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ROBUST PID CONTROL TUNING FOR THE UNCERTAIN NONLINEAR DYNAMIC MODEL OF THE UNMANNED AERIAL VEHICLE

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Abstract—*The two-step procedure is suggested to tune the robust PID control for the uncertain nonlinear dynamic system of the unmanned aerial vehicle based on the nonlinear optimization and output error method in the time domain. The suggested procedure is applied to design the robust PID control for the Skywalker X8 flying wing roll and lateral channels. For evaluating the suggested procedure, the robust PID for the Skywalker X8 flying wing roll and lateral channels is also designed using "systune" command in the Matlab Software. Suggested method can be used to design robust control with known structures for uncertain nonlinear systems.*

Index Terms—Robust PID control; unmanned aerial vehicle; uncertain aerodynamic coefficient; linear and nonlinear uncertain dynamic model; output error method; optimization; Simulated Annealing algorithm; flying wing.

I. INTRODUCTION

PID controllers are used in a variety of areas such as industrial, automotive, aerospace, electrical motors, and so on. More than 90% of the control loops have PID control. The reasons for using the PID control are as follows:

1) Easy to understand and use.
2) Acceptable performance.
3) Easy to tune, though not optimal.
4) Traditional tendency.
5) Need for support and maintenance in different systems.

Despite the widespread application, but especially with the improvement of the electric and digital technology, the attention to the PID controller was reduced. But recently, there has been the tendency to design the automated, optimal and robust PID controllers. The PID controller works well with integral windup and actuator saturation. Manual adjustment without modeling is still used. Experienced engineers use controller parameter control methods based on model and impulse test. Two methods of internal model control and lambda method are often used to tune the PID controller's parameters. According to researches, only 20 to 30 percent of the PID controllers used in the industry are optimal or near optimal and work satisfactorily, [1] – [4].

Control engineers are faced with a wide range of design requirements. These requirements such as reference tracking, disturbance rejection, robustness, noise attenuation and implementation constraints are contradictory. The complexity and contradictory of the requirements make it difficult to design the control system. Additionally, in real-world applications, there is a tendency to use simple controllers such as PIDs and known structures to facilitate implementation, validation, and re-tuning, [5], [6].

In order to adjust the parameters of the robust controller with a known structure such as PID, the uncertain dynamics of the system and the requirements of the closed-loop system are determined at first. The design requirements of the closed-loop system include the efficiency, robustness to the uncertainty of system parameters and external disturbances, and the low sensitivity to noise. One reasonable method for finding the parameters of the controller is to satisfy these requirements based on the optimization of a criterion function.

Regardless of the implementation limitations, LMI-based methods have been developed to design a control system for multiple design requirements. These techniques lead to sophisticated controllers that are necessary to reduce order, delete fast dynamics, etc. for implementing. This controller simplification is a difficult problem and sometimes complex controllers cannot be implemented. So recently, research has been conducted on finding optimal parameters for simple controller and PIDs. Different methods for adjusting of the parameters for PID and robust PID controllers such as Ziggler Nichols method, Kappa Tauing Tuning, pole placement, design based on gain and phase margins, interval polynomial method, QFD method, Kharitanov-based methods, Nyquist-based methods, tuning based on the genetic algorithm, loop shaping, and so on were developed, [1] – [4], [7] – [17].

In order to achieve the good robustness of the single-input single-output systems, the gain and phase margins are used. Robustness of the multi-input multi-output systems can be applied through the infinity norm of the sensitivity and complementary transfer functions of the closed-loop system:

$$\|S(j\omega)\|_{\infty} \leq M_S, \; \|T(j\omega)\|_{\infty} \leq M_T, \quad \forall \omega \in \Re^{+}, \tag{1}$$

where ω, the frequency is measured in rad/s and $\Re^{+}$ includes all non-negative frequencies. The above conditions are equivalent to the non-entrance to the circles related to M_S and M_T values in the Nyquist diagram of the open loop system. By decreasing the values of the M_S and M_T, the radius of the circles and level of the robustness are increased. The values from 1.2 to 2.0 for M_S and M_T provide acceptable robustness and they are equivalent to the gain margin from 6 to 2, and the phase margin from 49 to 29 degrees, respectively, [1].

According to Kharitanov's theorem, for the investigation of the stability of uncertain linear systems with an interval characteristic equation (in which the coefficients of the characteristic equation can independently change), instead of the stability check for all different combinations of the coefficients, it is enough to check the stability for the four Kharitanov's equations, [18]. Each of the uncertain coefficients of the characteristic equation is a function of the uncertain aerodynamic coefficients of the unmanned aerial vehicle (UAV). Every uncertain coefficient of the characteristic equation depends on some uncertain aerodynamic coefficients, so the intervals of the uncertain coefficients in the characteristic equation are not independent. Kharitanov's theorem can only be used to determine the intervals (space) of the parameters that stabilize the uncertain linear system. These acceptable intervals of parameters can be used as a limitation to determine their optimal value using constraint optimization for the criterion function obtained in accordance with the design requirements. Therefore, the use of the Khiryatanov's theorem for the linear system with uncertain aerodynamic coefficients of an UAV is conservative, and it is not possible to use the Kharitonov's theorem for nonlinear systems with uncertain aerodynamic coefficients. Therefore, stabilizing robust controller can be better achieved for the non-linear dynamical system of the UAV with using parametric optimization and nonlinear criterion function. But the optimization of this function is a complicated criterion for achieving an optimal global minimum. Therefore, in the first phase of the suggested approach, the robust control system is designed with a reduced dimension of uncertain aerodynamic coefficients vector based on the output sensitivity to them to reduce the order of the problem of optimization and increase the probability of achieving the optimal global minimum.

