UDC 629.7.027 (045)
DOI:10.18372/1990-5548.53.12138

[1]O. A. Sushchenko,
[2]Y. M. Bezkorovainyi,
[3]N. D. Novytska

ASSESSMENT OF ACCURACY OF NONORTHOGONAL REDUNDANT INERTIAL MEASURING INSTRUMENTS

Educational & Research Institute of Air Navigation, National Aviation University, Kyiv, Ukraine
E-mails: [1]sushoa@ukr.net, [2]yurii.bezkor3@gmail.com, [3]n-m@ukr.net

Abstract*—The paper deals with features of accuracy assessment of nonorthogonal redundant measuring instruments based on inertial sensors. The nonorthogonal redundant measuring instrumental based on the inertial units are developed. Approaches to assessment procedure both for measuring instruments based on single inertial sensors and triaxial inertial units are represented. Tables of directional cosines between the inertial and measuring reference frames are obtained. Analysis of measuring accuracy of the nonorthogonal redundant measuring instruments is carried out. Experimental results are given. The represented results can be applied in control systems of unmanned moving vehicles.*

Index Terms—Directional cosines; inertial sensor; measuring accuracy; nonorthogonal measuring instrument; redundancy.

I. INTRODUCTION

Development of modern navigation systems is accompanied with rising requirements to accuracy and reliability of navigation information measurements. One of ways to solve such a problem is using of the nonorthogonal redundant measuring instruments based on inertial sensors [1], [2].

It is known that increase of reliability can be provided by means of redundancy. If inertial sensors are located along axes of the orthogonal measuring reference frame, redundancy is achieved by means of increasing quantity of inertial sensors located along a given orthogonal axis. Creation of the nonorthogonal redundant measuring instruments foresees location of inertial sensors along axes of some geometrical figure (a nonorthogonal measuring reference frame). This provides the most redundancy of a measured navigation parameter (the angular rate, the acceleration) projections onto axes of the orthogonal navigation reference frame.

The nonorthogonal redundant measuring instruments are the most important for application in navigation systems operating on the unmanned moving vehicles. It is caused by rigid requirements to reliability of such navigation measuring instruments due to their long operation without the possibility to be changed or repaired. For example, the nonorthogonal redundant measuring instruments based on the float rate gyroscopes are used in navigation systems of artificial satellites.

Nowadays achievements in MEMS-technologies provide the wide application of these sensors in the nonorthogonal redundant measuring instruments. There are some additional advantages of such measuring units application taking into consideration features of rate gyroscopes and accelerations based on MEMS-technologies. These measuring instruments can be used in autopilots of unmanned aerial vehicles (UAVs). Another application is connected with satellites too. The high reliable nonorthogonal measuring units of low cost can be used in rockets applied for putting a satellite into the orbit.

In general, the nonorthogonal redundant navigation measuring instruments based on rate gyroscopes have the following advantages.

In the first place, such measuring instruments provide decreasing bias of inertial sensors. It should be noted that presence of bias is one of the most important problems of inertial sensors operation. So, usage of the nonorthogonal redundant measuring instruments improves accuracy of navigation information measurements. In the second place, reliability of navigation information increases efficiently. In the third place, such measuring instruments allow using the bigger quantity of sensors in the same dimensions of the constructive unit. This is useful even taking into account miniaturization of modern inertial sensors.

II. REVIEW OF RESEARCHES

Architecture of the nonorthogonal redundant inertial measuring instruments based on single inertial sensors is described in many papers [1], [2]. Similar problems relative to sensors based on MEMS-technologies are given in papers [3], [4]. Paper [5] deals with the nonorthogonal redundant inertial measuring instruments applied in UAVs.

Nowadays inertial measuring units are the most widespread. In this case, three one-axis sensors (gyroscopes and accelerometers) directed along the appropriate orthogonal axes are mounted in the single case. Approaches to assessment of accuracy of navigation instruments based both on single inertial sensors and inertial measuring units are of great importance for inertial navigation.

III. PROBLEM STATEMENT

There are known some ways to create redundant navigation measuring instruments using the nonorthogonal measuring reference frames [1], [2]:

1) use of a cone as a figure of symmetry and arrangement of the navigational sensors along the cone's generatrices as it is shown in Fig. 1;

2) use of a cone as a figure of symmetry and arrangement of the navigational sensors along the cone's generatrices and the axis of symmetry as it is shown in Fig. 2;

3) arrangement of sensor sensitivity axes perpendicularly to faces of regular polyhedrons in accordance with Figs 3 and 4.

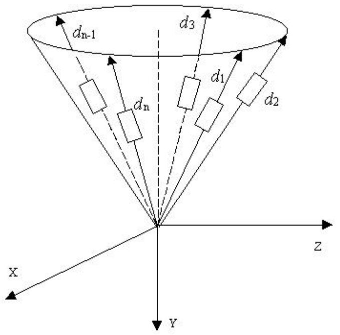

Fig. 1. Location of sensors along cone generatrices

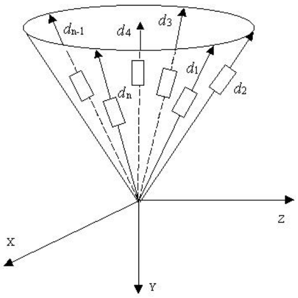

Fig. 2 Location of sensors along cone symmetry axis and generatrices

Problem statement is determination of approaches to assessment of accuracy of the nonorthogonal redundant measuring navigation instruments based on both single inertial sensors and triaxial inertial units.

