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FLUID MOTION MATHEMATICAL MODEL DEVELOPMENT IN A PARTIALLY FILLED CAVITY

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***Abstract**—The solution of the problem of oscillations of a fluid in the cavities with a rectilinear generator at different levels of completion. Parameter calculation results obtained pendulum model for longitudinal fluid oscillations.*

Index Terms—Potential velocity of the liquid; oscillation frequency; fluid vibrations; mathematical pendulum; tank.

I. INTRODUCTION

Problems of the dynamics of bodies with cavities containing liquid, to date, are still relevant. These tasks many papers.

Interest in the problems of the dynamics of bodies with cavities containing fluid significantly increased due to the rapid development of rocket and space technology. The stock of liquid fuel available on board the rockets, satellites and space vehicles, in some cases, can have a significant impact on the movement of the aircraft. Similar problems arise in the theory of motion of aircraft and spacecraft, as well as in other technical matters. Thus, the problem of the dynamics of bodies that have a cavity containing a liquid, is of undoubted practical value. These problems are also of fundamental theoretical interest.

Problems of the dynamics of bodies with cavities containing liquid, are among the hardest classical problems of mechanics. Their study was initiated in the last century by Stokes and other scientists. Questions of stability of a rigid body with a cavity containing a liquid, also has long attracted the attention of researchers.

Effect of fluctuations in fuel dynamics of the aircraft is determined, first, the mass ratio of liquid in the partially filled tanks and weight of the apparatus itself and, secondly, the level of traffic control automation machine center of mass. For vehicles missile scheme in which an overload in the unperturbed motion is directed along the longitudinal axis of the apparatus, conducted extensive studies [1], [2] which solved the problem of determining the mathematical models of fluid flow in partially filled tanks.

For vehicles of rocket scheme, these questions are virtually unexplored, and their relevance increases with increasing load ratio of aircraft and automation in the management role. And for firefighters aircraft, where in addition to fluctuations in fuel and there is still hesitation in fully extinguishing liquid-filled tanks, this problem is very important.

II. PROBLEM STATEMENT

When solving practical problems with the dynamics of the object of interest liquid filling both the detection fluid in the cavities, and wherein the resulting forces acting on the aircraft by the vibrating the housing fluid. No less important is the study of the impact of the additional degrees of freedom possessed by the fluid on the stability of the unperturbed motion of the aircraft. Excluding the additional degrees of freedom of the liquid, it does not always goes to the margin of stability, so, for example, stable object according to the calculations without taking into account the fluid mobility, may in fact be unstable. The task is complicated by the fact that the proper damping of oscillations of liquid fuels in large rockets too little, as inversely proportional to the diameter of the tank. Accounting for additional degrees of freedom of liquids is reduced to solving a rather complicated boundary value problems of mathematical physics (Neumann problem).

In order to solve the problem on its own linear oscillations of a fluid in the cavities should be fixed to make such assumptions:

– fluid is ideal and incompressible, and its movement without turbulence;

– vibrations displacement and particle velocity in the fluid are considered small;

– to consider linear motion axis plane *OX*;

– aircraft does not make rotating movements.

Considering that the irrotational movement which depends on the: $\varphi\,(x, y, z, t)$ is the potential unsteady motion velocity of the fluid.

With these assumptions is valid for transient liquid irrotational movement Lagrange integral that determines the pressure at any point of the liquid:

$$p = p_0 - \rho \frac{\partial \varphi}{\partial t} - \rho \frac{v^2}{2};$$

$$v_x = \frac{d\varphi}{dx}; \quad v_y = \frac{d\varphi}{dy}; \quad v_z = \frac{d\varphi}{dz};$$

$$v^2 = v_x^2 + v_y^2 + v_z^2,$$

where ρ is the mass density of the liquid; v is the particles speed; p_0 is the fluid pressure ∞, where $v = 0$; p is the fluid pressure at any point.

