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OPTIMAL PRINCIPLE FOR DYNAMICAL SYSTEM WITH ALTERATIVE ORBITING

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Abstract*—*Lagrange problem with the account of functional limitation at any functional limitations at any moment in a given interval is presented. The required conditions for optimal trajectories of the determinated dynamic system synthesized as space vehicle trajectories have been obtained.

Index Terms—Optimization; Dynamic system; Branching path; Optimal trajectory.

I. INTRODUCTION

At present the problem is considered when the dynamic system trajectories should satisfy not only the main trajectories but a number of alternative ones [1].

Trajectory alternativity consists in that at any moment of dynamic system movement on this trajectory there are conditions for another variant of movement the objective of which excludes the main objective of this system movement. To such class of trajectories one may relate for example flying vehicle landing trajectory from any point of which go-around fly is possible; trajectories of flying vehicle orbital injection enabling the maneuver of return in the event of malfunction [2], vehicle flight trajectory from any point of given section of which accident-free cargo dropping is possible [3]. The designed possibility of dynamic system trajectory branching at any moment in a given interval gives grounds to refer to such a trajectory as quasi-branched trajectory or trajectory with an alternative (alternative variant of movement).

II. PROBLEM STATEMENT

The proposed in [1] Boltz problem formulation contemplates dynamic system movement main trajectory optimization provided that redirecting and (or) system movement dynamics change requiring the construction of auxiliary trajectory and resulting in main trajectory branching may occur at a finite number of points of the main trajectory. But in a whole number of technical systems the command of change to auxiliary trajectory may come at any current moment in a given interval [2]. This means that it is necessary to solve branched trajectory optimization problem with not fixed time moment but with fixed time interval that is with infinite number of fixed points of main trajectory branching.

The derivation of the required conditions of optimality for main trajectory movement of the dynamic system with regard to infinite number of auxiliary trajectories is set out in this paper. The method used as the basis for mentioned required condition derivation lies in change over to the problem with finite number of probable time moments of trajectory branching into main and auxiliary paths and in formulation of its solution [4] as well as in applying the principle of limiting transformation to this solution [5].

III. PROBLEM SOLUTION

In working out algorithms of trajectory control we have to solve optimization problems in which dynamic object control should provide functional extremum not only at given but also at free time moments of the beginning and the end of the process and also at any current time moment [8].

For the problem

$$I = S(x(t_0),t_0); x(t), t; x(t_f), t_f) + \int_{t_0}^{t} \Phi(x,u,\tau) d\tau \to \min, \quad (1)$$

$$\dot{x} = f(x,u,t), t \in [t_0, t_f], \quad (2)$$

$$\varphi(x(t_0),t_0); x(t_f), t_f) = 0, \quad (3)$$

where
$x \in E^n$, $u \in \Omega \subset E^m$, $\varphi : E^{2n+2} \to E^r \, (r < 2n+2)$, $S(\cdot)-$ is smooth scalar function of variables $x(t_0)$, $x(t)$, $x(t_f)$, $\Phi(\cdot)-$ is a continuous reflection of $E^n \times \Omega \times E^1 \to E^1$ together with matrix $\partial\Phi / \partial x$, the solution may be found by a trivial way: that is on the basis of the principle of optimality [6] it follows that the section of optimal trajectory itself is an optimal trajectory. Hence it is clear that the solution of problem (1) – (3) coincides with the solution of a problem (1) – (1) for $t = t_f$.

The problem becomes considerably more complicated for the case of disintegrated system with current moment of disintegration.

We consider the problem of optimization for disintegrated system

$$I = S(x(t_0), t_0);\ x(t_1), t_1;\ x(t_N), t_N;\ x(t_f), t_f) + \int_{t_0}^{\tau} \Phi(x,u,t)dt + \int_{\tau}^{t_f^\tau} \Phi^0(x^0, u^0, \eta)d\eta \to \min, \quad (4)$$

$$\dot{x} = f(x,u,t), t \in [t_0, \tau], \quad (5)$$

$$\dot{x}^0 = f^0(x^0, u^0, \eta), \eta \in [\tau, t_f^\tau],\ \ x(t) = x^0(t), \quad (6)$$