Optimal orientation of inertial sensors as components of a redundant navigation measuring instrument can be determined based on optimization of a chosen criterion. Such a criterion must provide the highest accuracy of navigation information measurement. It means that an error of measurement of angular rate vector must be minimized [1].

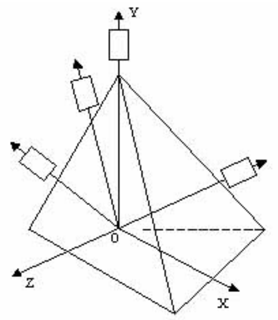

Fig. 3. Location of sensors perpendicularly to tetrahedron faces

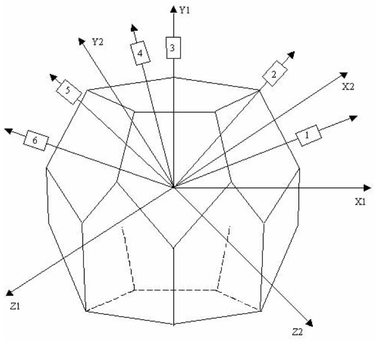

Fig. 4. Location of sensors perpendicularly to dodecahedron faces

The correlation matrix of errors can be chosen as one of accuracy characteristics. The optimization criterion depends both on inertial sensors orientation and on a way of information processing. To choose the optimization criterion correctly it is necessary to take into consideration the following requirements:

1) the highest accuracy and reliability;

2) simplicity of testing of measuring instruments, diagnostics of failures, reconfiguration and adaptation of algorithms for information processing;

3) minimum losses of computer time for information processing.

Accuracy of measurements depends on quantity and quality of the primary navigation information. It is also necessary to take into consideration the way of redundant information processing.

The least square method can be used for processing of redundant information [1], [2]. The minimum trace of the correlation matrix of errors can be chosen as the optimization criterion for this method. In this case, the statistic characteristics are believed to be independent and the mathematical expectation – equal to zero. The trace of the correlation matrix of errors can be determined by the expression [2]

$$\mathbf{D} = [\mathbf{H}^{\mathrm{T}}\mathbf{H}]^{-1}, \tag{1}$$

where **H** is a matrix of the directional cosines between the nonorthogonal measuring reference frame and the orthogonal navigation frame.

Trace of the matrix **D** represents a sum of its diagonal elements that is variances of errors

$$\mathrm{tr}(\mathbf{D}) = \sum_{i=1}^{n} d_{ii} \tag{2}$$

where d_{ii} are diagonal elements of the matrix **D**; n is quantity of sensors.

Algorithm of the least square method can be represented in the matrix form [2]

$$\mathbf{W} = \mathbf{DH}^{\mathrm{T}}\mathbf{E} = \mathbf{ME}, \tag{3}$$

where **W** is the vector of assessments; **E** is the vector of measurements; **M** is the matrix of transformation of the vector of redundant measurements **E** into the vector **W**.

Based on expressions (1) – (3) it is possible to make the following conclusion. The correlation matrix of errors of the vector redundant measuring instrument depends on orientation of separate measuring instruments.

IV. NONORTHOGONAL REDUNDANT MEASURING INSTRUMENTS BASED ON SINGLE INERTIAL SENSORS

To estimate accuracy of the nonorthogonal redundant measuring instruments based on single sensors it is necessary to obtain matrices of directional cosines. Projections of the navigation reference frame we will denote $\omega_x, \omega_y, \omega_z$. Projections of the measuring reference frame – $d_1, d_2, d_3, d_4, d_5, d_6$ respectively.

Directional cosines of the nonorthogonal measuring instruments for location of measuring axes of four, five and six sensors along generatrices of the cone are represented in Tables I, II, and III.

TABLE I. LOCATION OF FOUR SENSORS ALONG CONE GENERATRICES

	ω_x	ω_y	ω_z
d_1	$-\cos\pi/4\ \sin\vartheta$	$-\cos\vartheta$	$\cos\pi/4\sin$
d_2	$-\cos\pi/4\ \sin\vartheta$	$-\cos\vartheta$	$-\cos\pi/4\sin$
d_3	$\cos\pi/4\ \sin$	$-\cos\vartheta$	$-\cos\pi/4\sin$
d_4	$\cos\pi/4\ \sin$	$-\cos\vartheta$	$\cos\pi/4\sin$

