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[1]N. F. Tupitsin,
[2]I. V. Chychkan

APPROXIMATE CALCULATION OF THE PROCESS CHARACTERISTICS OF THE UAV LANDING ON ROPE

[1]Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine
[2]Department of Intellectual and Information systems, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
E-mails: [1]nift@mail.ru, [2]i.v.chychkan@gmail.com

Abstract—*Analytical expressions, which connected of the unmanned aerial vehicle parameters and characteristics of landing's rope, are obtained. In particular, the relationship between the necessary length of rope and the value of its stretching during the landing of the unmanned aerial vehicle is determined. Meanwhile, additional dampers for the rope are not considered. The mathematical model of the deceleration process of unmanned aerial vehicle during its landing on rope is based on Hooke's law and Newton's 2nd law. One of the main assumptions at the development of the mathematical model is that of the braking of the unmanned aerial vehicle after its coupling with the rope occurs with constant acceleration.*

Index Terms—Unmanned aerial vehicle landing; rope stretching; Hooke's law; unmanned aerial vehicle overload.

I. INTRODUCTION

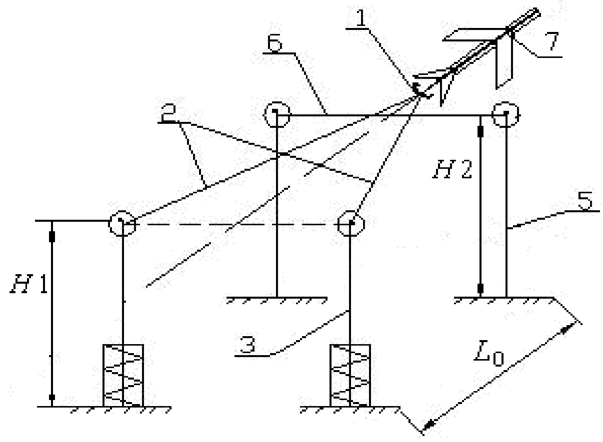
The unmanned aerial vehicles, launched from the hands or from the starting guide track installation (catapult), must have defined means for landing. Sometimes, the Unmanned Aerial Vehicle (UAV) launched in this way are equipped with a parachute, with the help of which they are down to earth.

In recent years, specially designed landing methods for UAVs without chassis have been developed and applied: landing on rope or in a special elastic net [1] – [3]. As a result of application of these methods is increasing economic and technical efficiency of UAV by increasing its payload due to the lack of chassis on board. In addition, the UAV landing with the help of devices, which are realized such methods, can be carried out on area of limited dimensions in a given place.

Known landing device [1], [3] contain: a rope stretched perpendicular to the trajectory of UAV on a place of landing, a brake hook and a shock absorber located on the UAV. The device [1] has also first and second gates. Schematically two main moments of UAV landing on this device is shown in Fig. 1. In Figure 1a is shown moment, when the kinetic energy of UAV is equal zero and in Fig. 1b is shown final moment of UAV landing, when the UAV is immoveable. The height of the rope of first gate $H1$ is greater than the height of second one $H2$ on the value $\Delta H = H1 - H2$, so that

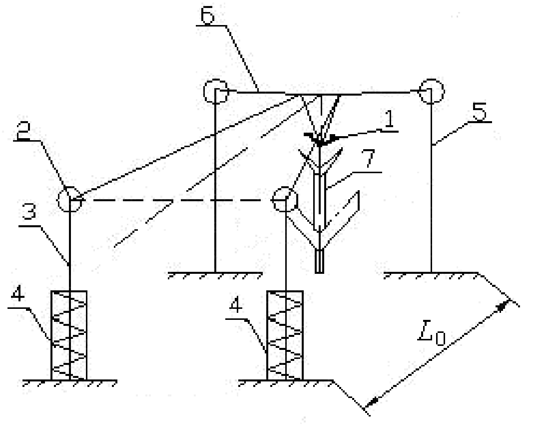
$$\Delta H \geq gL_0^2 / \left(2V_{FV}^2\right),$$

where g is the acceleration of gravity; L is the distance between landing gates, which depends on the rigidity of the brake rope and the characteristics of the shock absorber; V_{FV} is the UAV landing speed.



