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<sup>1</sup>A. S. Yurchenko,  
<sup>2</sup>A. P. Kozlov**TO THE QUESTION OF ESTIMATING THE PARAMETERS OF THE DYNAMIC MEMORY ALLOCATION SYSTEM**Educational & Research Institute of Information and Diagnostic Systems, National Aviation University,  
Kyiv, UkraineE-mails: <sup>1</sup>ayurchenko@yahoo.com, <sup>2</sup>ap\_kozlov@ukr.net

**Abstract**—An analytical model of the dynamic memory allocation is given in this article. In addition, analytical estimates of memory fragmentation and temporal system costs are obtained, which take place in the case of segmented memory allocation. The estimates obtained are based on the well-known rule of fifty percent, described by D. Knuth.

**Index Terms**—Dynamic memory allocation; first-fit; best-fit; fragmentation; modeling.

**I. INTRODUCTION**

Virtual memory is a graceful solution of the problem of dynamic memory allocation (DMA). Over the past forty years there has been two main approaches to implement virtual memory. These approaches are segmentation and paging memory allocation, which are reviewed and compared in [1], [2] where reasons of their development are found. Besides these approaches, there is intensively studied “twins” memory allocation algorithm (featuring small system delays).

At present time, in computing systems the emphasis is on the paged memory allocation, although there are a very good systems with a “clear” segmentation. To fill the gap in the literature concerning dynamic allocation of the non-paged memory (DANM), in this study an attempt is made to analyze both existing and new algorithms of DANM, consisting of segment allocation and allocation of memory by “twins” algorithms.

**II. PROBLEM STATEMENT**

This study uses the notion of external, internal, and full fragmentation, which is given in [1], where an overview of existing algorithms DANM is also given. Below, the term fragmentation (if its type is not specified) will denote the full fragmentation.

In the process of solving problems on computers, it's needed to allocate random access memory (RAM) for data, commands, results of intermediate calculations, and so on. We assume that for solving of any problem only one segment is enough, and for different tasks sizes of the corresponding segments will be different. Before solving the problem with the help of some algorithm of providing RAM, there will be allocated such area of memory, into which there can placed a segment of this problem. When the task solution ends, memory, occupied by segment, is

given to the reserve of free memory (i.e. memory, which may be used for other tasks) using an algorithm of releasing of used memory. The process of impact of some area of random access memory into the free memory reserve will be called memory release. All investigated in this study algorithms DANM belong to the allocation of random access memory, so the word “memory” or “allocated memory” will mean random access memory.

In addition, for ease of explanation as a synonym to the word “task” will be used the word “request”.

**III. ANALYTIC MODEL OF DANM**

Let us denote by  $S_{N,M}$  the memory state, in which  $N$  occupied segments and  $M$  free segments ( $S_{0,1}$  is the state of the memory in which all memory is free). Let us denote by  $P_{N,M}$  the probability that at time  $t$  system the of the memory is in the state

$$S_{N,M} (N = 0, 1, 2, \dots, N_{\max}; M = 1, 2, \dots, N_{\max} + 1),$$

where  $N_{\max}$  is the maximum number of segments of information that can be placed in memory.

For the servicing system under consideration, the state graph  $S_{N,M}$  is shown in Fig. 1, which presents possible options for occupying free segments, as well as the release of occupied free segments. Let us fix the moment  $t$  and find the probability  $P_{N,M}(t + \Delta t)$  that at the time  $t + \Delta t$  the system will be in a state  $S_{N,M}$ . Since the system can remain in the same state or only pass to neighboring states, then

$$P_{N,M}(t + \Delta t) = P(A) + P(B) + P(C) + P(D) + P(E),$$

where  $A, B, C, D, E$  are incompatible events. Event  $A$  means that the system during time  $\Delta t$  has not changed its state  $S_{N,M}$ , and events  $B, C, D, E$  are the

transition to  $S_{N,M}$  was occurred respectively, from the states  $S_{N-1,M}$ ,  $S_{N-1,M+1}$ ,  $S_{N+1,M+1}$ ,  $S_{N+1,M}$ .