TABLE II. LOCATION OF FIVE SENSORS ALONG CONE GENERATRICES

	ω_x	ω_y	ω_z
d_1	$\sin\vartheta$	$-\cos\vartheta$	0
d_2	$-\cos2\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$\sin2\pi/5\sin\vartheta$
d_3	$-\cos\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$\sin2\pi/5\sin\vartheta$
d_4	$-\cos\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$-\sin2\pi/5\sin\vartheta$
d_5	$\cos2\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$-\sin2\pi/5\sin\vartheta$

TABLE III. LOCATION OF SIX SENSORS ALONG CONE GENERATRICES

	ω_x	ω_y	ω_z
d_1	$\sin\vartheta$	$-\cos\vartheta$	0
d_2	$\cos\pi/3\ \sin\vartheta$	$-\cos$	$\sin\pi/3\sin\vartheta$
d_3	$-\cos\pi/3\sin\vartheta$	$-\cos\vartheta$	$\sin\pi/3\sin\vartheta$
d_4	$\sin\vartheta$	$-\cos\vartheta$	0
d_5	$\cos\pi/3\ \sin\vartheta$	$-\cos\vartheta$	$-\sin\pi/3\sin\vartheta$
d_6	$-\cos\pi/3\sin\vartheta$	$-\cos\vartheta$	$-\sin\pi/3\sin\vartheta$

Measuring axes of the sensors are tilted to the cone symmetry axis at angle ϑ [1].

If three, four, and five sensors are located along cone generatrices and one sensor – along cone symmetry axis, directional cosines look like it is represented in Tables IV, V, VI [1].

TABLE IV. LOCATION OF FOUR SENSORS

	ω_x	ω_y	ω_z
d_1	$\sin\vartheta$	$-\cos\vartheta$	$\cos\pi/4\sin\vartheta$
d_2	$-\cos\pi/3\ \sin\vartheta$	$-\cos\vartheta$	$-\cos\pi/4\sin\vartheta$
d_3	$\cos\pi/3\ \sin\vartheta$	$-\cos\vartheta$	$-\cos\pi/4\sin\vartheta$
d_4	0	-1	$\cos\pi/4\sin\vartheta$

TABLE V. LOCATION OF FIVE SENSORS

	ω_x	ω_y	ω_z
d_1	$-\cos\pi/4\sin\vartheta$	$-\cos\vartheta$	$\cos\pi/4\sin\vartheta$
d_2	$-\cos\pi/4\sin\vartheta$	$-\cos\vartheta$	$-\cos\pi/4\sin\vartheta$
d_3	$\cos\pi/4\sin\vartheta$	$-\cos\vartheta$	$-\cos\pi/4\sin\vartheta$
d_4	$\cos\pi/4\sin\vartheta$	$-\cos\vartheta$	$\cos\pi/4\sin\vartheta$
d_5	0	-1	0

TABLE VI. LOCATION OF SIX SENSORS

	ω_x	ω_y	ω_z
d_1	$\sin\vartheta$	$-\cos\vartheta$	0
d_2	$\cos2\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$\sin2\pi/5\ \sin\vartheta$
d_3	$-\cos\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$\sin\pi/5\ \sin\vartheta$
d_4	$-\cos\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$-\sin\pi/5\ \sin\vartheta$
d_5	$\cos2\pi/5\ \sin\vartheta$	$-\cos\vartheta$	$-\sin2\pi/5\ \sin\vartheta$
d_6	0	-1	0

Matrix of directional cosines for the dodecahedron is represented in Table VII. The angle between measuring axes of sensors is equal to $31^{\circ}43'$ [1].

TABLE VII. LOCATION OF SIX SENSORS ALONG DODECAHEDRON

	ω_x	ω_y	ω_z
d_1	cosγ	–sinγ	0
d_2	cosγ	sinγ	0
d_3	0	cosγ	–sinγ
d_4	0	cosγ	sinγ
d_5	–sinγ	0	cosγ
d_6	sinγ	0	cosγ

V. NONORTHOGONAL REDUNDANT MEASURING INSTRUMENTS BASED ON TRIAXIAL INERTIAL UNITS

Nowadays inertial measuring units are the most widespread in practical applications. Taking into consideration the rigid requirements to accuracy and reliability of navigation information it is important to create the new nonorthogonal redundant measuring instruments based on inertial units.

As stated above, the nonorthogonal redundant measuring instruments can be based on the triaxial inertial units located perpendicularly to faces of regular polyhedrons. In this case, it is possible to choose such geometrical figures as tetrahedron (triangular pyramid) and octahedron. From the point of view of construction implementation and dimension restrictions it is convenient to use half of the octahedron (tetragonal pyramid).

Digital triaxial MEMS-units MPU-6050 can be used as inertial measuring units of the primary navigation information (the vehicle angular rate) [6].

MPU-6050 is the integrated 6-axis motion tracking device that combines a 3-axis gyroscope, a 3-axis accelerometer, and a digital motion processor (DMP) united in a small package [6]. The MPU-6050 includes three 16-bit analog-to-digital converters (ADCs) for digitizing the gyroscope scope outputs and three 16-bit ADCs for digitizing the accelerometer outputs. For precision tracking of both fast and slow motions it is possible to use a user-programmable gyroscope in the full scale range of ± 250, ± 500, ± 1000, and ± 2000°/s and a user-programmable accelerometer in the full scale range of ± 2 g, ± 4 g, ± 8 g, and ± 16 g [6]. It is includes also the temperature sensor.

