[1]**O. Y. Krasnousova,**
[2]**Y. A. Opanasiuk,**
[3]**B. I. Dmytrenko**

PROBABILISTIC METHODS OF ZERO-DEFECTS CHECKING FOR NONHOMOGENEOUS SAMPLINGS

[1,2,3]Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine
E-mails: [1]o.krasnousova@gmail.com, [2]Yuriy.opanasiuk@gmail.com, [3]dmbog@ukr.net

Abstract—A short outline of the modified probabilistic methods of zero-defects checking is given and the applicability of these methods for nonhomogeneous sampling is substantiated. The main task of this paper is to substantiate the proposed modification of the zero-defect checking for obviously heterogeneous sampling. The proposed modification of the classical zero-defects methods allows: essentially decrease required volumes of testing; reduce testing terms in comparison with repeated fulfillments of the classical zero-defects method of tests; visibly declined testing costs. The proposed modifications of the classical zero-defects methods are fitted to work for all cases of essentially inhomogeneous samples. Given substantiation of the proposed method may be essentially used also for the modification of Bernoulli binomial testing scheme to reconcile it with the real situation in the accepting sampling. Our methods as a classical ones are based on application of Bernoulli's binominal testing scheme.

Index Terms—Probabilistic methods; nonhomogeneous sampling; zero-defects methods; testing scheme; ergodic systems; inferential statistic; Bernoulli binomial testing scheme.

I. INTRODUCTION & PROBLEM STATEMENT

It is customary to describe various aspects of checking procedures and connected problems in terms of inferential statistic. More and yet more of interest and activity in statistics today, particularly as it relates of scientific activity and experimentation, concerns inferences rather than just description. The engineer who accumulates data on a sample of computer system will ultimately wish to draw conclusions about all such systems. The medical team that develops a new vaccine for disease treatment in a particular population is interested in what would happen if the vaccine where administrated to all people in the population. The marketing expert may test a product in a few "representative" areas; from the resulting information he will draw conclusions about what would happen if the product were made available to all potential purchases. The main focus of this paper is on presenting and illustrating methods of inferential statistics that are useful in accepting sampling and in similar areas [1]. The classical probabilistic method of the zero-defects checking [2] envisages a verification of a special hypothesis on the probability of successful working, p, which satisfies inequality $P \geq P_T$ (P_T – some minimum value) with an admissible low risk β_0 under results of a string of N_0, successful successive testing:

$$\beta_0 = P_T^{N_0},\qquad(1)$$

It should be admitted is it a value of a defect failure error $q = 1 - p$ is implied to be study for each taken separately testing. In this sense the samples should be homogeneous and identical with each other for all testing conditions. The same demand, q = const. is envisaged in the classical methods of accepting sampling, which are based on application of Bernoulli's binominal testing scheme (BTS). In the frame of this approach the adoption of their bodies hypothesis $P \geq P_T$ is possible with risk β only with presence of no more than k failures in a testing for the length of $N_k > N_0$, [2]–[6]. However, the postulate q = const. which possibly holds in some trivial cases but is evidently inapplicable to the complicated technical and ergodic systems including a man. Really, it is not possible to assume q = const. under an automatic airplane landing at different atmospheric conditions. Also it is not possible at the time of complicated medical operation, success of which depends on many factors including health conditions, patient age and so on. So, there is a problem of using off the classical probabilistic methods at the presents of nonhomogeneous samplings which correspond to different working and testing conditions. For example, under frequent using of zero-defects methods (separately for every different condition) it is necessary:

– define and settle number of n groups of different possible homogeneous or inhomogeneous testing conditions;

– define or settle a frequency of occurrence S_i, $i = 1...n$ of these conditions (or, which is the same, apriori probabilities);

– select required value of $P_{i,T}$ (aposteriori probabilities);

– settles the admissible risks of erroneous assumption for hypothesis $P_i \geq P_{i,T}$;

– realize carrying out of tests with the total volume $N_0 = \sum_{i=1}^{n} N_{0,1}$, where particular volumes may be defined from relation $\beta_{0,i} = (p_{i,T})^{N_{0,i}}$ [3] – [6];

– substantiate the assumption of the decision $P \geq P_T$ as well as the value of risk β_0, which is adapted for this hypothesis.