In this research, all of the PID controllers are designed simultaneously for the non-linear uncertain dynamic model of the UAV in two stages, based on the optimization of the criterion function in the time domain. To reduce the size of the problem of optimization based on the sensitivity of the output vector to the aerodynamic coefficients, the aerodynamic coefficients are classified into two groups with high and low sensitivity. In the first stage, nominal values for the aerodynamic coefficients with low-sensitivity, and upper and lower limit values for coefficients with high-sensitivity are used. In the second step, the upper and lower limits for all coefficients are considered and coefficients of the controller are evaluated and re-adjusted. Simulating annealing optimization algorithm (SAOA) as a powerful algorithm is used for nonlinear optimization.

"Systune" command in the Matlab software is also addressed to design robust PID control for the equivalent uncertain linear model of the UAV.

Problem is given in the second section. Robust PID controller for the uncertain nonlinear dynamic of the UAV is discussed in the third section. Fourth section presents robust PID controller using the "systune" command. Parameters of the designed robust PID controllers and simulation results are given in the fifth section. Conclusion and suggested future works are given in final section.

II. PROBLEM STATEMENT

The UAV dynamics model can be considered as a dynamic model with uncertainty. Because the aerodynamic model or the aerodynamic coefficients cannot be accurately calculated and are always associated with uncertainty. The UAV also has various flight modes and operates at a range of airspeed. In other words, the UAV dynamic system has different operating points. Based on the uncertainty of the aerodynamic coefficients as well as the range of changes in the airspeed of the UAV, an interval dynamic model can be considered for it.

The goal is to suggest a procedure to design the optimal robust PID controllers for uncertain nonlinear dynamic model of the UAV and then apply this approach to the roll and lateral uncertain dynamic model of the Skywalker X8 flying wing. The Skywalker X8 flying wing, its airframe and aerodynamic parameters were shown in Fig. 1, Table and Table respectively [19], [20]. Estimated aerodynamic parameters and their uncertainties are used in the uncertain nonlinear model of the UAV for designing robust PID controller.

Fig. 1. Skywalker X8 flying wing with the mini autopilot developed in at the National Aerospace University (KhAI) in Kharkov, Ukraine

Nonlinear dynamical equations of motion equations in the presence of the wind can be write as, [21]:

$$\begin{aligned}
&\left[\dot{V}_A\right]_B = \frac{f_{B-\text{aero}} + f_{B-\text{thrust}}(\delta_T)}{m(t)} + H_I^B g_I - \omega_B \times [V_I]_B, \\
&f_{B-\text{aero}} = f_{B-\text{aero}}\left(|V_A|, \alpha-\alpha_W, \beta-\beta_W, \omega_B, \delta_A, \delta_E, \delta_R\right), \\
&\frac{d\omega_B}{dt} = I_B^{-1}\left(m_{B-\text{aero}} + m_{B-\text{thrust}}(\delta_T) - \omega_B \times I_B\omega_B\right), \\
&m_{B-\text{aero}} = m_{B-\text{aero}}\left(|V_A|, \alpha-\alpha_W, \beta-\beta_W, \omega_B, \delta_A, \delta_E, \delta_R\right), \\
&\frac{d\Theta}{dt} = L_B^E\omega_B,
\end{aligned} \tag{2}$$

where $f_{B-\text{aero}}, f_{B-\text{thrust}}, m_{B-\text{aero}}$ and $m_{B-\text{thrust}}$ are aerodynamic and thrust forces and moments in the UAV Body Frame, α, β are angles of attack and side slip, α_W, β_W are angles of attack and side slip due to wind, V_A, V_I are airspeed and ground speed, ω_B is angular velocity vector; $\delta_A, \delta_E, \delta_R, \delta_T$ are control signals for aileron, elevator, rudder and thrust, g_I is the gravity vector in the Inertial Frame; I_B is matrix of inertial moments, Θ is vector of the Euler angles, H_I^B and L_B^E are the corresponding rotation matrixes. It means that aerodynamic forces and moments depend on airspeed where:

$$\begin{aligned}
&V_A = V_I - W = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_I - \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}_I, \\
&[V_A]_B = \begin{bmatrix} u_A \\ v_A \\ w_A \end{bmatrix} = H_I^B V_A = V_B - H_I^B W, \\
&\begin{bmatrix} |V_A| \\ \beta_A \\ \alpha_A \end{bmatrix} = \begin{bmatrix} \sqrt{u_A^{\;2} + v_A^{\;2} + w_A^{\;2}} \\ \sin^{-1}\left(v_A / V_A\right) \\ \tan^{-1}\left(w_A / u_A\right) \end{bmatrix},
\end{aligned} \tag{3}$$

where W is the wind vector in the Inertial Frame.