Appearance of the sensor is represented in Fig. 5.

Two constructions of researched inertial measuring unit are represented in the paper. They use the triangular and tetragonal pyramids as constructive units. Appearance of the inertial measuring unit based on the tetragonal pyramid is represented in Fig. 6. The unit is mounted at the tested bench.

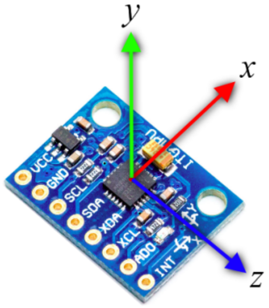

Fig. 5. `Appearance of the inertial unit and location of its axes

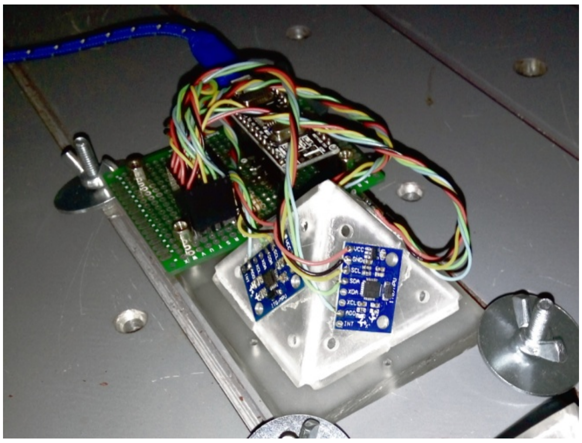

Fig. 6. Appearance of inertial measuring unit based on the tetragonal pyramid

Directions of axes are chosen to make angles between them as wide as possible. This leads to decrease of bias influence on determination of angular rate projections onto axes of the navigation reference frame.

To obtain navigation information using the nonorthogonal redundant measuring instruments it is necessary to determine mutual location between the navigation reference frame and appropriate reference frames of the inertial measuring units.

Axes of the navigation references frame, along which projections of the vehicle angular rate are determined, are chosen as follows. The axis *Oy* is up-directed along the pyramid symmetry axes. Axes *Ox*, *Oz* of the navigation reference frames coincide with appropriate axes of the inertial measuring unit located at the pyramid base (triangular or rectangular). Directions of axes are opposite to increase reliability of navigation information.

To determine matrices of directional cosines it is possible using two ways. The first way lies in obtaining projections of the unit vector using geometrical transformations. The second way is determination of directional cosines between navigation reference frame and reference frames of inertial measuring units. This process can be implemented by successive turns on definite angles.

The first approach can be implemented by less transformations and calculations respectively. Advantage of the second way is clearness. Additional complication of calculations can be compensated by the possibility to automate all necessary calculations by means of Matlab.

Directional cosines of the nonorthogonal configuration based on the triangular pyramid look like

$$\begin{aligned} \mathbf{D}_1 &= \mathbf{A}_x; \\ \mathbf{D}_2 &= \mathbf{A}_{y1}\mathbf{A}_z\mathbf{A}_y; \\ \mathbf{D}_3 &= \mathbf{A}_{y2}\mathbf{A}_z\mathbf{A}_y; \\ \mathbf{D}_4 &= \mathbf{A}_{y3}\mathbf{A}_z\mathbf{A}_y, \end{aligned} \tag{4}$$

here $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$ are matrices of directional cosines between axes of the navigational reference frame and the reference frame of the inertial measuring unit. The matrix $\mathbf{A}_x$ defines axes of the inertial measuring unit located at the base of the triangular pyramid. The matrix $\mathbf{A}_x$ characterizes tilt of inertial measuring units located on the side faces relative to the horizontal plane. Matrices $\mathbf{A}_{y1}, \mathbf{A}_{y2}, \mathbf{A}_{y3}$ define location of axes of inertial measuring units relative to previous axes. The matrix $\mathbf{A}_y$ is defined by axes of inertial measuring units located on the side faces along their medians (equal to 120°). For the triangular pyramid the angle between the base and side face is equal to 70.5°. Matrices in (4) look like

$$\mathbf{A}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix}, \mathbf{A}_y = \begin{bmatrix} \cos\psi_0 & 0 & -\sin\psi_0 \\ 0 & 1 & 0 \\ \sin\psi_0 & 0 & \cos\psi_0 \end{bmatrix},$$

$$\mathbf{A}_{y_i} = \begin{bmatrix} \cos\psi_i & 0 & -\sin\psi_i \\ 0 & 1 & 0 \\ \sin\psi_i & 0 & \cos\psi_i \end{bmatrix}, \mathbf{A}_z = \begin{bmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

here $i = 1, 2\,3$; $\gamma =$ 180°; $\psi_0 =$ 120°; $\psi_1 =$ 0°; $\psi_2 =$ 120°; $\psi_3 =$ 240°; $\vartheta = 70.5°$; γ is defined for pyramid base; ψ_i defines turns of lateral faces; ψ_i defines turns relative to normal axis of the face; ϑ is the angle of the face slope.