a

Fig. 1. The two moments of UAV landing: *1* is the hook; *2* is the elastic brake rope; *3* is the first gate; *4* are dampers; *5* is the second gate; *6* is the nonelastic rope; *7* is the UAV



b

Fig. 1. Ending. (See also p. 67)

II. PROBLEM STATEMENT

The purpose of this work is development of calculation method of the UAV parameters landing on the elastic brake rope.

Assume that trajectory of the UAV before landing is in horizontal plane and at the UAV landing on the brake rope; its kinetic energy is parried by elastic ropes only.

Let the elastic rope between the supports of the brake gates A and B have length L before UAV landing and coupling of hook to rope is executed at point O, as it shown in Fig. 2.

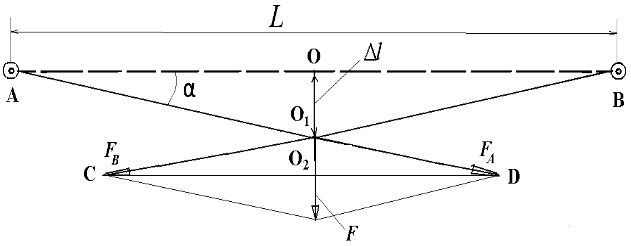


Fig. 2. To calculation of rope stretching during UAV landing: Δl is the transversal lengthening of rope; vector F is the force, which is imposed to point O by the UAV at landing; vector F_A, F_B are the forces, which lengthen out the rope from the left side and the right sides

III. PROBLEM SOLUTION

Assume that when the rope is hooked, an absolutely inelastic impact occurs, i.e. the speeds of the UAV and of the rope at the point of coupling become equal. Then a uniformly decelerated motion of the UAV with the rope on distance Δl during time t occurs. As a result, the UAV speed is reduced to zero. Conditionally the complete solution of the problem can be divided into two parts:

1) determination of the kinematic parameters of the UAV motion while landing on the rope;

2) determination of the dynamic parameters of the UAV motion.

To calculate the kinematic parameters of the UAV motion, we write two equations with two unknowns:

$$\begin{cases} V_{FV}(t) = at, \\ \Delta l = \dfrac{at^2}{2}, \end{cases} \qquad (1)$$

where a is the acceleration of breaking action of UAVs at its coupling with a rope; $V_{FV}(t) = V_{FV}$ is the landing speed of UAV.

For given values of V_{FV} and Δl solution (1) is trivial and does not depend on the second part of the complete solution of the problem.

Suppose that the engagement by the break hook is performing in the middle of the rope. In this case $F_A = F_B$ and each half of the rope will be stretched by a value

$$\Delta l_1 = \frac{\sqrt{\left(L^2 + 4(\Delta l)^2 - L\right)}}{2}. \qquad (2)$$

The magnitude of the stretching of half the rope Δl_1 according to Hooke's law is determined as

$$\Delta l_1 = \frac{F_1}{SE}\left(\frac{L}{2}\right), \quad \Delta l_1 = \frac{F_1}{SE}\left(\frac{L}{2}\right), \qquad (3)$$

where F_1 is a tensile force of half the rope, $F_1 = F_A = F_B$; S is a cross-sectional area of the rope, E is the Young's modulus.

According to Newton's 2nd law, the magnitude of the rope stretching force $F = 2F_A \sin\alpha$, where $\sin\alpha = \dfrac{\Delta l}{\sqrt{L^2 + (\Delta l)^2}}$, is determined by the product of the mass of the UAV on acceleration at braking, i.e.

$$F = m_{FV} a. \qquad (4)$$

Equating the right-hand sides of equations (2) and (3) with account (4), we obtain an equation for the quantity L at $\sin\alpha = \dfrac{\Delta l}{\sqrt{L^2 + (\Delta l)^2}} \simeq \dfrac{\Delta l}{L}$ in the form

$$\sqrt{L^2 + 4(\Delta l)^2} - L = \frac{\eta L^2}{\Delta l}, \qquad (5)$$

where $\eta = \dfrac{m_{FV} a}{4SE}$ is an accessory parameter.