The linearized lateral equations of motion, including the effect of the wind gusts, are:

$$\begin{aligned}
&\Delta x_{latB} = \begin{bmatrix} \Delta v & \Delta p & \Delta r & \Delta\phi \end{bmatrix}^{\mathrm{T}}, \\
&\Delta u_{lat} = \Delta\delta_A, \quad \Delta w_{lat} = \Delta v_W, \\
&\Delta\dot{x}_{latB} = A_{lat}\Delta x_{latB} + B_{lat}\Delta u_{lat} + E_{lat}\Delta w_{lat}, \\
&\Delta y_{latB} = C_{lat}\Delta x_{latB} + D_{lat}\Delta u_{lat} + F_{lat}\Delta w_{lat},
\end{aligned} \tag{4}$$

where matrixes A_{lat}, B_{lat}, E_{lat}:

$$\begin{aligned}
&A_{lat} = \begin{bmatrix} Y_v & Y_p + w_0 & Y_r - u_0 & g_I\cos\theta_0 \\ L_v & L_p & L_r & L_\phi \\ N_v + Y_v N_{\dot{v}} & N_p + (Y_p + w_0)N_{\dot{v}} & N_r + (Y_r - u_0)N_{\dot{v}} & 0 \\ 0 & 1 & -\sin\theta_0 & 0 \end{bmatrix}, \\
&B_{lat} = \begin{bmatrix} Y_{\delta A} \\ L_{\delta A} \\ N_{\delta A} + Y_{\delta A}N_{\dot{v}} \\ 0 \end{bmatrix}, \quad E_{lat} = \begin{bmatrix} -A_{lat}(1,1) \\ -A_{lat}(2,1) \\ -A_{lat}(3,1) \\ 0 \end{bmatrix} = \begin{bmatrix} -Y_v \\ -L_v \\ -(N_v + Y_v N_{\dot{v}}) \\ 0 \end{bmatrix},
\end{aligned} \tag{5}$$

where all of the parameters Y_v, Y_p, Y_r, L_v, L_p, L_r, L_ϕ, N_v, N_p, N_r and $N_{\dot{v}}$ depend on aerodynamic ones and matrixes C_{lat}, D_{lat}, F_{lat}:

$$C_{lat} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D_{lat} = F_{lat} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}. \tag{6}$$

TABLE I. AIRFRAME PARAMETERS FOR SKYWALKER X8 FLYING WING

Parameter	Sky walker X8
Mass: $\mathrm{m}[\mathrm{kg}]$	4.5
Moment of inertia: $J_x\left[\mathrm{kg\cdot m^2}\right]$	0.45
Moment of inertia: $J_y\left[\mathrm{kg\cdot m^2}\right]$	0.325
Moment of inertia: $J_z\left[\mathrm{kg\cdot m^2}\right]$	0.75
Moment of inertia: $J_{xz}\left[\mathrm{kg\cdot m^2}\right]$	0.06
Wing area: $S\left[\mathrm{m^2}\right]$	0.75
Wing span: $b[\mathrm{m}]$	2.12
Mean aerodynamic chord: $c[\mathrm{m}]$	0.3571
Propeller area: $S_{\mathrm{prop}}\left[\mathrm{m^2}\right]$	0.1018
Air density: $\rho\left[\mathrm{kg/m^3}\right]$	1.2682
Motor constant: K_{Motor}	40
Propeller aerodynamic coef.: C_{prop}	0.5

TABLE II. AERODYNAMIC PARAMETERS ESTIMATES AND THEIR UNCERTAINTIES FOR SX8FW

Parame-ters	Values	
	Estimated values (Uncertainty)	***Theoretically values***
C_{Y_β}	–0.0811 (60%)	–0.1949
$C_{Y_{\delta_a}}$	–0.0729 (80%)	–0.0696
C_{l_β}	–0.1170 (50%)	–0.0765
C_{l_p}	–0.3507 (30%)	–0.4018
C_{l_r}	0.0830 (90%)	0.0250
$C_{l_{\delta_a}}$	0.3087 (40%)	0.2987
C_{n_β}	0.0583 (60%)	0.0403
C_{n_p}	–0.0154 (50%)	–0.0247
C_{n_r}	–0.1023 (30%)	–0.1252
$C_{n_{\delta_a}}$	-0.002 (80%)	-0.0076

The failure of one of the elven actuators for the flying wing also can be considered as an uncertainty in the design of the control system.

In the case of the wide range of changes for the airspeed, robust adaptive control can be designed based on the airspeed of the UAV. In the roll channel, the reduction of the disturbance effect is of paramount importance, since it is a regulation problem for the roll angle. It is desirable that the roll angle in the presence of internal and external disturbances will not be further increased. In other words, the problem of the reference tracking for the roll angle does not have a priority, but for the lateral channel, the precision of command tracking for the course angle is of prime importance, which is, of course, done through the roll angle.

The objective is to optimize the parameters of robust PID controllers for the uncertain nonlinear dynamic model of the UAV based on the two-stage approach.

Design requirements for the uncertain dynamic model of the UAV roll and lateral channels include:

1) Good reference course angle, χ tracking.

2) Disturbance rejection for the roll and course angles.

3) Minimum energy consumption to include angle and rate limits of the actuator.