$$\tau \in [t_1, t_N] \subset [t_0, t_f],\ \varphi^0(x(t_0), t_0) = 0,$$

$$\varphi^f(x(t_f), t_f) = 0,\ \ \varphi^{f^0}(x^0(t_f^\tau), t_f^\tau) = 0,$$

$$x \in E^n,\ x^0 \in E^n, u \in \Omega \in E^m, u^0 \in \Omega_0 \subset E^{m_0},$$

where with respect to $S(\cdot)$, $\varphi^0(\cdot), \varphi^f(\cdot), \varphi^{f^0}(\cdot)$, $\Phi(\cdot)$, $f(\cdot)$, $f^0(\cdot)$, $\partial f / \partial x, \partial f^0 / \partial x^0$ the assumptions are performed that are similar to those assumptions made for corresponding functions of the problem (1)-(3); t_f – time moment at which system (5) reached finite variety $Q_f = \{(x(t_f), t_f) : \varphi^f(x(t_f), t_f) = 0)\}$ provided that the change of the dynamics of its movement in time interval $[t_1, t_N]$ did not take place; t_f^τ is the time moment of finite variety movement $Q_{f^0} = \{(x^0(t_f^\tau), t_f^\tau) : \varphi^{(f^0)}(x^0(t_f^\tau), t_f^\tau) = 0\}$ of system (6). In physical meaning the problem (4) – (6) may be interpreted as a problem of dynamic system optimal trajectory construction with probable malfunctions in time interval $[t_1, t_N]$ resulting in this system movement dynamics change. With this despite probable malfunctions the dynamic system should accomplish its main and auxiliary task [1]–[4]. Assuming that the system dynamics hasn't been subjected to changes at time interval $[t_1, t_N]$ or that it could be changed at any time moment $t \in [t_1, t_N]$ we come to the next auxiliary problem of vector optimization [7]–[10]

$$I^W = col\left[I_0 \to \min, I_1 \to \min, ..., I_N \to \min\right], \quad (7)$$

$$I_0 = S(x(t_0), t_0); x(t_1), t_1; x(t_N), t_N; x(t_f), t_f) + \int_{t_0}^{t_f} \Phi(x,u,\tau)d\tau,$$

$$I_i = S\left(x(t_0), t_0;\ x(t_1), t_1;\ x(t_N), t_N; x(t_f), t_f\right) + \int_{t_0}^{t_i} \Phi(x,u,\tau)d\tau + \int_{t_i}^{t_f^i} \Phi^0(x^0, u^0, \eta)d\eta,\ (i = \overline{1,N}),$$

$$\dot{x} = f(x,u,t),\quad t \in [t_0, t_i], \quad (8)$$

$$\dot{x}^0 = f^0(x^0, u^0, \eta), \eta \in [t_i, t_f^i], \quad (9)$$

$$\varphi^0(x(t_0), t_0) = 0, \qquad \varphi^f(x(t_f), t_f) = 0,$$

$$\varphi^{f^0}(x^0(t_f^i), t_f^i) = 0, \quad (10)$$

$$x \in E^n, x^0 \in E^n, u \in \Omega \in E^m, u^0 \in \Omega_0 \subset E^{m_0},$$

$$x(t_i) = x^0(t_i),\ (i = \overline{1,N}),\qquad t_1 < t_2 < . < ... < t_{N-1} < t_N,$$

where t_f^i is the time moment at which finite variety Q_{f^0} is reached by system (9), that began its movement at time moment t_i, and this problem differs from the problem (4) – (6) by that fact that the condition allowing infinitely large numberof time moments of system dynamics change was substituted by less rigid condition assuming finite number of points $t_i (i = \overline{1,N})$ of system dynamics change. As the requirements (7) – (10) are less rigid than (4) – (6) each feasible process $x(t)$, $u(t)$, $x^0(\eta)$, $u^0(\eta)$, t_0, t_f, t_1, t_N of the problem (4) – (6) will be admissible as well as in the problem (7) – (10) at arbitrary chosen $t_i (i = \overline{2, N\text{-}1})$ and $t_{i-1} < t_i (i = \overline{1,N})$. We shall introduce the notion of steady optimal process and establish its features as in [5], [8].