It is known that the problem of proper linear oscillations of an incompressible fluid in the cavities formed fixed as follows:

$$\Delta\varphi = 0;$$

$$\frac{\partial\varphi}{\partial n} = 0;$$

$$\frac{\partial^2 y}{\partial t^2} + j\frac{\partial\varphi}{\partial y} = 0,$$

where φ is the potential velocity of the liquid, $\varphi = \Phi(xyz)e^{i\omega t}$; ω is the oscillation frequency.

The velocity potential φ satisfies the Laplace equation

$$\Delta\varphi = 0,$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, Δ is the Laplace operator.

III. PROBLEM SOLUTION

The solution of this boundary value problem is reduced to the calculation of the frequency of the *i*-th tone and fluid vibrations corresponding to the tone of the velocity potential $\Phi_i(xyz)$ speeds.

The assumption of smallness free surface deviations from the undisturbed state. For this potential φ compiled a partial differential equation (Laplace equation). The study of this equation, together with a system of ordinary differential equations describing the motion of the aircraft shell, is very difficult and can be brought to an end only for some of the simplest forms of tank: sphere, cylinder, parallelepiped (Fig. 1).

Obtained in this way are rather complicated algorithms and to implement them, as a rule, it requires a powerful computer software. A simpler solution to the problem, which has an accuracy sufficient for engineering calculations, can be obtained by using mechanical models of fluid fluctuations in the tanks. The principle of construction of these models is based on a certain analogy between the fluid vibrations more mass forces and vibrations of mechanical models. The model parameters selected such that the forces and moments acting on the part of models on the tank when it is small fluctuations, coincided with a similar impact on the liquid side. The most wide circulation was received pendulum and spring-mass models of fluid oscillations in tanks. The structure and parameters of the mechanical model depends on the form of the tank of fluid properties. In the mathematical description of fluid fluctuations by solutions of the Neumann problem for the Laplace equation, the wave process, appearing on its free surface, represented as a superposition infinite set of vibrations tones. To every tone has to conform to its mechanical model, however, due to slight influence compared to the first tone, the higher vibrations tones is usually neglected. This is because the fluid mass participating in the oscillations decreases with increasing tone.
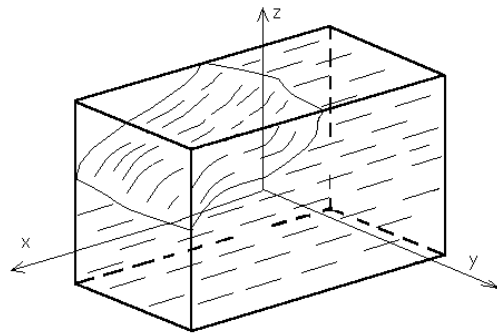

Fig. 1. The tank in the form of parallelepiped

For tank, balanced state is characterized by the free liquid surface perpendicularity to the longitudinal axis of the tank, the mechanical model of the first tone vibrations resulting from the plane motion of the tank are shown in the following figure (Fig. 2).
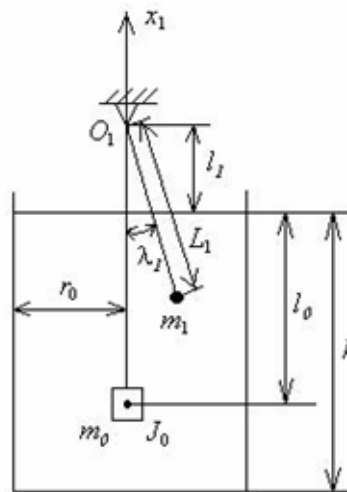

Fig. 2. Pendulum model

The model is presented in the figure is a mathematical pendulum m_1, which owned by the suspen-

sion point of the tank axis and located at a distance l_1 from the free surface of a liquid (m_1 simulates fluid mass that participates in the oscillations 1 tone). The angle λ_1 of deviation of the pendulum simulates the slope of the slope of the wave vibrations in one tone. Mechanical model takes into account the fact that some of the liquid does not oscillate with respect to the tank.