III. ROBUST PID CONTROLLER FOR THE UNCERTAIN NONLINEAR DYNAMIC OF THE UAV

In practice, for the design of the UAV control system, the successful control loop closure is usually used. In this way, the controller design is initially designed for the inner control loop in the presence of the angle and angular velocity saturation of the actuator. In other words, the limitation of the angle and angular rate of the actuator limits the bandwidth and thus the function of the inner loop. Then, it is assumed that the performance of this control loop is successful, the inner loop is replaced with the unit gain and the second control loop is designed. It's worth noting that the speed or bandwidth of the inner control loop should be 5 to 10 times faster than the speed or bandwidth of the outer control loop. Again, it is assumed that the performance of the second control loop is successful, this loop also is replaced with the unit gain and the controller of the third control loop is designed. For example, in Fig. 2, the successive loop closure method as an usual and practical autopilot design method is shown in which $P_1(s)$ to $P_3(s)$ are transfer functions of the UAV and $C_1(s)$ to $C_3(s)$ are the controllers that must be designed sequentially. In this research, the parameters of internal and external loop controllers for the dynamical system are simultaneously found using optimization in the frequency and time domains, [19].

In this research, all of the PID controllers are designed simultaneously for the non-linear uncertain dynamic model of the UAV in two stages, based on the optimization of the criterion function in the time domain.

To achieve optimal performance, several functions can be used to optimize in the time domain:

1) Integrated Absolute Error.

2) Integrated Square Error.

3) Integrated Time Weighted Absolute Error.

4) Integrated Time-Weighted Square Error.

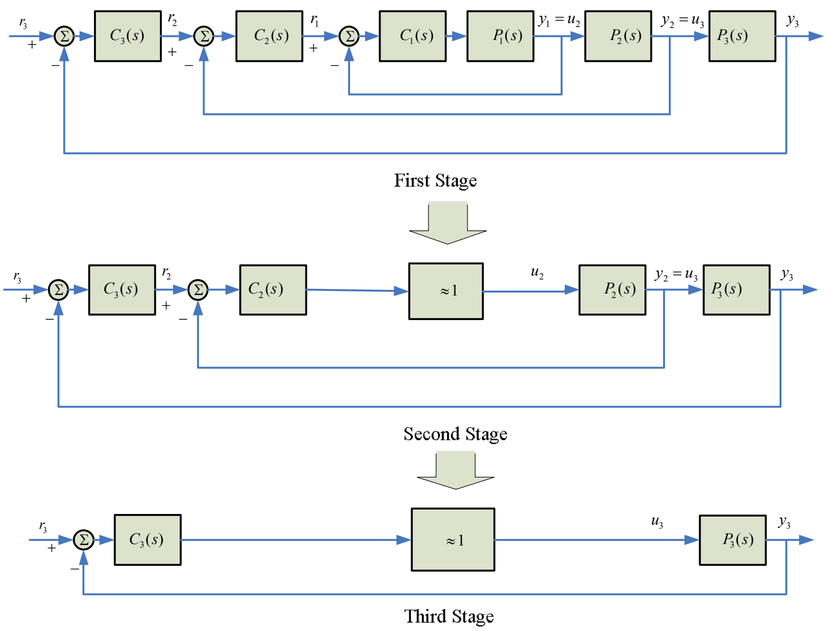

Fig. 2. Successive loop closure method for autopilot design

To achieve zero steady state error, time-weighted functions are used.

According to Kharitonov's theorem, it is enough to examine the stability of the four Kharitanov's equations for studying the stability of an interval characteristic equation. According to the Kharitonov's theorem, it is no longer necessary to examine the stability of all the different combinations of the uncertain parameters separately.

An interval polynomial is a polynomial that:

$$p(s) = p_0 s^0 + p_1 s^1 + \ldots + p_{n-1} s^{n-1} + p_n s^n , \quad (7)$$

which the polynomial coefficients can be independently changed in their intervals:

$$p_i \in \left[p_i^- \quad p_i^+ \right], \quad i = 1, 2, \ldots, n. \quad (8)$$

Then Kharitonov's theorem states that all the polynomials of the interval equation are stables if and only if these four Kharitonov's equations are stable:

$$\begin{aligned} \Delta_1(s) &= p_0^- s^0 + p_1^- s^1 + p_2^+ s^0 + p_3^+ s^1 + p_4^- s^1 + \ldots \\ \Delta_2(s) &= p_0^- s^0 + p_1^+ s^1 + p_2^+ s^0 + p_3^- s^1 + p_4^- s^1 + \ldots \\ \Delta_3(s) &= p_0^+ s^0 + p_1^- s^1 + p_2^- s^0 + p_3^+ s^1 + p_4^+ s^1 + \ldots \\ \Delta_4(s) &= p_0^+ s^0 + p_1^+ s^1 + p_2^- s^0 + p_3^- s^1 + p_4^+ s^1 + \ldots \end{aligned} \quad (9)$$

The generalized Kharitonov's theorem also has been developed as the box theorem for single input-multi outputs or multi inputs-single output, [22].

The uncertain linear dynamic model of the UAV, based on the uncertain aerodynamic coefficients, leads to the transfer functions with dependent coefficients. Therefore, the use of Khiryatanov's theorem leads to a conservative approach for describing uncertain coefficients of the polynomials. However, using the Kharitonov's generalized theorem, initial range for the parameters of the stabilizing PID controllers can be obtained, since the coefficients of the polynomial are independent of each other. Also, Kharitonov's theorem cannot be used for the non-linear uncertain dynamic model of the UAV. Therefore, a two-stage non-linear optimization approach is proposed for designing the robust PID controller.

