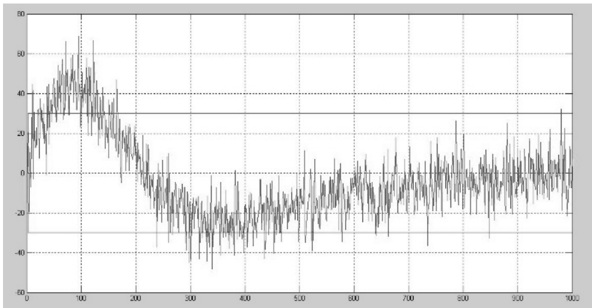

Fig. 5. Difference between input signal and filtered signal with smoothing factor α = 0.01

III. Adaptation of the Filter by Using the Method of Least Squares

At first, to understand how we can get information about filtration and forecast quality, let us shortly describe the working principle of the filter in a few formulas. In our research we used two filters connected in series. As a result, we can represent the formula which describes the value of forecast filtered signal in the $k + m$ time moment, where k is the value of the present time moment from the time moment when measurement has begun and m is the steps amount of forecast.

$$\hat{\hat{x}}(k+m) = \hat{\hat{x}}(k) + m\,\Delta t\,\hat{\hat{\dot{x}}}(k), \qquad (1)$$

where $\hat{\hat{x}}(k)$ is the double smoothed signal; $\hat{\hat{x}}(k+m)$ is the double smoothed forecast of the signal for m steps; Δt is the sample time; $\hat{\hat{\dot{x}}}(k)$ is the derivative of double smoothed signal.

Considering the facts mentioned above the first task for us was how to estimate the quality of the filtering process. It is very difficult to create the data processing system that is appropriate for different systems. That is why during our research we simplified few factors that prevented from getting information about quality of filtering process. Let us describe these few statements.

A. The trajectory of the tracking signal is not changing during the estimating process or it might be changed inconsiderably

We accept this simplification since we need to compare filtered signal with some other signal to get information about filtration quality and error. The main question is how to build this other signal. This signal can be described with the function. The degree of the function polynomial can be different that is why we must accept this simplification. Due to this statement we can try to get the first-order function which approximately describes the estimated part of the tracking signal. To sum up, we will get the set of simplified parts of the tracking signal which can be compared with filtered signal.

There is one more task we get here. How to get that first-order polynomial? In this case we can use least squares. This method gives us an opportunity to determine the approximated polynomial function using filtered signal values. The result of approximation can be inaccurate comparing with original tracking signal but the advantage we get from this process is that we have the linear signal which can be compared with filtered signal.

It is quite clear that the main information that can be used in the adaptation is the root-mean-square (rms) error. In our research we need to know only the value of the difference between filtered signal and approximated function and it is not necessary to know the sign. That is why we use rms error as the main information that describes quality of the filtering process.

B. The value of the permissible error should be defined

The algorithm of adaptation proposed in our research is simple. At first, we must initialize filter parameters. They are smoothing factor, the number of signal measurements which we accept to estimate filtration quality, $\Delta\alpha$ the magnitude of the smoothing factor changes after the filtration quality evaluation. After the filtering process has started it is necessary to wait until we get enough signal measurements to make our first estimation. As it was mentioned above the first step will be getting the first-order polynomial and building the approximated signal from the filtered signal data using least square. After we get rms error it should be compared with one more parameter we have not mentioned before. This parameter is the permissible error. This is the quantity that characterizes the acceptable quality of the filtration. During our research we did not use real signal from some measuring or control system, but the signal simulated in the Matlab. That is why to define a real permissible error it is necessary to get the information from the real system and carry out few more researches. In our case we chose the value of the permissible error which is equal to 0.03.

The final step of the adaptation is comparison of the real rms error and the permissible error. If the value of rms error is higher than permissible error, then smoothing factor will be decreased on $\Delta\alpha$. In this case the fixed $\Delta\alpha$ value cannot be used because the smoothing factor should lie within 0 and 1. If $\Delta\alpha$ is fixed than the smoothing factor can be decreased below zero that will crash the filtering algorithm.