This is readily be observed this way is not available. The sufficient reasons for these are inappropriate volumes of testing, their costs and needed terms to carry out of the tests. So, the main task of this paper is to substantiate the proposed modification of the zero-defect checking for obviously heterogeneous sampling. This modification is probably only partial solution of the problem. To solve the problem some particular methods of inferential statistics [1] – [5], including the result of papers [6], [7] where used.

II. THE BASIC POINTS OF THE PROPOSED MODIFICATIONS

– The postulate q = const. which is customary to describe various aspects of checking procedures (especially in the acceptance sampling) is not used to completely or partially. In the last case it is possible to assume constant for every different value of q_i, for example, for n homogeneous or quasihomogenous groups of the working or testing conditions.

– The notion of the total success probability P_T is used.

– The notion of the total success probability in all N_0 tests $P_{T,S}$.

– Heterogeneous proportional samples are used; in this case the total volume $N_0 = \sum_{i=1}^{n} N_{0,i}$, where

$N_{0,i} = S_i N_0$, and S_i is so called "occurrence" (frequency of occurrence). The situation when $n = N_0$, i.e. $S_i = N_0^{-1}$ is possible. Also possible is the case of the 'weighted' samples, when, e.g., occurrence of the complicated testing conditions, S_2 is equal to a occurrence of S_1 of the normal ones.

– Setting of the required areas of $q_{iT}, P_{i,T}$ is not obligatory.

III. THE SUBSTANTIATION OF THE PROPOSED MODIFICATION

The total success probability P_T of an ahead operation is equal to [1] – [4]

$$P_T = \sum_{i=1}^{n} S_i P_{iT} , \qquad (2)$$

with S_i is the apriori occurrence of i-conditions; P_{iT} is the aposteriori (conditional) required probabilities of success; n is the number of groups of different testing conditions, where every group has its special value of P_{iT} = const.

At a pinch it is assumed

$$P_T = N_0^{-1} \sum_{i=1}^{n} P_{iT} , \qquad (3)$$

i.e. every ith condition is assumed to be equally likely and sample consist of a solitary test for every ith conditions.

It is easy to see that (3) is arithmetic mean (AM) and (2) is average weighted arithmetic value (AWA).

Let us consider now expression $P_{T,S}$ of the total success probability in all N_0 tests. As is readily be observed, that expression β_0, (1), gives value $P_{T,S} = \beta_0$ in the case of homogeneous sample, when for success of every testing is needed probability $P = P_T$ = const.

Singling out of n layers in an inhomogeneous sample yields

$$P_{T,S} = \beta_0^* = \prod_{i=1}^{n} P_i^{S_i N_0} = \left(\prod_{i=1}^{n} P_i^{S_i} \right)^{N_0} , \qquad (4)$$

As is easy to see that the parentheses in (4) contain the average weighted geometrical mean (AWGM) of P_i. Therefore, the expression (4) can be rewritten:

$$\beta_0^* = (AWGM)^{N_0} ,$$

In the case, when $n = N_0, S_i = N_0^{-1}$, and (4) takes the form

$$P_{T,S} = \beta_0^* = \left(\prod_{i=1}^{N_0} (P_i) \right)^{N_0^{-1}} , \qquad (5)$$

where the parenthesis's contain the geometrical mean (GM) of P_i, and in this case (5) may be rewritten as:

$$P_{T,S} = \beta_0^* = (GM)^{N_0},$$

It is not possible to realize a single-valued choice of the required values for $P_{i,T}$ even in the simplest case. For example, if $n = 2$, $S_1 = S_2 = 0.5$, then the infinite number of pairs (P_1, P_2) correspond to every value of P_T. On the other hand it is known that

$$(GM) \leq (AM) \text{ and } (AWGM) \leq (AWAM), \quad (6)$$

Using the relations (3) – (6) let us set someone value for P_T, may be according to an agreement with purveyors (manufacturers) and consumers. Then, don't pick up – it is the main point of the proposed method – let us compare possible values of $P_{T,S}$ (4) with the expression $\beta_0 = (P_T)^{N_0}$. The last expression gives the probability values of the total success of N_0 testing under the homogeneous condition, which are characterized by constant success probability in every testing, and more of this, which is equal to the value P_T inhomogeneous sample. And in such a way we'll always should be have $P_{T,S} \leq P_T^{N_0}$ and also will be heaven

$$\beta_0^* \leq \beta_0. \quad (7)$$

Naturally, positive results of testing (absence of failures), which are obtained at the using of weighted sample should be even so more true with respect to the situations when the present complicated situations are less complicated in comparison with ones of the discussed methods. So, the essence of the proposed modifications of the zero-defects method infers:

The choice of the inquired value of $P_{n,T}$ the complete (total) probability of success and the admissible value of risk $\beta_0^* \leq \beta_0$ of an acceptance of hypothesis $P_n \geq P_{n,T}$; the total volume of testing is defined according to formula (1).