For the design of a robust PID controller for the entire UAV, the procedure described in Fig. 3 is proposed. It is worth noting that based on the actuator limitation and generalized Kharitonov's theorem for families of interval linear systems, it is possible to obtain an acceptable approximation of the stabilizer coefficients range and then, by optimizing find the optimum parameters of the robust PID controllers for the uncertain nonlinear system.

For the lateral and roll channel based on the sensitivity of the output vector to the aerodynamic coefficients, the two groups of high-sensitivity and low sensitivity coefficients are classified as follows:

$$\begin{aligned} &\text{High sensitive parameters:} \\ &C_{l_p}, C_{n_p}, C_{l_{\delta_a}}, C_{n_\beta}, C_{Y_\beta}. \\ &\text{Low sensitive parameters:} \\ &C_{l_\beta}, C_{n_{\delta_a}}, C_{n_r}, C_{Y_{\delta_a}}, C_{l_r}. \end{aligned} \quad (10)$$

Two aerodynamic coefficients C_{Y_p} and C_{Y_r} are usually neglected due to very small quantities, [19].

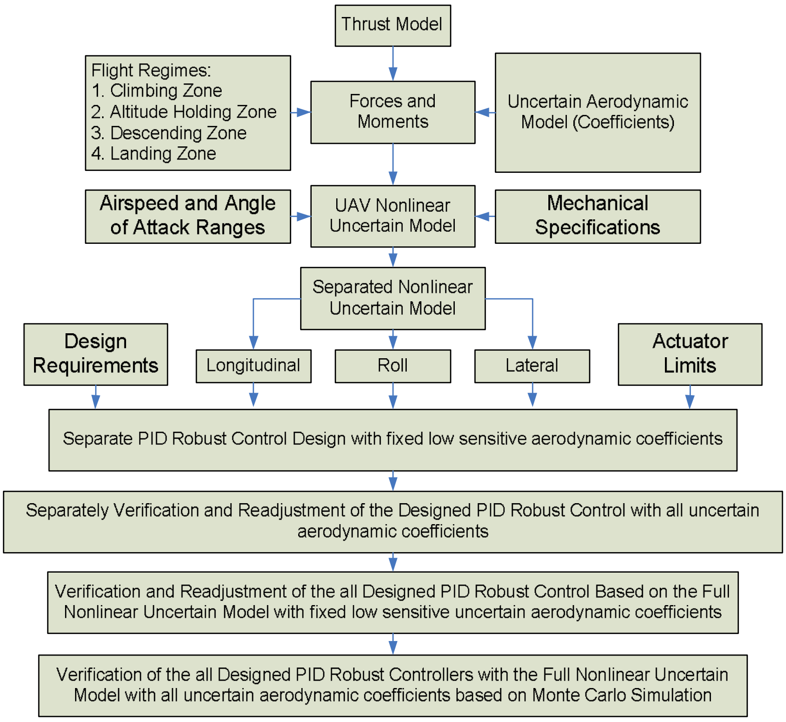

Fig. 3. Proposed procedure for design robust PID controllers for the uncertain nonlinear dynamic model of the UAV in the time domain

Then, based on the optimization algorithm for time responses to minimize disturbance effects on the outputs of the roll and course angles, tracking of the reference input as well as minimizing the control signal in the presence of system uncertainties, the optimal values of the parameters of the PID controllers are obtained. In the solution of the problem, the parameters of the PID controllers of the inner and outer loops are obtained based on optimization and using the nonlinear model of the roll and lateral channel. To perform optimization, the Integrated Time-Weighted Square Error function is used to apply the integrator of the PID controllers for removing the steady state error:

$$J=\sum_{i=1}^{r}\sum_{\substack{j=1\\ t_0\le t(j)<t_1}}^{n}\big(t(j)-t_0\big)\big[f(j)\big]+\sum_{i=1}^{r}\sum_{\substack{j=1\\ t_1\le t(j)<t_2}}^{n}\big(t(j)-t_1\big)\big[f(j)\big]+\sum_{i=1}^{r}\sum_{\substack{j=1\\ t_2\le t(j)<t_3}}^{n}\big(t(j)-t_2\big)\big[f(j)\big]$$
$$+\sum_{i=1}^{r}\sum_{\substack{j=1\\ t_3\le t(j)}}^{n}\big(t(j)-t_3\big)\big[f(j)\big], \tag{11}$$

and

$$f(j)=\frac{w_1}{\phi_{\max}}\big(\phi_i(j)\big)^2+\frac{w_2}{\chi_{\max}}\big(\chi_{\mathrm{ref}}(j)-\chi_i(j)\big)^2+\frac{w_3}{u_{\max}}\big(u_i(j)\big)^2+\frac{w_4}{\dot u_{\max}}\big(\dot u_i(j)\big)^2, \tag{12}$$

where $u_i(j)$ is elevon control signal, $\dot u_i(j)$ is the derivative of the control signal, $\phi_i(j)$ is roll angle, $\chi_{\mathbf{ref}}(j)$ is reference course angle, $\chi_i(j)$ is course angle, n is the total number of signal values in the discrete form, $r=2^m$ is the total number of upper and lower limits for uncertain aerodynamic coefficients, and $u_{\max}$, $\phi_{\max}$, $\chi_{\max}$ are the maximum values of the corresponding signals. The parameters w_1 to w_4 are weighting values used to weigh the presence of different signals in the criterion function. In Figure 4 is shown the block diagram used to design optimal robust PID controllers. Parameters of PID parameters are $C_1(s, K_{p2}, K_{i2}, K_{d2})$ and $C_2(s, K_{p2}, K_{i2}, K_{d2})$ obtained in the optimization process. SAOA is used for nonlinear constrained optimization. In SAOA, the tendency to get trapped in a local minimum is avoided by adding randomness to the acceptance of a better direction in optimization procedure and initial acceptance of direction with worse value for cost function. The optimization procedure can be restarted from obtained minimum with a high probability of worse values acceptance of cost function.