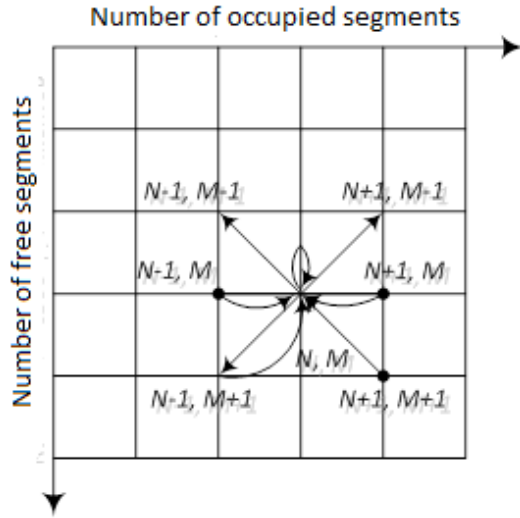


Fig. 1. Status graph

IV. DYNAMIC SEGMENT ALLOCATION OF MEMORY AND RULE OF "FIFTY PERCENT"

For the dynamic segment allocation D. Knuth [3] proved the "fifty percent" rule, which sets the ratio between the number of occupied and free segments. According to this rule, if the memory system tends to the equilibrium state, in which system has an average of  $N$  occupied segments, than the average number of free segments  $M$  is approximately  $p \frac{N}{2}$ , where  $p$  is the probability, that another memory request is provided due to the free segment, the difference between the size of which and required larger than  $\Delta I_{min}$  words ( $\Delta I_{min}$  is introduced into [1] and is equal to the maximum acceptable internal fragmentation on a occupied segment and the minimum size of the free segment).

With the help of this rule for  $p = 1$  were obtained some estimates of fragmentation in dynamic segment allocation. In this study, a number of estimates for the General case of  $p \neq 1$  will be received.

If we denote  $M_0$  is the size of the allocated memory in words, and  $S_0$  is the average size of the used segment (the law of the allocation of occupied segment and the rule of determination of  $S_0$  are supposed to be the same as in [3]), so for the relative losses of the internal  $f_i$ , external  $f_e$ , and complete  $f_1$  fragmentation when  $p \neq 1$  have the following expression:

$$f_i = N(1-p) \frac{\Delta I_{min}}{2M_0}, \tag{1}$$

$$f_e = \frac{NpK_{av} \cdot S_0}{2M_0}, \tag{2}$$

$$f_1 = f_e + f_i, \tag{3}$$

where  $K_{av}$  is the ratio between the average size of the free segment and  $S_0$ , based on the "fifty percent" rule:

$$M_0 = NpS_0 + N(1-p) \left( S_0 + \frac{\Delta I_{min}}{2} \right) + \frac{N}{2} pK_{av} \cdot S_0 = N \left[ S_0 p \left( 1 + \frac{K_{av}}{2} \right) + (1-p) \left( S_0 + \frac{\Delta I_{min}}{2} \right) \right]. \tag{4}$$

Expressions for the relative losses on external and complete fragmentation can be easy obtained from qualitative arguments:

$$f_1 = \frac{M_0 - NS_0}{M_0}, \tag{5}$$

$$f_e = f_1 - f_i = \frac{(M_0 - NS_0) - N(1-p) \frac{\Delta I_{min}}{2}}{M_0}. \tag{6}$$

The other expression is also true for  $f_e$ :

$$f_e = \frac{M_0 - pNS_0 - (1-p)N \left( S_0 + \frac{\Delta I_{min}}{2} \right)}{M_0}, \tag{7}$$

which after simple transformations becomes (6). Substituting  $K_{av}$  from (4) in (2) it is easy to show that expressions (2) and (3) becomes (5), (6), respectively.

IV. COMPRESSION OF MEMORY

Let's consider some aspects of garbage collection in the case of a system for processing lists and obtain estimates of system costs in this case.

The method of garbage collection uses in each word a special field of one bit, which is called a "marking bit" or simply a "marker". In this case, the idea is that almost all algorithms do not return words to the list of free memory and the program works carelessly until the entire list is exhausted, then the "garbage collection" algorithm, using the marking bits, returns all words to free memory, which are currently unavailable to the program, after which the program continues to work.

In addition to the unpleasant loss of one bit in each word, the difficulty of the garbage collection method is that it is extremely slow when the memory load reaches the limit. In such cases, the amount of free words obtained by the collection process does not pay for the effort. Those programs that grab memory and

this happens with many unworked programs, often waste a lot of time, repeatedly and almost fruitlessly causing the garbage collector just before the memory is exhausted.

Thus, the problem of assigning some optimal memory size for each program performed in order to minimize the system costs spent for the execution of this program is important.

Let  $R$  is the size of the memory allocated to the program;  $M_0$  is the size of the entire memory;  $T$  is time required to execute the program (useful time);  $X$  is the size of the memory occupied by the program;  $A(X)$  is the amount of memory that must be retained after compaction.

The compression of memory in the area of size  $R$  with the amount of required data  $A(X)$  is performed in time  $aA+bR$  ( $a, b$  are some constants).

The number of memory compressions, required when executing a program with a allocated memory size  $R$  and total time, spent on compaction of memory, are calculated in [3].

Suppose that when a program is executed, data of size  $D$  is generated using an area for storing data of size  $R$ . Then the approximate values of the number of memory compaction required for program execution and the total time for compaction are equal respectively.

$$N_{\text{comp}} = \int_0^D \frac{dx}{R - A(X)} - 1,$$

$$t_{\text{comp}} = \int_0^D \frac{aX(x) + BR}{R - A(X)} dx - BR.$$

In addition, it was shown in [2] that the total time taken for compaction is a decreasing function of the size  $R$ . These results are used here to obtain an expression for the size  $R$  at which the system costs for program execution are minimal.