The number n of different homogeneous or quasihomogeneous conditions of testing is appointed.

The value of S_i occurrences of homogeneous or quasihomogeneous conditions is appointed for example, $S_i = 0.25$, when $i = \overline{1,4}$.

The volume of testing's for every of the conditions is appointed as $N_{0,i} = S_i N_0$.

If number n is comparable with N_0, then $n = N_0$, $S_i = N_0^{-1} S_i = N_0^{-1}$ are chosen.

All testing's $N_0 = \sum_{i=1}^{n} N_{0,i}$ are successively carried out (in any succession), or all N_0 solitary testing are performed at the proper conditions.

All defects and failures are absent in all appointed test, when hypothesis $P_n \geq P_{n,T}$ is accepted with risk $\beta_0^* \leq \beta_0$.

All defects and failures were found at the time of testing, then it is necessary either reject the accepted hypothesis or continue testing according to BTS. But proceeding with BTS also needs modifications, at least, for instance similar to those, which earlier described. At a pinch it is possible also a lowering of the $P_{T,S}$ – level and, accordingly, an increase of the risk β_0^*.

IV. CONCLUSIONS

1) The proposed modifications of the classical zero-defects methods are fitted to work for all cases of essentially inhomogeneous samples.

2) The proposed modification of the classical zero-defects methods allow:

– essentially decrease required volumes of testing;

– reduce testing terms in comparison with repeated fulfillments of the classical zero-defects method of test;

– visibly declined testing costs.

3) Obtained results may be used in the accepting sampling of complicated technical and ergodic systems as well as in the estimation of correspondence with prescribed requirements for specialist in different areas including science, industry, engineering, finance, medicine, aviation and armament.

4) Given substantiation of the proposed method may be essentially used also for the modification of Bernoulli binomial testing scheme to reconcile it with the real situation in the accepting sampling.

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Krasnousova Olga. Candidate of Science (Engineering).
Aviation Computer-Integrated Complexes Department, Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Institute of Civil Aviation Engineers, Kyiv, Ukraine. (1995).
Research interests: statistical methods of radioelectronic systems quality control.
Publications: 25.
E-mail: o.krasnousova@gmail.com

Opanasiuk Yuriy. Assistant Professor.
Aviation Computer-Integrated Complexes Department, Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.
Education: Kiev State University named T. G. Shevchenko, Kyiv, Ukraine (1973).
Research interests: gravitation and general theory of relativity, astrophysics and cosmology, relativistic dynamics, air navigation.
Publications: more than 50.
E-mail: yuriy.opanasiuk@gmail.com

Dmytrenko Bohdan. Teaching Fellow.
Aviation Computer-Integrated Complexes Department, Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.
Education: National Technical University of Ukraine "Kyiv Politechnic Institute," (1999).
Research area: automated systems, CAD, CAM, windenergy.
Publication: 21.
E-mail: dmbog@ukr.net

О. Ю. Красноусова, Ю. А. Опанасюк, Б. І. Дмитренко. Ймовірнісні методи бездефектного контролю у разі застосування неоднорідної вибірки

Наведено короткий опис модифікованих ймовірнісних методів перевірки нульових дефектів та обґрунтовано придатність застосування цих методів для неоднорідної вибірки. Основним завданням даної роботи є обґрунтування запропонованої модифікації перевірки дефектів з явно неоднорідною вибіркою. Запропонована модифікація класичних методів нульових дефектів дозволяє істотно зменшити необхідні обсяги тестування; скоротити терміни випробувань порівнянно з повторними тестуваннями класичного методу бездефектного контролю; помітно знизилися витрати на тестування. Запропоновані модифікації класичних методів бездефектного контролю придатні для роботи у всіх випадках суттєво неоднорідних зразків. Враховуючи обґрунтування запропонованого методу, можна, по суті, використати також модифікацію схеми біномиальних випробувань Бернуллі, щоб узгодити її з реальною ситуацією в приймаючій вибірці. Розглянуті методи, як і класичні, базуються на застосуванні схеми біноміального тестування Бернуллі.
Ключові слова: ймовірнісні методи; неоднорідна вибірка; методи нульових дефектів; схема тестування; ергодичні системи; інферентна статистика; біноміальна схема Бернуллі.