Substituting matrices (5) in (4) it is possible to determine mutual location of navigation and measuring reference frames. Finally, the table of directional cosines between the navigation reference frame and references frames of inertial measuring units located on faces of the triangular pyramid can be defined in the following way (Table VIII).

TABLE VIII. TABLE OF DIRECTIONAL COSINES (TRIANGULAR PYRAMID)

	ω_x	ω_y	ω_z
$d_1 = \omega_x^1$	1	0	0
$d_2 = \omega_y^1$	0	$\cos\gamma$	$-\sin\gamma$
$d_3 = \omega_z^1$	0	$\sin\gamma$	$\cos\gamma$
$d_4 = \omega_x^2$	$-\sin\psi_0 \sin\psi_1 +$ $\cos\psi_0 \cos\psi_1 \cos\vartheta$	$-\sin\vartheta \cos\psi_1$	$\sin\psi_0 \cos\psi_1 \cos\vartheta +$ $\sin\psi_1 \cos\psi_0$
$d_5 = \omega_y^2$	$\sin\vartheta \cos\psi_0$	$\cos\vartheta$	$\sin\psi_0 \sin\vartheta$
$d_6 = \omega_z^2$	$-\sin\psi_0 \cos\psi_1 -$ $\sin\psi_1 \cos\psi_0 \cos\vartheta$	$\sin\psi_1 \sin\vartheta$	$-\sin\psi_0 \sin\psi_1 \cos\vartheta +$ $\cos\psi_0 \cos\psi_1$
$d_7 = \omega_x^3$	$-\sin\psi_0 \sin\psi_2 +$ $\cos\psi_0 \cos\psi_2 \cos\vartheta$	$-\sin\vartheta \cos\psi_2$	$\sin\psi_0 \cos\psi_2 \cos\vartheta +$ $\sin\psi_2 \cos\psi_0$
$d_8 = \omega_y^3$	$\sin\vartheta \cos\psi_0$	$\cos\vartheta$	$\sin\psi_0 \sin\vartheta$
$d_9 = \omega_z^3$	$-\sin\psi_0 \cos\psi_2 -$ $\sin\psi_2 \cos\psi_0 \cos\vartheta$	$\sin\psi_2 \sin\vartheta$	$-\sin\psi_0 \sin\psi_2 \cos\vartheta +$ $\cos\psi_0 \cos\psi_2$
$d_{10} = \omega_x^4$	$-\sin\psi_0 \sin\psi_3 +$ $\cos\psi_0 \cos\psi_3 \cos\vartheta$	$-\sin\vartheta \cos\psi_3$	$\sin\psi_0 \cos\psi_3 \cos\vartheta +$ $\sin\psi_3 \cos\psi_0$
$d_{11} = \omega_y^4$	$\sin\vartheta \cos\psi_0$	$\cos\vartheta$	$\sin\psi_0 \sin\vartheta$
$d_{12} = \omega_z^4$	$-\sin\psi_0 \cos\psi_3 -$ $\sin\psi_3 \cos\psi_0 \cos\vartheta$	$\sin\psi_3 \sin\vartheta$	$-\sin\psi_0 \sin\psi_3 \cos\vartheta +$ $\cos\psi_0 \cos\psi_3$

Directional cosines of the nonorthogonal redundant inertial measuring instruments based on the tetragonal pyramid can be determined in the similar way taking into consideration a slope between the base and the side face, which is equal to 54.74°.

The appropriate matrices of directional cosines can be obtained in the following way

$$\mathbf{D}_1 = \mathbf{A}_x;\ \mathbf{D}_2 = \mathbf{A}_{y1}\mathbf{A}_z\mathbf{A}_y;\ \mathbf{D}_3 = \mathbf{A}_{y2}\mathbf{A}_z\mathbf{A}_y;$$

$$\mathbf{D}_4 = \mathbf{A}_{y3}\mathbf{A}_z\mathbf{A}_y;\ \mathbf{D}_5 = \mathbf{A}_{y4}\mathbf{A}_z\mathbf{A}_y.$$

The matrix $\mathbf{A}_x$ defines location of the inertial measuring unit located at the base of the tetragonal pyramid. The angle ϑ is equal to 54.74°. It defines tilt of the side face to the base of the tetragonal pyramid. Angles ψ_i define location of axes of the measuring reference units on the side faces. They are equal to 0°, 90°, 180°, 270° respectively.

Table of directional cosines (numerical data) for inertial measuring instrument, which uses the tetragonal pyramid as constructive element, is given in Table IX.