Process $x(t), u(t), x^0(\eta), u^0(\eta), t_0, t_f, t_1, t_N$ optimal for the problem (4) – (6) is considered as steady optimal if natural N_0 exists and it is so that for countable variety of values $N > N_0$ the admissible process $x(\tau), u(\tau), x^0(\eta), u^0(\eta), T, t_0, t_f$, having vector $T = (t_1, t_2, ..., t_{N-1}, t_N)$ consisting of fixed values $t_1 < t_2 < t_3 < ... < t_{N-1} < t_N$ is also optimal in problem (7) – (10). We recognize that $N > N_0$ and pass to the branched trajectory optimization problem using the principle of quasi-branching of system trajectory (9) at time moments $t_i (i = \overline{1,N})$ changing from trajectory of system (8), moving from original variety $Q_0 = \{(x(t_0), t_0) : \varphi^0(x(t_0), t_0) = 0\}$ to finite variety Q_f. This principle is based on the following considerations.

Suppose that system dynamics (8) will change at time moment $\tau = t_1$. In this case dynamic system trajectory should consist of two sections optimally joint together by condition of stop change [4]. In the interval $[t_0, t_1]$ trajectory is given by equation (8), and in the interval $[t_1, t_f]$ – by equation (9).

Suppose that for reaching time moment t_1 system dynamics change didn't occur. Then system movement trajectory continues to be described by equation (8) until the next hypothetical moment t_2 of its dynamics change and change-over to the system moment trajectory description by equation (9). However trajectory section in time interval $[t_1, t_N]$ should be optimally joint at time moment t_1 with two above mentioned sections. Hence it follows that at time moment t_1 it is necessary to observe the condition of step change for systems moving on branched trajectories [4] but not for disintegrated systems [2], [5].

Viewing in symilar way the condition of trajectory step change at time moments t_2, t_3 and so on until t_N we come to the problem of optimization of branched trajectory with criterion of

$$\dot{I} = \nu^N \left[S(x(t_0), t_0), ..., x(t_f), t_f) + \int_{t_0}^{t_f} \Phi(x, u, t) dt \right] + \sum_{i=1}^{N} \left[\mu_i^N \int_{t_i}^{t_f^i} \Phi^0(x^0, u^0, \eta) d\eta + \xi_i^{NT} \varphi^{(f^0)}(x^0(t_f^i), t_f^i) \right]. \tag{11}$$

According to [4] for process optimum $\hat{x}(t), \hat{u}(t), \hat{x}^0(\eta), \hat{u}^0(\eta), T, \hat{t}_0, \hat{t}_f$ the problems (8) – (11) there are solutions $\lambda^N(t), \lambda^{0N}(\eta)$ of abjoint vector equations

$$\begin{gathered} \nu^N = \left[\lambda^N(t) + \frac{\partial H\left(\hat{x}(t), \hat{u}(t), \lambda^N(t), t\right)}{\partial \hat{x}(t)} \right] = 0, \\ \mu_i^N \left[\lambda^{0N}(\eta) + \frac{\partial H^0(\hat{x}^0(\eta), \hat{u}^0(\eta), \lambda^0(\eta), \eta)}{\partial \hat{x}^0(\eta)} \right] = 0, \end{gathered} \tag{12}$$

such that the conditions following beneath are valid:

(1^0) of transversality

$$\nu^N \left[\left. \frac{\partial S}{\partial x(t_0)} \right|_{\wedge} + \lambda^N(\hat{t}_0) \right] + \left. \frac{\partial \varphi^{(0)\grave{o}}}{\partial \tilde{o}(t_0)} \right|_{\wedge} \xi_0^{\ N} = 0, \tag{13}$$

$$\nu^N \left[\left. \frac{\partial S}{\partial t_0} \right|_{\wedge} - H\left(\hat{x}(\hat{t}_0), \hat{u}(\hat{t}_0), \lambda^N(\hat{t}_0), \hat{t}_0\right) \right] + \left. \frac{\partial \varphi^{(0)\grave{o}}}{\partial t_0} \right|_{\wedge} \xi_0^{\ N} = 0, \tag{14}$$

$$\nu^N \left[\left. \frac{\partial S}{\partial x(t_f)} \right|_{\wedge} - \lambda^N(\hat{t}_f) \right] + \left. \frac{\partial \varphi^{(f)\grave{o}}}{\partial \tilde{o}(t_f)} \right|_{\wedge} \xi_f^{\ N} = 0, \tag{15}$$

$$\mu^N \lambda^{0N}(\hat{t}_f^i) + \left. \frac{\partial \varphi^{(0)\grave{o}}}{\partial \tilde{o}(t_f^i)} \right|_{\wedge} \xi_i^{\ N} = 0, \tag{16}$$