Optimal persistent and practical reference and disturbance inputs similar to those applied in the system identification are used in the procedure of the optimal robust PID controllers, Fig. 5, [23], [24].

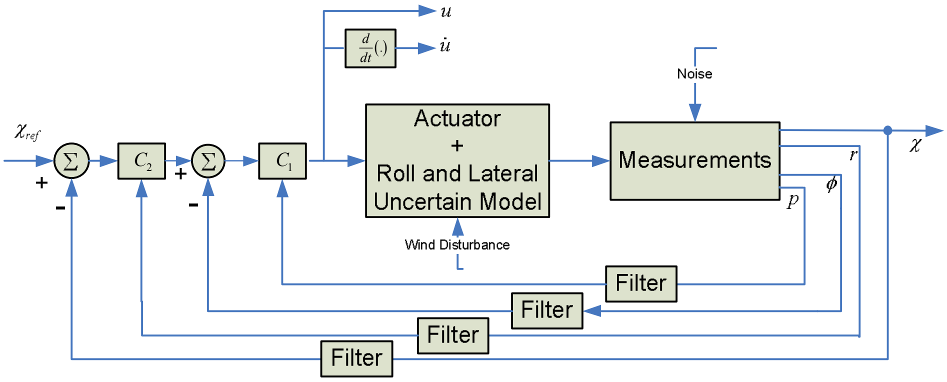

Fig. 4. Block diagram used to design optimal robust PID Controllers for roll and lateral channels of the UAV

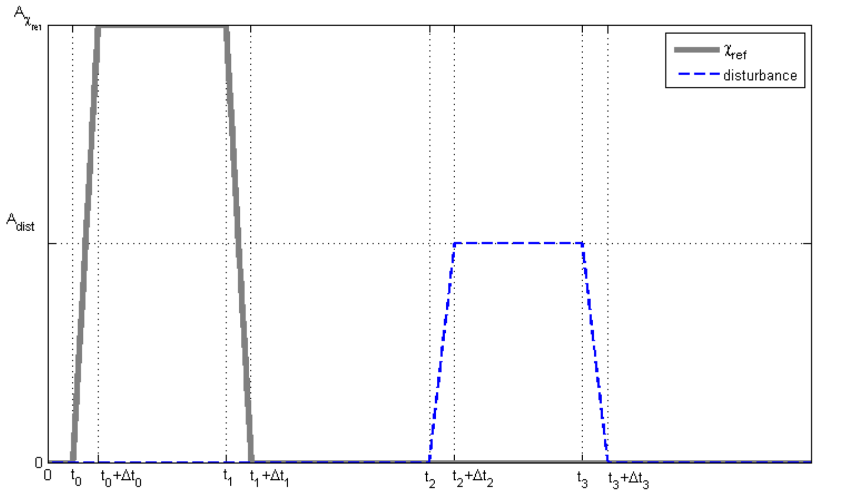

Fig. 5. Optimal course angle reference and disturbance inputs used for designing optimal robust PID controllers

For the roll angle output signal in the optimization algorithm, two different strategies can be used:

1) Minimize the angle of the roll only to eliminate the disturbance effect. In other words, in order to achieve the appropriate course angle, the roll angle can take required values. There are two disadvantages in this strategy:

– it is unacceptable for some UAV applications such as filming and photography to take large roll angles;

– large roll angle reduces aerodynamic normal force to hold the height.

2) The minimization of the roll angle is applied not only to attenuate disturbance but also to have the small roll angle during the course angle tracking. In this case, the weight, w_1 is considered non-zero for the whole time of the simulation.

IV. ROBUST PID CONTROLLER USING THE "SYSTUNE" COMMAND

The command "systune" in the Matlab software is also used to design robust PID controllers for the uncertain linear dynamic model of the UAV roll and lateral channels. Parameters of the robust PID controllers and simulation results for nonlinear uncertain roll and lateral dynamics of the UAV using two methods are presented for comparison.

In order to use the command "systune" for robust PID controller design for the roll and lateral channels of the UAV, soft and hard design requirements and linear uncertain model are determined. In Fig. 6 block diagram of the problem is displayed. The family of linear uncertain models is displayed with $G_i(s)$ where $i = 1,...,N_G$. The vector of inputs and outputs are $\left[w_1,\ldots,w_{N_I}\right]$ and $\left[z_1,\ldots,z_{N_O}\right]$ respectively and N_I and N_O are the dimensions of the input and output vectors. The input control input and measurement output vectors are u and y respectively. The fixed structure of the robust control is $C(s,K)$ that its parameters vector, K must be found using optimization. The controller parameters are found based on optimization to satisfy the following conditions, [5], [6], [13], [25] – [30]:

$$\begin{gathered}\left\|T_{ij}\left(C(s,K)\right)\right\|\le 1,\\ i=1,\ldots,N_I,\ j=1,\ldots,N_O.\end{gathered} \tag{13}$$

In which $\|.\|$ is used for H_2 or H_∞. T_{ij} is used to express the closed-loop transfer functions from each design input, w_i to each design output, z_j. Various filters can be used and their parameters, together with the controller ones can be found through the optimization basen on "systune" command.