Красноусова Ольга Юрьївна. Кандидат технічних наук.
Кафедра авіаційних комп'ютерно-інтегрованих комплексів, Навчально-науковий інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.
Освіта: Київський інститут інженерів цивільної авіації, Київ, Україна (1995).
Напрям наукової діяльності: статистичні методи контролю якості радіоелектронних систем.
Кількість публікацій: 25.
E-mail: o.krasnousova@gmail.com

Опанасюк Юрій Арценович. Старший викладач.
Кафедра авіаційних комп'ютерно-інтегрованих комплексів, Навчально-науковий інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.

Освіта: Національний університет ім. Тараса Шевченка, Київ, Україна, (1973).

Напрям наукової діяльності: теорія тяжіння та загальна теорія відносності, астрофізика та космологія, релятивістська динаміка, аеронавігація.

Кількість публікацій: більше 50.

E-mail: yuriy.opanasiuk@gmail.com

Дмитренко Богдан Іванович. Асистент.

Кафедра авіаційних комп'ютерно-інтегрованих комплексів, Навчально-науковий інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.

Освіта: Національний технічний Університет України «Київський політехнічний інститут», (1999).

Напрям наукової діяльності: автоматизовані системи, вітроенергетика.

Кількість публікацій: 21.

E-mail: dmbog@ukr.net

О. Ю. Красноусова, Ю. А. Опанасюк, Б. И. Дмитренко. Вероятностные методы бездефектного контроля при использовании неоднородной выборки

Приведено краткое описание модифицированных вероятностных методов проверки нулевых дефектов и обоснованно пригодность применения этих методов для неоднородной выборки. Основной задачей данной работы является обоснование предложенной модификации проверки дефектов с явно неоднородной выборкой. Предложенная модификация классических методов нулевых дефектов позволяет существенно уменьшить необходимые объемы тестирования; сократить сроки испытаний по сравнению с повторными испытаниями классического метода бездефектного контроля; заметно снизились расходы на тестирование. Предложенные модификации классических методов бездефектного контроля пригодны для работы во всех случаях существенно неоднородных образцов. Учитывая обоснование предложенного метода, можно, по сути, использовать также модификацию схемы биномиальных испытаний Бернулли, чтобы согласовать ее с реальной ситуацией в принимающей выборке. Рассмотрены методы, как и классические, основанные на применении схемы биномиального тестирования Бернулли.

Ключевые слова: вероятностные методы; неоднородная выборка; методы нулевых дефектов; схема тестирования; эргодической системы; инферентная статистика; биномиальная схема Бернулли.

Красноусова Ольга Юрьевна. Кандидат технических наук.

Кафедра авиационных компьютерно-интегрированных комплексов, Учебно-научный институт информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.

Образование: Киевский институт инженеров гражданской авиации, (1995).

Направление научной деятельности: статистические методы контроля качества радиоэлектронных систем.

Количество публикаций: 25.

E-mail: o.krasnousova@gmail.com

Опанасюк Юрий Арценович. Старший преподаватель.

Кафедра авиационных компьютерно-интегрированных комплексов, Учебно-научный институт информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.

Образование: Национальный университет им. Тараса Шевченко, Киев, Украина, (1973).

Направление научной деятельности: теория тяготения и общая теория относительности, астрофизика и космология, релятивистская динамика, аэронавигация.

Количество публикаций: более 50.

E-mail: yuriy.opanasiuk@gmail.com

Дмитренко Богдан Иванович. Ассистент.

Кафедра авиационных компьютерно-интегрированных комплексов, Учебно-научный институт информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.

Образование: Национальный технический Университет Украины «Киевский политехнический институ», (1999).

Направление научной деятельности: автоматизированные системы, ветроэнергетика.

Количество публикаций: 21.

E-mail: dmbog@ukr.net