After transformation (5) we can write this equation in the form

$$\eta^2 \xi^4 + 2\eta \xi^3 - 4 = 0, \qquad (6)$$

where variable $\xi = L / \Delta l$.

If we multiply (6) on η^2 and introduce the designation $x = \eta\xi$, then receive

$$x^4 + 2x^3 - 4\eta^2 = 0. \qquad (7)$$

Using the solution of Descartes–Euler with help by substitution $x = y - 0.5$, we reduce (7) to “incomplete” type

$$y^4 - 1.5y^2 + y - \left(\frac{3}{16} + 4\eta^2\right) = 0. \qquad (8)$$

The roots of equation (8) equal one of the following expressions $\pm\sqrt{z_1} \pm \sqrt{z_2} \pm \sqrt{z_3}$, and z_1, z_2, z_3 are roots of cubic

$$z^3 - 0.75z^2 + C_1 z - \frac{1}{64} = 0, \qquad (9)$$

where. $C_1 = \frac{3}{16} + \eta^2$.

After replacement $z = p + 0.25$ and transformation (9) we receive

$$p^3 + \eta^2 p + \eta^2 / 4 = 0. \qquad (10)$$

After Vieta’s replacing $p = \omega - \eta^2 / (3\omega)$ and transformation (10) we receive a biquadratic equation

$$\varepsilon^2 + \frac{\eta^2 \varepsilon}{4} - \frac{\eta^6}{27} = 0, \qquad (11)$$

where $\varepsilon = \omega^3$.

The roots of equation (11) are

$$\varepsilon_{1,2} = -\frac{\eta^2}{8\left(1 \pm \sqrt{1 + 64\eta^2 / 27}\right)}. \qquad (12)$$

But $x = \frac{\eta L}{\Delta l} = y - 0.5 > 0$, then must be inequality $y > 0.5$, i.e. $\sqrt{z} > 0.5$ and therefore $p > 0$. Numerical solution (6) at different meanings η (Fig. 3) indicates that: 1) at considered values of parameters for the UAV and the rope exists only the one decision (6); 2) it is necessary to take the absolute value x when we find a solution of the task from (8) – (12).

During boarding on the rope, the UAV will experience an overload, magnitude of which is equal

$$n_x = \frac{m_{FV} a}{m_{FV} g} = \frac{a}{g}. \qquad (13)$$

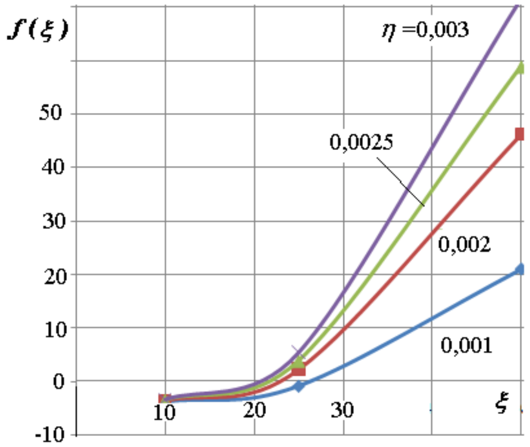


Fig. 3. Graph of function $\eta^2\xi^4 + 2\eta\xi^3 - 4 = 0$ at different meanings η

Assuming that during the UAV brakes, its kinetic energy passes into the potential energy of the stretched rope, we can write

$$\frac{m_{FV} V_{FV}^2}{2} = K_T \frac{(2\Delta l_1)^2}{2}. \qquad (14)$$

where K_T is the coefficient of rigidity of the brake gates rope.

The value of K_T depends both on the properties of the material and on the dimensions of the rope.