Let's consider the objective function of the cost of the program.

$$F = t_{\text{comp}}M_0 + RT,$$

and find the minimum of this function  $F$ . The necessary condition for the minimum of this function is

$$\frac{dF}{dR} = - \left( \int_0^D \frac{A(X)(a+b)}{(R - A(x))^2} dx + b \right) M_0 + T = 0.$$

It is easy to see that the last expression is transformed as follows

$$\int_0^D \frac{(T - M_0)}{D} dx = \int_0^D \frac{A(X)(a+b)}{(R - A(x))^2} dx.$$

From here

$$R = A(X) + \sqrt{\frac{A(X)(a+b)}{T - bM_0}}.$$

It is easy to show that there is also a sufficient condition for the minimum of the function  $F$ , that is condition

$$\frac{d^2F}{dR^2} > 0.$$

Thus, an expression is obtained for the size of  $R$ , at which the minimum of the function  $F$  is reached.

## V. CONCLUSION

A mathematical model of hierarchical memory systems is used to solve the problem of minimizing the average time to access memory hierarchy which is restricted by the total cost of the memory system.

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**Yurchenko Alexander.** Candidate of Engineering. Assistant professor.

Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.

Education: Moscow Physics-technical Institute, Moscow, Russia (1975).

Research area: operating system, dynamic allocation memory.

Publication: 58.

E-mail: ayurchenko@yahoo.com

**Kozlov Anatoliy Pavlovich.** Candidate of Engineering. Associate Professor.

Educational & Research Institute of Information and Diagnostic Systems, National Aviation University, Kyiv, Ukraine.

Education: Kyiv State University named T. G. Shevchenko, Kyiv, Ukraine (1965).

Research interests: Capacitive transducers with non-uniform electromagnetic field. Capacitive meters of parameters small altitude of the flight aircraft. The use of capacitive transducers in automatic control small-altitude of the flight aircraft.

Publications: 50.

E-mail: ap\_kozlov@ukr.net

**О. С. Юрченко, А. П. Козлов. До питання оцінки параметрів системи динамічного розподілу пам'яті**  
Описано математичну модель динамічного розподілу пам'яті. Приведено аналітичне оцінювання фрагментації пам'яті і системних витрат, які мають місце в разі сегментного розподілу пам'яті. Отримане оцінювання засновано на добре відомому правилі п'ятдесяти відсотків, описаному Д. Кнудом.

**Ключові слова:** динамічний розподіл пам'яті; first-fit; best-fit; фрагментація; моделювання.

**Юрченко Олександр Сергійович.** Кандидат технічних наук. Доцент.

Навчально-науковий Інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.

Освіта: Московський фізико-технічний інститут, Москва, Росія (1975).

Напрямок наукової діяльності: операційні системи, динамічний розподіл пам'яті.

Кількість публікацій: 58.

E-mail: ayurchenko@yahoo.com

**Козлов Анатолій Павлович .** Кандидат технічних наук . Доцент.

Навчально-науковий Інститут інформаційно-діагностичних систем, Національний авіаційний університет, Київ, Україна.

Освіта: Київський державний університет імені Т. Г. Шевченка, Київ, Україна ( 1965).

Напрямок наукових інтересів: Ємнісні перетворювачі з неоднорідним електромагнітним полем. Ємнісні прилади вимірювання геометричних параметрів мало висотного польоту повітряного судна. Використання ємнісних перетворювачів в системах автоматичного управління маловисотним польотом повітряного судна.

Публікації: 50.

E-mail: ap\_kozlov@ukr.net

**А. С. Юрченко, А. П. Козлов. К вопросу оценивания параметров системы динамического распределения памяти**

Описана математическая модель динамического распределения памяти. Приведены аналитические оценки фрагментации памяти и системных затрат, которые имеют место в случае сегментного распределения памяти. Полученные оценки основаны на хорошо известном правиле пятидесяти процентов, описанном Д. Кнудом.

**Ключевые слова:** динамическое распределение памяти; first-fit; best-fit; фрагментация; моделирование.

**Юрченко Александр Сергеевич.** Кандидат технических наук. Доцент.

Учебно-научный институт Информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.

Образование: Московский физико-технический институт, Москва, Россия (1975).

Направление научной деятельности: операционные системы, динамическое распределение памяти.

Количество публикаций: 58.

E-mail: ayurchenko@yahoo.com

**Козлов Анатолий Павлович.** Кандидат технических наук. Доцент.

Учебно-научный институт Информационно-диагностических систем, Национальный авиационный университет, Киев, Украина.

Образование: Киевский государственный университет имени Т. Г. Шевченко, Киев, Украина (1965).

Область научных интересов: Емкостные преобразователи с неоднородным электромагнитным полем. Емкостные устройства измерения геометрических параметров мало высотного полета воздушного судна. Использование емкостных преобразователей в системах автоматического управления маловысотным полетом воздушного судна.

Публикации: 50.

E-mail: ap\_kozlov@ukr.net