TABLE IX. TABLE OF DIRECTIONAL COSINES (TETRAGONAL PYRAMID)

	ω_x	ω_y	ω_z
$d_1 = \omega_x^1$	1	0	0
$d_2 = \omega_y^1$	0	–1	0
$d_3 = \omega_z^1$	0	0	–1
$d_4 = \omega_x^2$	–0.28868	–0.81650	–0.50000
$d_5 = \omega_y^2$	–0.40825	0.57735	–0.70711
$d_6 = \omega_z^2$	0.86603	0	0.5
$d_7 = \omega_x^3$	0.86603	0	0.5
$d_8 = \omega_y^3$	–0.40825	0.57735	–0.70711
$d_9 = \omega_z^3$	0.28868	0.81650	0.5
$d_{10} = \omega_x^4$	0.86603	0	0.5
$d_{11} = \omega_y^4$	–0.40825	0.57735	–0.70711
$d_{12} = \omega_z^4$	0.28868	0.81650	0.5
$d_{13} = \omega_x^5$	–0.86603	0	0.5
$d_{14} = \omega_y^5$	–0.40825	0.57735	–0.70711
$d_{15} = \omega_z^5$	–0.28868	–0.81650	–0.5

It should be noted that expressions for these direction cosines determination are simpler of similar than expressions for the directional cosines of measuring instrument based on the triangular pyramid.

VI. EXPERIMENTAL RESULTS OF NONORTHOGONAL REDUDANT INERTIAL MEASURING INSTRUMENTS

Results of experimental researches of the traxial inertial sensor are given in Fig. 7.

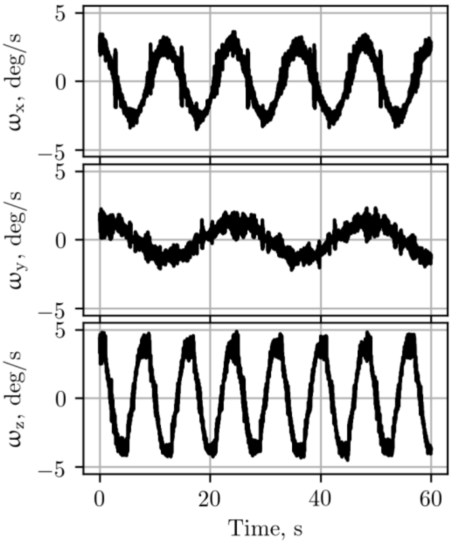

Fig. 7. Graphs of angular rates measured with an inertial triaxial sensor mounted at the base of the nonorthogonal measuring instrument

Errors of angular rate determination for the nonorthogonal inertial measuring units based on such mounting surfaces as faces of triangular and tetragonal pyramids are represented in Figs 8 and 9.

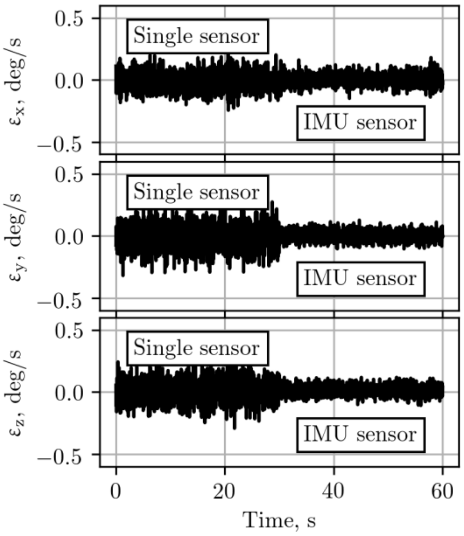

Fig. 8. Errors of angular rate determination for measuring instruments based on the triangular pyramid

To carry out experimental researches of different nonorthogonal redundant measuring units based on the triaxial inertial units, the three-degree-of-freedom dynamic bench of angular motions has been used. Such a bench provides the possibility of simulation of angular motion in three directions, which correspond to axes of the navigation reference frame.

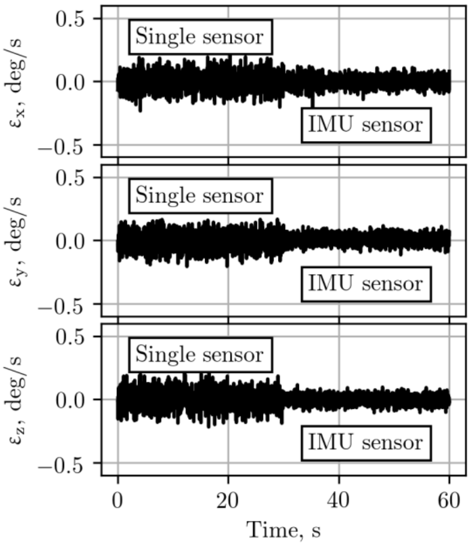

Fig. 9. Errors of angular rate determination for measuring instruments based on the tetragonal pyramid

Angular sinusoidal motions implemented by means of the bench can be described in the following form

$$x(t) = 5\sin(\pi / 6t);$$
$$y(t) = 5\sin(\pi / 12t); \qquad (6)$$
$$z(t) = 5\sin(\pi / 4t).$$

The inertial measuring unit before test starting was keeping in the immovable state during (15…20) min. This was carried out for temperature stabilization of MEMS-sensors and levelling of the zero drift. Angular motions of the three-degree-of-freedom bench (6) were given during 5 min after temperature stabilization.