Dependence of the value K_T from the rope dimensions and Young's modulus may be express in the form

$$K_T = \frac{ES}{L}.$$

Using (14) instead of (3) does not simplify a solution of the task.

Let consider case of UAV landing on rope, when $AO \neq OB$. For calculation values of forces F_A and F_B we apply the principle of virtual displacements [4], as it shown in Fig. 4, where is an angle of infinitesimal rotation round point A.

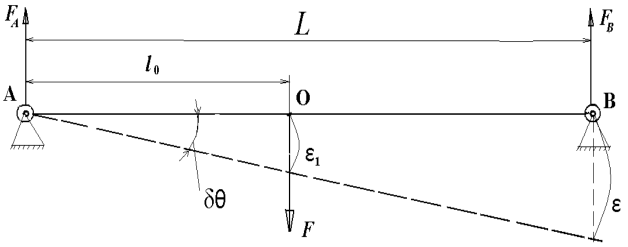


Fig. 4. Case of UAV landing on rope, when $AO \neq OB$

If we take away right-hand support by imposing on the rope the reaction F_B, then the rope acquires an one degree of freedom – an angular displacement round support A. In this case we can write

$$\begin{cases} -F_{\text{B}}\varepsilon + F\varepsilon_1 = 0, \\ \varepsilon = L \cdot \delta\theta, \\ \varepsilon_1 = l_0 \cdot \delta\theta. \end{cases} \qquad (15)$$

We receive from system (15) $F_{\text{B}} = F\dfrac{l_0}{L}$.

Analogical we can receive the value $F_{\text{A}} = F\dfrac{(L - l_0)}{L}$.

IV. CALCULATION OF THE UAV PARAMETERS LANDING ON ROPE

As an example, we will choose the UAV, which has a takeoff weight of 20.00 kg and a flight speed of 30 m/s. We choose the following changes of value Δl: in the range from 0.1 m to 1.5 m.

Solving system (1) and using equation (13) for given data, we will receive numerical values for *t, a* and *n*, which are resulted in Table I. As follows from the calculations, the braking distance $\Delta l = 0.1$ m is insufficient for UAVs of this type since it is possible to destroy it at an overload of more than 458 units.

TABLE I

CALCULATED VALUES OF TIME, ACCELERATION AND OVERLOAD

Transversal lengthening of rope Δl, m / Parameters of the UAV motion	0.1	1	1.5
t, s	0.0067	0.067	0.1
a, m/s^2	4500	450	300
n	458.72	45.87	30.58

Calculations were carried out for ropes with a cross section S = 8 cm^2 = $8 \cdot 10^{-4}$ m^2 (Table II), made of silk threads (E_s = 13.0×10^9 Pa [5]) and of rubber (E_r = 0.9×10^9 Pa). Calculations showed that the required length of the rope made of silk threads is in several times longer than the rope made of rubber at same value Δl.

TABLE II

CALCULATED VALUES OF THE LENGTH OF THE BRAKING ROPE AND ITS EXTENSION DURING LANDING OF THE UAV

Rope material	Rubber		Silk thread	
Δl, m	1	1.5	1	1.5
$\eta \cdot 10^{-4}$	31.25	20.83	2.16	1.44
L, m	4.29	7.37	10.49	18.02
Δl_1, m	0.222	0.293	0.0945	0.124

V. CONCLUSION

In the paper analytical expressions are obtained, which allow making an approximate calculation of parameters of the UAV landing on the rope without additional dampers. The above calculations showed that the required braking ropes, without additional cushioning devices (springs), may be used in practice. But, during landing on a rope the UAV can experience significant longitudinal overloads, therefore it is necessary carefully select the characteristics of rope.

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Tupitsin Nikolay. Candidate of Science (Engineering). Assosiate Professor.

Aviation Computer-Integrated Complexes Department, Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.

Education: Moscow Phisics-thechnical Institute, Moscow, Russia (1975).

Research area: dynamic of flight, experimental methods of aerodynamic, aviation simulators.

Publications: 92.