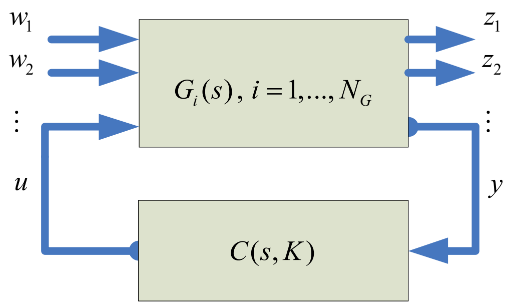

Fig. 6. Robust PID Controllers for an interval dynamic model (family of the dynamic models) to meet a set of design requirements

In a complex approach, design requirements can be classified into two categories: soft and hard requirements. Hard design requirements are those that have high priority and must be met. Hard design requirements act as a limiting factor in the process of optimizing soft design requirements. Soft design requirements are those that have a lower priority than hard design requirements, and their maximum achievable level is required in the optimization process. that's mean, [5], [6], [13], [25] – [30]:

$$\begin{gathered}\min_{\mathbf{K}}\max_{\mathbf{ij}}\left\{\left\|T_{ij}\left(C(s,K)\right)\right\|\right\}\\ i=1,\ldots,N_I, j=1,\ldots,\mathrm{N}_O\\ \textbf{Subject to}\left\{\left\|T_{k,l}\left(C(s,K)\right)\right\|\right\}\le 1\\ k=1,\ldots,N_I, l=1,\ldots,\mathrm{N}_O\end{gathered} \tag{14}$$

Hard design requirements with limit less than one must be met and act as a limiting factor in the optimization process. For linear systems, the use of the frequency domain provides a more powerful tool. The concepts of the performance and robustness in the frequency domain can be generalized for multi-inputs multi-outputs systems. Design requirements may include:

– use soft to express average gain;
– use soft for maximum gain;
– rate of decade;
– system damping;
– natural frequency of the system;
– loop shaping;
– controller Dynamic Stability;
– stability margin for single-input single-output and multiple-inputs multi-outputs systems.

V. SIMULATION RESULTS AND COMPARISON

For the airspeed range $V_a = 18 \pm 4.5$ m/s and in the presence of the aerodynamic coefficients uncertainty, optimal robust PID controllers are designed using the proposed procedure for the UAV nonlinear dynamics of the roll and lateral channels. In Fig. 7 and Fig. 8, the roll and course angles and the control signal for designed optimal robust PID controllers are shown with the "systune" command and the first and second stages of the proposed time domain method. In Table III, the values of the PID controllers parameters for the internal and external loops of the first stage (SM#1) and second stage (SM#2) of the suggested approach and "systune" command are displayed. It is also observed that the parameters obtained in the second stage of design are not significantly different from that of the first stage. Simulation results are very similar to each other using parameters of robust PID controllers for the first and second stages of the proposed method. This confirms the effectiveness of the suggested approach.

TABLE III. PARAMETERS OF PID CONTROLLERS WITH PROPOSED METHOD AND "SYSTUNE" COMMAND

Method	SM#01	SM#02	"systune"
K_{p1}	1.3276	1.2464	0.9498
K_{i1}	1.8707	1.7469	2.8841
K_{d1}	0.0083	0.0078	0.1318
K_{p2}	1.3868	1.3017	1.0759
K_{i2}	0.0066	0.0062	0.0012
K_{d2}	0.0181	0.0229	0.0000

In Fig. 9, the roll and course angles and the control signal are shown for optimal robust PID controllers designed with the "systune" method. It is worth noting that the design parameters in a two-step method of adjusting the parameters of robust PID controllers include the weighting, the model reference time response of the course angle and the optimal input signals for course angle reference signal and disturbance. It is possible to achieve optimal design by using the nonlinear model in the two-stage method for robust PID controllers design. In the first stage, the controller is designed by fixing the aerodynamic coefficients with low sensitivity, so the number of the combinations for the lower and upper limits of the uncertain aerodynamic coefficients is decreased. It is worth noting that the aerodynamic coefficients C_{Y_p} and C_{Y_r} are usually ignored. As a result, the optimization problem accelerates 16 times. As can be seen, the values of the derivative parameters of the designed controllers are very small and close to zero with both methods. It is worth noting that in practice, for the following reasons, the PI control is also used instead of the PID:

1) Due to the damping of the system, PI controllers are usually sufficient in practice.

2) The derivative part of the PID controller can lead to an increase in the control signal and the sensitivity to noise. this can damage the actuator.