$$\nu^N \left[\left. \frac{\partial S}{\partial t_f} \right|_{\wedge} + H(\cdot, \hat{t}_f)\Big|_{\wedge} \right] + \left. \frac{\partial \varphi^{(f)\grave{o}}}{\partial t_f} \right|_{\wedge} \xi_f^{\ N} = 0, \tag{17}$$

$$\mu_i^N H^0(\cdot, \hat{t}_f) + \left. \frac{\partial \varphi^{(f^0)\grave{o}}}{\partial t_f^i} \right|_{\wedge} \xi_i^{\ N} = 0. \tag{18}$$

(2^0) of step change

$$\nu^N \left[\left. \frac{\partial S}{\partial x(t_1)} \right|_{\wedge} + \lambda^N(t_1 + 0) - \lambda^N(t_1 - 0) \right] + \mu_1^N \lambda^{0N}(t_1 + 0) = 0, \tag{19}$$

$$\nu^N \left[\left. \frac{\partial S}{\partial x(t_N)} \right|_{\wedge} + \lambda^N(t_N + 0) - \lambda^N(t_N - 0) \right] + \mu_N^N \lambda^{0N}(t_N + 0) = 0,$$

$$\nu^N \left[\lambda^N(t_i + 0) - \lambda^N(t_i - 0) \right] + \mu_i^N \lambda^{0N}(t_i + 0) = 0. \tag{20}$$

(3^0) of Hamiltonian minimum

$$H(\hat{x}(t), \hat{u}(t), \lambda^N(t), t) = \min_{u(t) \in \Omega, t \in [\hat{t}_0, \hat{t}_f]} H\left(\hat{x}(t), u(t), \lambda^N(t), t\right), \tag{21}$$

$$H^0\left(\hat{x}^0(\eta), \hat{u}^0(\eta), \lambda^{0N}(\eta), \eta\right) = \min_{u^0(\eta) \in \Omega^0, \eta \in [t_i, t_f^i]} H^0\left(\hat{x}^0(\eta), u^0(\eta), \lambda^{0N}(\eta), \eta\right). \tag{22}$$

(4^0) of nontriviality and nonnegativity:

$$\nu^N + \sum_{i=1}^{N} \mu_i^N = 1; \sum_{j=1}^{r^{(0)}} \xi_{0j}^N + \sum_{j=1}^{r^{(f)}} \xi_{fj}^N + \sum_{i=1}^{N} \sum_{j=1}^{r^{(f^0)}} \xi_{fj}^N = 1, \tag{23}$$

$$\begin{gathered} \xi_{0j}^N \ge 0 \, (j = \overline{1, r^{(0)}}), \ \xi_{fj}^N \ge 0, \ (j = \overline{1, r^{(f)}}), \ \mu_i^N \ge 0, \\ \xi_{ij}^N \ge 0, \ (j = \overline{1, r^{(f^0)}}; \ i = \overline{1, N}). \end{gathered} \tag{24}$$

Here marks “^” means optimum variables and parameters:

$$H(\cdot)=\Phi\big(x(t),u(t)+\lambda^{NT}(t)f(x(t),u(t),t\big),$$

$$H^0(\cdot)=\Phi^0\big(x^0(\eta),u^0(\eta),\eta\big)$$

$$+\lambda^{0NT}(\eta)f^0\big(x^0(\eta),u^0(\eta),\eta\big)$$

Let’s set on numerical axis step functions $\mu^N(\tau)$ и $\xi_j{}^N(\tau)$ $\big(j=\overline{1,r^{(f^0)}}\big)$ with step change correspondingly $\mu_1{}^N,\ldots,\mu_N{}^N$, $\xi_{j1}{}^N,\ldots,\xi_{jN}{}^N$ at points $t_1,\ldots,t_N$. At $\tau<t_1$ assume that $\mu^N(\tau)=\xi_j^N(\tau)=0$. In time interval $[t_1,t_N]$ the adjoint equation (14) together with the condition of step change (20) is written in equivalent integral form including Stieltjes integral,

$$\lambda^N(\tau)=\nu^N\int_\tau^{t^N}\frac{\partial H(\hat{x}(t),\hat{u}(t),\lambda(t),t)}{\partial x(t)dt}$$
$$+\int_\tau^{t^N}\lambda^{0N}(t)d\mu^N(t)+\nu^N\left[\left.\frac{\partial S}{\partial x(t)}\right|_\wedge+\lambda^N(t_N+0)\right], \tag{25}$$