Results represented in Figs 8 and 9 are averaged readings of angular rates, which have been measured in the nonorthogonal instruments based on such constructive units as the triangular and tetragonal pyramids respectively. Obtained results of measurements have been converted in projections of the angular rates onto axes of navigation reference frame using matrices of the directional cosines given in Tables VIII and IX.

Assessment of accuracy of nonorthogonal measuring instruments can be done based on the expression (1).

Results of comparative analysis of the different nonorthogonal redundant measuring instruments based on single inertial sensors are given in Table X.

TABLE X. RESULTS OF ACCURACY COMPARATIVE ANALYSIS FOR MEASURING INSTRUMENTS BASED ON SINGLE INERTIAL SENSORS

Characteristic of instrument	**Quantity of failed sensors**		
	0	**2**	**3**
5 sensors along the cone’s generators	2.21	3.20	3.92
4 sensors along the cone’s generators, 1 along the axis of symmetry	1.93	3.15	5.00
6 sensors along the cone’s generators	1.79	2.13	4.50
5 sensors along the cone’s generators, 1 along the axis of symmetry	1.70	2.18	3.35
6 sensors along the dodecahedron	1.50	2.00	3.00

Table X includes information about errors of navigation information measurement for different types of the nonorthogonal redundant measuring instruments based on single inertial sensors. Results take into consideration different situations of sensor failures.

Assessments of accuracy of the nonorthogonal redundant measuring instruments based on triaxial inertial measuring units are given in Table XI.

Analysis of results given in Table XI shows that the nonorthogonal redundant measuring instrument based on five inertial measuring units located on faces of tetragonal pyramid is the most preferable by accuracy.

TABLE XI. ASSESSMENT OF ACCURACY FOR MEASURING INSTRUMENTS BASED ON INERTIAL UNITS (TRACE OF CORRELATION MATRIX)

Characteristic of instrument	**Number of sensors**	**Trace of the matrix**
Triangular pyramid	4	0.75
Tetragonal pyramid	4	0.75
Tetragonal Pyramid	5	0.6

Accuracy assessment has been carried out also based on the normalized value of root-mean-square (RMS) relative to RMS of the single MEMS-sensor, which has been calculated during measurement interval. To make perception easier, assessment results are given in Table XII.

TABLE XII. ROOT MEAN SQUARE ASSESSMENTS OF ACCURACY FOR MEASURING INSTRUMENTS BASED ON INERTIAL UNITS

Type of constructive units	$\sigma_{rel} = \sigma_i / \sigma_i^0$		
	x	y	z
The triangular pyramid	0.66511	0.41626	0.48451
The tetragonal pyramid	0.54809	0.50939	0.44596

VII. CONCLUSIONS

Comparative analysis of accuracy of the nonorthogonal measuring instruments based on both the single inertial sensors and the triaxial inertial units was carried out. Directional cosines for the nonorthogonal redundant measuring units were derived.

The nonorthogonal inertial measuring units based on the triaxial MEMS-sensors and such construction units as the triangular and tetragonal pyramids were described in details.

Tests on accuracy assessment of the developed nonorthogonal instruments based on the triaxial inertial measuring units were carried out. The three-degree-of-freedom bench has been used for test implementation. Test results expressed in RMS values were represented.

Comparative analysis of the nonorthogonal measuring instrument accuracy of both single inertial sensors and inertial measuring units was carried out.

Research of the nonorthogonal measuring instruments of single inertial sensors and inertial measuring triaxial units proves the possibility of their using for implementation of the high-accurate and reliable measurements.

So, even taking into account the possibility of two sensors failure, the nonorthogonal redundant measuring instruments of sensors are characterized by the better efficiency of information processing.

Comparison of the different nonorthogonal redundant measuring instruments based on the triaxial inertial units by measuring accuracy shows that measuring instrument using the tetragonal pyramid is more preferable owing to improved total accuracy, more level of redundancy and simplicity of manufacturing.

REFERENCES

[1] A.D. Epifanov, *Redundant Systems of Aircraft Control*, Moscow: Mashinostroenie, 1978, 144 p. (in Russian)

[2] A.D. Epifanov, *Reliability of Control Systems*, Moscow: Mashinostroenie, 1975, 180 p. (in Russian)

[3] O.G. Gordin and V.O. Zekina, "Functionally redundant inertila measuring unit," *Radioelectronic and computer systems*, no. 2(54), 2012, pp. 35–43. (in Russian)

[4] S.N. Firsov, Kuok Tuan Do, and O.Y. Zlatkin, "Creation of fault-tolerance measuring unit of UAV SINS accelerometers," *Aviation and space engineering and technology*, no. 1(48), 2008, pp. 5–10. (in Ukrainian)

[5] V.V. Alyoshkin, M.V Alyoshkin, A.S. Sokilskii, A.S. Matveev, "Research of information processing algorithms for redundant unit of micromechanical accelerometers," *proceedings of SSTU*, no. 1(21), issue 1, 2007, pp. 95–106. (in Russian)