E-mail: nift@mail.ru

Chychkan Ivan. Candidate of Science (Physics and Mathematics). Associate professor.

Department of Intellectual and Information systems. Taras Shevchenko National University of Kyiv, Kyiv, Ukraine.

Education: Taras Shevchenko State University of Kyiv, Kyiv, Ukraine (1975).

Research area: Information systems; Information security in computer networks.

Publications: 106.
E-mail: I.V.Chychkan@gmail.com

М. Ф. Тупіцин, І. В. Чичкань. Наближені обчислення характеристик процесу посадки БПЛА на трос

Отримано аналітичні вирази, що зв'язують параметри безпілотного літального апарата і характеристики троса, на який він здійснює посадку. Зокрема, визначається співвідношення між необхідною довжиною кабелю і значенням його величини розтягування під час посадки безпілотного літального апарата. При цьому, додаткові амортизатори для троса не розглядаються. Математичну модель процесу посадки безпілотного літального апарата на трос засновано на законі Гука і 2-му законі Ньютона. Одним з основних припущень під час розробки математичної моделі є те, що гальмування безпілотного літального апарата, після його зчіпки з тросом, відбувається з постійним прискоренням.

Ключові слова: посадка безпілотного літального апарата; розтягування каната; закон Гука; перевантаження безпілотного літального апарата.

Тупіцин Микола Федорович. Кандидат технічних наук. Доцент.
Кафедра авіаційних комп'ютерно-інтегрованих комплексів, Навчально-науковий інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.
Освіта: Московський фізико-технічний інститут, Москва, Росія (1975).
Напрям наукових досліджень: динаміка польоту, експериментальні методи аеродинаміки.
Кількість публікацій: 92.
E-mail: nift@mail.ru

Чичкань Іван Васильович. Кандидат фізико-математичних наук. Доцент.
Кафедра інтелектуальних та інформаційних систем факультету інформаційних технологій, Київський національний університет ім. Тараса Шевченка, Київ, Україна.
Освіта: Київський державний університет ім. Т. Г. Шевченка, Київ, Україна (1975).
Напрям наукових досліджень: інформаційні системи; захист інформації в комп'ютерних мережах.
Кількість публікацій: 106.
E-mail: I.V.Chychkan@gmail.com

Н. Ф. Тупицин, И. В. Чичкань. Приближенные вычисления характеристик процесса посадки БПЛА на трос

Получены аналитические выражения, связывающие параметры беспилотного летательного аппарата и характеристики троса, на который он осуществляет посадку. В частности, определяется соотношение между необходимой длиной кабеля и значением его величины растяжения во время посадки беспилотного летательного аппарата. При этом, дополнительные амортизаторы для троса не рассматриваются. Математическая модель процесса посадки беспилотного летательного аппарата основана на законе Гука и 2-м законе Ньютона. Одним из основных допущений при разработке математической модели является то, что торможение беспилотного летательного аппарата, после его сцепки с тросом, происходит с постоянным ускорением.

Ключевые слова: посадка беспилотного летательного аппарата; растяжения каната; закон Гука; перегрузка беспилотного летательного аппарата.

Тупицин Николай Федорович. Кандидат технических наук. Доцент.
Кафедра авиационных компьютерно-интегрированных комплексов, Учебно-научный институт информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.
Образование: Московский физико-технический институт, Москва, Россия (1975).
Направление научной деятельности: динамика полета, экспериментальные методы аэродинамики.
Количество публикаций: 92.
E-mail: nift@mail.ru

Чичкань Иван Васильевич. Кандидат физико-математических наук. Доцент.
Кафедра интеллектуальных и информационных систем факультета информационных технологий, Киевский национальный университет им. Тараса Шевченко, Киев, Украина.
Образование: Киевский государственный университет им. Т. Г. Шевченко, Киев, Украина (1975).
Направление научной деятельности: информационные системы; защита информации в компьютерных сетях.
Количество публикаций: 106.
E-mail: I.V.Chychkan@gmail.com