IV. RESULTS

The results of the proposed system were modeled in the Matlab and represented on Figs 6–9.

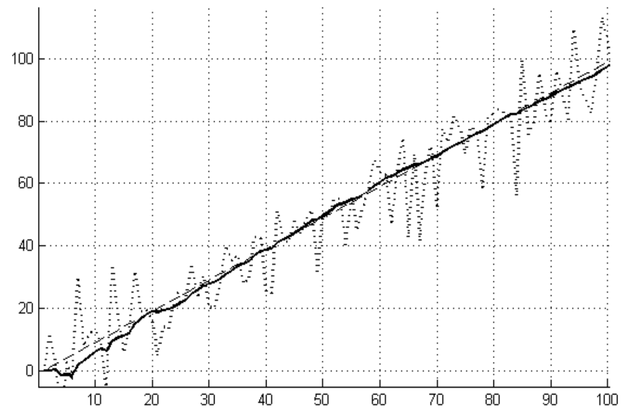

Fig. 6. Filtered signal, tracking signal and input signal at the beginning of the adaptation process

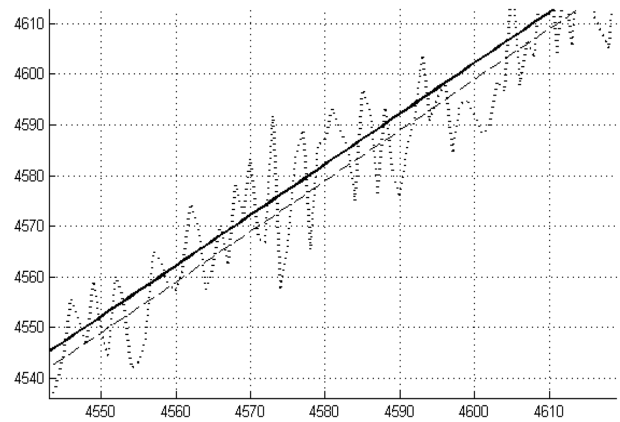

Fig. 7. Filtered signal, tracking signal and input signal after the adaptation process is finished

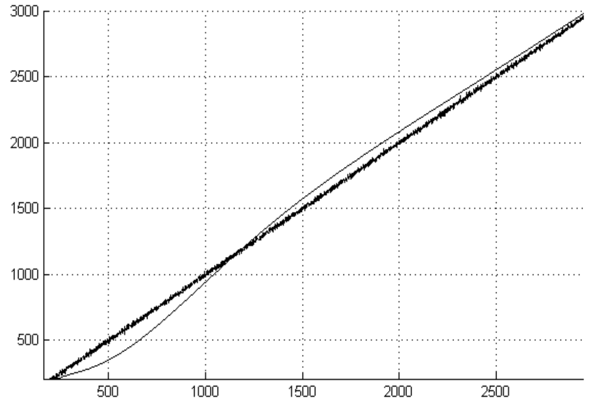

Fig. 8. Filtered signal, tracking signal and input signal during the adaptation process

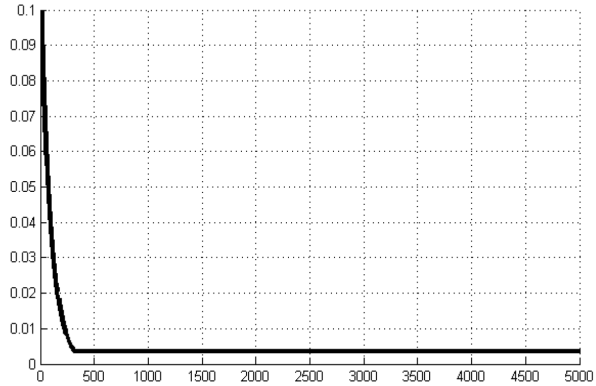

Fig. 9. The process of the adaptation of the smoothing factor

This system has its advantages and drawbacks. Let us start from disadvantages.