[6] MPU-6000 and MPU-6050 Product Specification. Access mode https://store.invensense.com/datasheets/invensense/MPU-6050_DataSheet_V3%204.pdf

Received March 02, 2017

Sushchenko Olha. Doctor of Engineering. Professor.
Aircraft Control Systems Department, Education & Research Institute of Air Navigation, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Polytechnic Institute, Kyiv, Ukraine. (1980)
Research interests: systems for stabilization of information-measuring devices operated at vehicles of the wide class.
Publications: 250.
E-mail: sushoa@ukr.net

Bezkorovainyi Yurii. Candidate of Engineering. Associate Professor.
Aircraft Control Systems Department, Education & Research Institute of Air Navigation, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, (2002).
Research interest: identification and control of complex systems.
Publications: 30.

Novytska Nataliia. Master. Assistant.
Aircraft Control Systems Department, Education & Research Institute of Air Navigation, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, (2008).
Research interest: control systems and processes.
Publications: 15.
E-mail: n-m@ukr.net

О. А. Сущенко, Ю. М. Безкоровайний, Н.Д. Новицька. Оцінка точності неортогональних надмірних інерціальних вимірювачів

Розглянуто особливості оцінювання точності неортогональних надмірних вимірювачів на основі інерціальних сенсорів. Розроблено неортогональні надмірні вимірювачі на основі інерціальних блоків. Представлено підходи до процедур оцінювання точності вимірювачів на основі як поодиноких сенсорів, так і триосних інерціальних блоків. Отримано таблиці напрямних косинусів між навігаційною та вимірювальною системами координат. Виконано аналіз вимірювальної точності неортогональних надмірних вимірювачів. Представлено результати експериментальних досліджень. Надані результати можуть бути використані в системах керування безпілотними рухомими об'єктами.

Ключові слова: напрямні косинуси; інерціальний датчик; вимірювальна точність; неортогональний вимірювач; надмірність.

Сущенко Ольга Андріївна. Доктор технічних наук. Професор.
Кафедра систем управління літальних апаратів, Навчально-науковий інститут аеронавігації, Національний авіаційний університет, Київ, Україна.
Освіта: Київський політехнічний інститут, Київ, Україна (1980).
Напрям наукової діяльності: системи стабілізації інформаційно-вимірювальних пристроїв, експлуатованих на рухомих об'єктах широкого класу.
Кількість публікацій: 250.
E-mail: sushoa@ukr.net

Безкоровайний Юрій Миколайович. Кандидат технічних наук. Доцент.
Кафедра систем управління літальних апаратів, Навчально-науковий інститут аеронавігації, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, (2002).
Напрям наукової діяльності: ідентифікація та управління складними системами.
Кількість публікацій: 30.
E-mail: yriy.bezkor@ukr.net

Новицька Наталія Дмитрівна. Магістр. Асистент.
Кафедра систем управління літальних апаратів, Навчально-науковий інститут аеронавігації, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, (2008).
Напрям наукової діяльності: системи та процеси управління.
Кількість публікацій: 15.
E-mail: n-m@ukr.net

О. А. Сущенко, Ю. Н. Безкоровайный, Н. Д. Новицкая. Оценка точности неортогональных избыточных инерциальных измерителей

Рассмотрены особенности оценивания точности неортогональных измерителей на основе инерциальных датчиков. Разработаны неортогональные избыточные измерители на основе инерциальных блоков. Представлены подходы к процедурам оценивания точности измерителей на основе, как одиночных датчиков, так и трехосных инерциальных измерительных блоков. Получены таблицы направляющих косинусов между навигационной и измерительной координатными системами координат. Выполнен анализ измерительной точности неортогональных избыточных измерителей. Представлены результаты экспериментальных исследований. Полученные результаты могут быть использованы в системах управления беспилотных подвижных объектов.

Ключевые слова: направляющие косинусы; инерциальный датчик; измерительная точность; неортогональный измеритель; избыточность.

Сущенко Ольга Андреевна. Доктор технических наук. Профессор.
Кафедра систем управления летательных аппаратов, Учебно-научный институт аэронавигации, Национальный авиационный университет, Киев, Украина.
Образование: Киевский политехнический институт, Киев, Украина, (1980).
Направление научной деятельности: системы стабилизации информационно-измерительных устройств, эксплуатируемых на подвижных объектах широкого класса.
Количество публикаций: 250.
E-mail: sushoa@ukr.net

Безкоровайный Юрий Николаевич. Кандидат технических наук. Доцент.
Кафедра систем управления летательных аппаратов, Учебно-научный институт аэронавигации, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, (2002).
Направление научной деятельности: идентификация и управление сложными системами.
Количество публикаций: 30.
E-mail: yriy.bezkor@ukr.net

Новицкая Наталия Дмитриевна. Магистр. Ассистент.
Кафедра систем управления летательных аппаратов, Учебно-научный институт аэронавигации, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, (2008).
Направление научной деятельности: системы и процессы управления.
Количество публикаций: 15.
E-mail: n-m@ukr.net