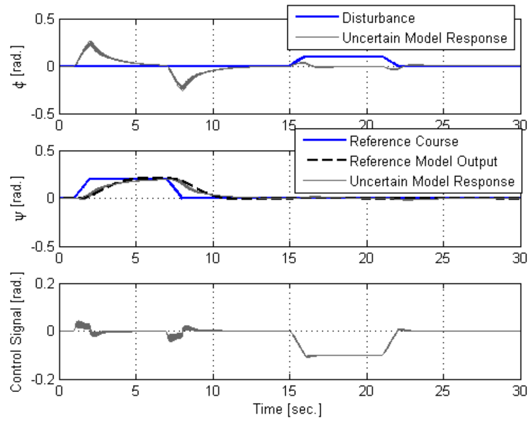

Fig. 7. Roll and course angles and channel signal for the nonlinear uncertain model with optimal robust PID controllers with the parameters obtained by the first stage of the proposed method

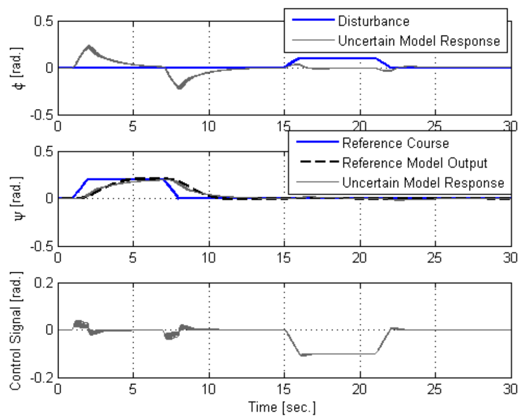

Fig. 8. Roll and course angles and channel signal for the nonlinear uncertain model with optimal robust PID controllers with the parameters obtained by the second stage of the proposed method

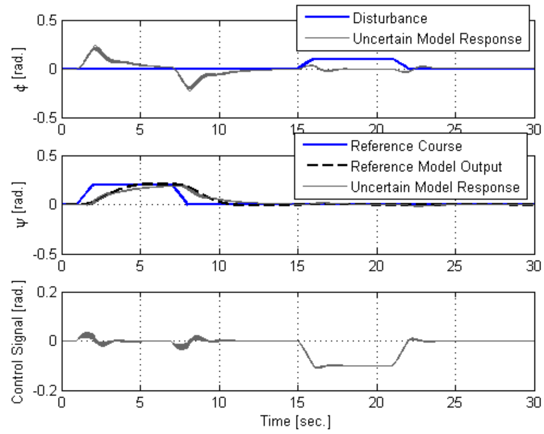

Fig. 9. Roll and course angles and channel signal for the nonlinear uncertain model with optimal robust PID controllers with the parameters obtained by the "systune" command

VI. Conclusions

A two-stage approach was proposed to design and optimize the parameters of robust PID controllers for the nonlinear uncertain dynamic model of the UAV based on nonlinear optimization in the time domain. In order to improve and accelerate nonlinear optimization, in the first stage, nominal values of aerodynamic coefficients with the low-sensitivity were used. For the uncertain nonlinear roll and lateral model of the Skywalker X8 flying wing, the robust PID controllers were designed based on the optimization of the criterion function to minimize the disturbance effect on the roll and course angles, the control signal, rate of the control signal and the tracking error of the course angle. The optimization of the criterion function was based on the nonlinear simulation of the roll and lateral channels. To reduce the effect of noise measurement, low pass filters were used in closed-loop system modeling. The Matlab command "systune" also was used to design robust PID controllers. Parameters of the robust PID controllers and closed-loop nonlinear simulation results were presented for the designed robust PID controllers.

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Р. М. Фархаді. Налаштування робастного ПІД-регулятора для невизначеної нелінійної динамічної моделі безпілотного літального апарату

Запропоновано двоетапну процедуру налаштування робастного ПІД-регулятора для невизначеної нелінійної динамічної системи безпілотного літального апарату на основі методу нелінійної оптимізації та помилки щодо виходу в тимчасовій області. Пропонована процедура використовується для налаштування робастного ПІД-регулятора моно-крила Skywalker X8 для каналів бічного і крену. Для оцінювання запропонованої процедури налаштування робастного ПІД-регулятора для каналів бічного і крену, також розроблений метод з використанням команди «systune» в програмному забезпеченні Matlab. Запропонований метод, можна використовувати для синтезу робастного керування з відомими структурами для невизначених нелінійних систем.

Ключові слова: робастний ПІД-регулятор; безпілотний літальний апарат; невизначений аеродинамічний коефіцієнт; лінійна і нелінійна невизначена динамічна модель; метод помилки щодо виходу; оптимізація; алгоритм імітації відпалу; літаюче крило.

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Р. М. Фархади. Настройка робастного ПИД-регулятора для неопределенной нелинейной динамической модели беспилотного летательного аппарата

Предложена двухэтапная процедура настройки робастного ПИД-регулятора для неопределенной нелинейной динамической системы беспилотного летательного аппарата на основе метода нелинейной оптимизации и ошибки по выходу во временной области. Предлагаемая процедура используется для настройки робастного ПИД-регулятора моно-крыла Skywalker X8 по каналам боковому и крена. Для оценки предложенной процедуры настройки робастного ПИД регулятора для каналов бокового и крена, также разработан метод с использованием команды «systune» в программном обеспечении Matlab. Предлагаемый метод, может быть использован для синтеза робастного управления с известными структурами для неопределенных нелинейных систем.

Ключевые слова: робастный ПИД-регулятор; беспилотный летательный аппарат; неопределенный аэродинамический коэффициент; линейная и нелинейная неопределенная динамическая модель; метод ошибки по выходу; оптимизация; алгоритм имитации отжига; летающее крыло.

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