Drawbacks

A. The limited integration

This one is a case when we cannot get enough measurements of the signal in a short period of time. The accuracy of definition of filtering quality depends on the value of signal measurements. As far as we get less information about the signal in some period of time the approximation will be inaccurate as well. This may relate to cases when the value of measurements can be changed during the working process. Due to this fact the value of measurements that was accepted as appropriate value for estimation should be changed.

The main thing is that this filtering algorithm can show good results being integrated in systems with such value of signal measurements that gives opportunity to break the tracking signal into small linear parts.

B. Non-reversible adaptation

The thing is that adaptation of the smoothing factor is carried out by method of decreasing the value of the smoothing factor during the filtering process until the rms error is lower than the permissible error. This effect is not a huge disadvantage if the distortion amplitude tends to grow during the working process. In the case when the noise amplitude can be decreased the smoothing factor will not be increased but will stay the same. It is well known that if the smoothing factor is low the lag in filtering process is high according to the exponential smoothing theory.

Advantages

A. The flexibility of filter settings

The main point here is that we can set different combinations of filter parameters and try to compensate some of drawbacks. For example, we can decrease the accepted value of measurements for evaluation if we cannot get enough of them but at the same time we can define the lower permissible error.

B. Simplicity and reliability

Talking about different filtration systems we can face some limitation connected with peculiarities of use of one or another system. In our case we used exponential smoothing which is quite simple and reliable. Even though our modified structure makes the filtering process a little bit more difficult it gives an opportunity to avoid few disadvantages connected with lags and errors.

V. CONCLUSIONS

The method of modified brown's exponential filter adaptation was modeled, researched and described in this article. This method can be used as integrated data processing algorithm to decrease negative influence of the disturbances.

The presented algorithm is a step in our research and might be improved for usage in real systems. The best variant to get rid of drawbacks of researched filtering systems is to test this filtering system on real object.

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Б. Р. Боряк, А. М. Сільвестров, В. В. Луцьо. Метод адаптації експоненціального фільтра Брауна із використанням методу найменших квадратів

Запропонована структура алгоритму фільтрації дає можливість уникнути деяких недоліків експоненціального згладжування. Основною метою запропонованого алгоритму є оцінка якості фільтрації та можливість зміни коефіцієнта згладжування під час роботи системи. Використано метод найменших квадратів для оцінки різниці між згладженим сигналом і сигналом, який був побудований з фільтрованого сигналу, отриманого після його апроксимації. Цей метод можна використовувати у випадку, якщо траєкторія сигналу, що відстежується не змінюється під час процесу оцінювання, або вона може змінюватись не значним чином. Даний алгоритм обробки інформації можна застосовувати як для фільтрації так і для отримання прогнозованого сигналу в системах із запізненнями.

Ключові слова: експоненціальне згладжування; шум; прогноз; сигнал відстеження; фільтр низьких частот; коефіцієнт згладжування; найменші квадрати.

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Б. Р. Боряк, А. М. Сильвестров, В. В. Луцьо. Метод адаптации экспоненциального фильтра Брауна с использованием метода наименьших квадратов

Предложенная структура алгоритма фильтрации дает возможность избежать некоторых недостатков экспоненциального сглаживания. Основной целью предложенного алгоритма является оценка качества фильтрации и возможность изменения коэффициента сглаживания при работе системы. Использован метод

наименьших квадратов для оценки разницы между сглаженным сигналом и сигналом, который был построен с фильтрованного сигнала, полученного после его аппроксимации. Этот метод может быть использован в случае, если траектория отслеживаемого сигнала, не меняется в процессе оценивания, или она может меняться не значительным образом. Данный алгоритм обработки информации может быть применен как для фильтрации, так и для получения прогнозируемого сигнала в системах с опозданиями.
Ключевые слова: экспоненциальное сглаживание; шум; прогноз; сигнал отслеживания; фильтр нижних частот; коэффициент сглаживания; наименьшие квадраты.

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