where $\tau\in[t_{1,}t_N]$, $\lambda^N(t_N+0)$–in the result of solution of equation (12) is in the interval of $\left[t_N,t_f\right]$ observance of limiting conditions (18). In this case the condition of step change (19) will take the form

$$\nu^N\left[\left.\frac{\partial S}{\partial x(t_1)}\right|_\wedge\lambda^N(t_1+0)-\lambda^N(t_1-0)\right]=0, \tag{26}$$

where $\lambda^N(t_1-0)$–is obtained as a result of the solution of equation (12) in the interval $[t_0,t_1]$ limiting condition (13).

Due to the condition (4^0) functions $\mu^N(\tau),\xi_j^N(\tau)$ are nonnegative, bounded and have bounded variation. Take arbitrary section J of numerical axis Including $[t_1,t_N]$ together with small neighbor hood and choose $N>N_0$ such, that $t_1,\ldots,t_N$ remained to be the points of continuity of control $u(t)$. From the sequence of functions $\{\mu^N(t):N>N_0\}$, $\{\xi_j^N(t):N>N_0\}$ one can isolate subsequences that by points on J converge to limiting functions $\mu(t),\xi_j(t)$ that is

$$\mu^N(t)\to\mu(t),\quad \xi_j^N(t)\to\xi_j(t),\quad \tau\in J\,. \tag{27}$$

Scalar ν^N and vectors ξ_0^N ξ_f^N, $N>N_0$ are also bounded in a set and therefore the sequences $\{\nu^N:N>N_0\}$, $\{\xi_0^N:N>N_0\}$, $\{\xi_f^N:N>N_0\}$ have convergent subconsequences, that is $\nu^N\to\nu$, $\xi_0^N\to\xi_0$, $\xi_f^N\to\xi_f$. Variations of functions $\mu(t)$ on J are bounded, hence in integral equation of Voltaire type

$$\lambda(\tau)=\nu\left[\left.\frac{\partial S}{\partial x(t_N)}\right|_\wedge+\lambda(t_N+0)\right]$$
$$+\nu\int_\tau^{t_N}\left.\frac{\partial H}{\partial x}\right|_\wedge dt+\int_t^{t_N}\lambda^0(t)d\mu(t), \tag{28}$$

the last summand has a meaning. The solution $\lambda(\tau)$ of this equation exists in a class of functions of bounded variation and therewith it is sole [9].

Due to (27) the solutions $\lambda^N(\tau)$ of equation (25) at each point $\tau\in J$ come to the solution $\lambda(\tau)$ of equation (28). With this function $\mu(\tau)$ as a limit (4^0) of nondecreasing nonnegative function is itself nondecreasing nonnegative function on J and hence it can be considered as a measure.

Passing to the limit of change to $N\big(N\to\infty,\ \max(t_{i-1}-t_i)\to0\big)$ in relationships (12) – (18), (21) – (24), (26), taking into consideration all above we obtain the following result.

Let $\hat{x}(t)$, $\hat{u}(t)$, $\hat{x}^0(\eta)$, $\hat{u}(\eta)$, $\hat{t}_0$, $\hat{t}_f$, $\hat{t}_1$, $\hat{t}_N$–too be a stable optimal process of problem (4) – (6).

In this case there are nonnegative numbers $\xi_{0j}\big(j=\overline{1,r^{(0)}}\big)$, $\xi_{fj}\big(j=\overline{1,r^{(f)}}\big)$, ν and nonnegative measures $\mu(t)$, $\xi_j(t)$ $\big(j=\overline{1,r^{(f^0)}}\big)$ of a bounded function that are concentrated on variety $M=\{t:t\in[t_1,t_N]\}$, there is vector function $\lambda(\tau)$ of bounded variation being the solution of integral equation (28) for $t\in[t_1,t_N]$ and of conventional differential equation

$$\dot{\lambda}(t)=-\left.\frac{\partial H}{\partial x}\right|_\wedge\quad\text{for}\quad t\in\left[t_0,t_f\right]\subset\left[t_1,t_N\right]$$

and there is a vector function $\lambda^0(\eta)$ of bounded variation being the solution of equation

$$\dot{\lambda}^0(\eta)=-\left.\frac{\partial H^0}{\partial x^0}\right|_\wedge,\quad \eta\in\left[\tau,t_f^\tau\right],$$

such that the following conditions are valid:

(1^0) of transversality

$$\nu\left[\left.\frac{\partial S}{\partial x(t_0)}\right|_{\wedge}+\lambda(\hat{t}_0)\right]+\left.\frac{\partial\varphi^{(0)\grave{O}}}{\partial\tilde{o}(t_0)}\right|_{\wedge}\xi_0=0,$$

$$\nu\left[\left.\frac{\partial S}{\partial t_0}\right|_{\wedge}-H(\hat{x}(\hat{t}_0),\hat{u}(\hat{t}_0),\lambda(\hat{t}_0),\hat{t}_0)\right]+\left.\frac{\partial\varphi^{(0)\grave{O}}}{\partial t_0}\right|_{\wedge}\xi_0=0,$$

$$\nu\left[\left.\frac{\partial S}{\partial x(t_f)}\right|_{\wedge}-\lambda(\hat{t}_f)\right]+\left.\frac{\partial\varphi^{(f)\grave{O}}}{\partial\tilde{o}(t_f)}\right|_{\wedge}\xi_f=0,$$

$$\mu(\tau)\lambda^0(\hat{t}_f^{\tau})+\left.\frac{\partial\varphi^{(0)\grave{O}}}{\partial\tilde{o}^0(t_f^{\tau})}\right|_{\wedge}d\xi(\tau)=0,$$

$$\nu\left[\left.\frac{\partial S}{\partial t_f}\right|_{\wedge}+H(\hat{x}(\hat{t}_f),\hat{u}(\hat{t}_f),\lambda(\hat{t}_f),\hat{t}_f,\hat{t}_f)\Big|_{\wedge}\right]$$
$$+\left.\frac{\partial\varphi^{(f)\grave{O}}}{\partial t^0}\right|_{\wedge}\xi_f=0.$$

(2^{o}) of step change

$$\left.\frac{\partial S}{\partial x(t_1)}\right|_{\wedge}+\lambda(t_1+0)-\lambda(t_1-0)=0.$$

(3^0) of Hamiltonian minimum

$$H\left(\hat{x}(t),\hat{u}(t),\lambda(t),t\right)=\min_{u(t)\in\Omega,}H\left(\hat{x}(t),u(t),\lambda(t),t\right),$$

$$H^0\left(\hat{x}^0(\eta),\hat{u}^0(\eta),\lambda^0(\eta),\eta\right)$$
$$=\min_{u^0(\eta)\in\Omega}H^0\left(\hat{x}^0(\eta),u^0(\eta),\lambda^0(\eta),\eta\right).$$

(4^0) of nontriviality

$$\nu+\int_{t_1}^{t_N}d\mu(t)=1,\ \sum_{j=1}^{r^{(0)}}\xi_{0j}+\sum_{j=1}^{r^{(f)}}\xi_{fj}+\int_{t_1}^{t_N}\sum_{j=1}^{r^{(f^0)}}d\xi_j(t)=1.$$

IV. CONCLUSION

The dynamic system's path complying with stated above conditions has the feature – it provides to dynamic system the additional abilities to proceed on new paths during fixed time. These paths allow performing auxiliary (additional/ alternative) task in case of system's dynamics changes due to faults and damages as well in case of on-line retargeting.

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О. І. Лисенко, О. М. Тачиніна. Принцип оптимальності для траєкторії виведення динамічної системи з альтернативою
Розглянуто задачу Лагранжа з урахуванням дії функціонального обмеження в кожен момент часу на заданому інтервалі. Отримано необхідні умови оптимальності траєкторії детермінованої динамічної системи, що інтерпретується як траєкторія виведення космічного літального апарату**.**
Ключові слова: оптимізація; динамічна система; розгалужені траєкторії; оптимальна траєкторія.

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А. И. Лысенко, Е. Н. Тачинина. Принцип оптимальности для траектории выведения динамической системы с альтернативой
Рассмотрена задача Лагранжа с учетом действия функционального ограничения в каждый момент времени на заданном интервале. Получены необходимые условия оптимальности траектории детерминированной динамической системы, интерпретируемой как траектория выведения космического летательного аппарата.
Ключевые слова: оптимизация; динамическая система; ветвящиеся траектории; оптимальная траектория.

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