Let setting λ_i, calculated at a filling level h, through $\lambda_i(h)$. To calculate $\lambda_i(h + \Delta h)$ use value $\lambda_i^*(h+\Delta h)$ for of volume $\tau + \tau^*$, where τ^* is the increment of volume, having a parallelepiped Fig. 1 shape with height Δh and square base equal to the area of the free surface of fluid at a rate of filling h.

If the width of the parallelepiped of τ^* is equal to $2R(h)$, and the length $a(h)$, the formula for calculating the value can be represented as [3]:

$$\lambda_i(h + \Delta h) = \frac{\lambda_1(h) + \dfrac{\pi^2}{a^2(h)}\Delta h}{1 + \lambda_1(h)\Delta h}.$$

We can now define $\lambda_i(h + \Delta h)$;

$$l_i(h + \Delta h) = l_1\lambda_1(h)\frac{1 + \dfrac{\Delta R}{R} + 2\dfrac{\Delta a}{a}}{\lambda_1(h) + \dfrac{\Delta R}{R}\lambda_1(h) + \dfrac{x^2}{a^2}\Delta h} \cdot \frac{1}{1 + \dfrac{\pi^2}{4}\dfrac{\Delta a}{a}}\left[1 + \frac{1}{\lambda_1(h)l_i(h)}\left(-\frac{\Delta R}{R} - \frac{\pi^2}{R^2}h\Delta h - \Delta a\frac{\pi^2}{4}\right)\right].$$

IV. RESULTS

The calculated parameters of the pendulum model of longitudinal oscillations of the liquid at different levels of filling are presented in Figs 3, 4, and 5.

If low depths filling the suspension point is located above the free liquid surface, then filling depth increases sharply and goes down at great depths filling located at a distance below the free liquid surface. If the position of the pendulum suspension point to determine the relative bottom of the tank, it is by increasing the relative depth of filling first descends and then increases practically linearly.
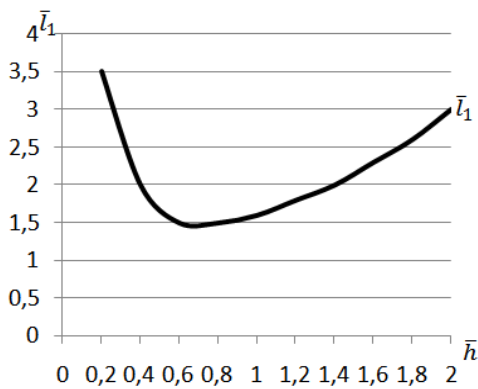
Fig. 3. The dependence on the length of the pendulum on the relative depth of filling

$$\lambda_i(h + \Delta h) = \frac{\lambda_i(h) + \dfrac{\pi^2}{a^2(h)}\Delta h}{1 + \lambda_i(h)\Delta h} \cdot \frac{1}{1 + \dfrac{\Delta R}{R(h)} + \dfrac{\Delta a}{a(h)}},$$

where

$$\Delta R = R(h + \Delta h) - R(h);$$

$$\Delta a = a(h + \Delta h) - a(h).$$

Knowing the parameter $\lambda_i(h)$ can determine the mass of the pendulum moving at an arbitrary fill level h

$$m_1(h) = \frac{\rho}{6}\lambda_1(h)R(h)a^3(h),$$

where ρ is the density of the liquid.

Coordinate l_i point of application of hydrodynamic forces is given by the case of longitudinal oscillations of a is defined by the equation
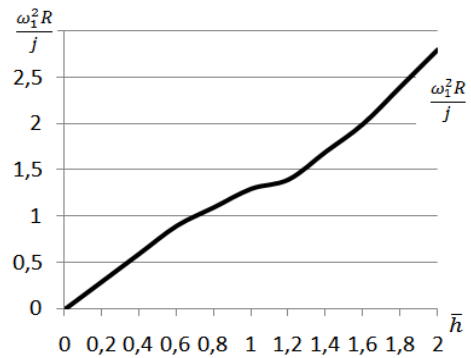
Fig. 4. The dependence of frequency of oscillation of the fluid filling the relative depth
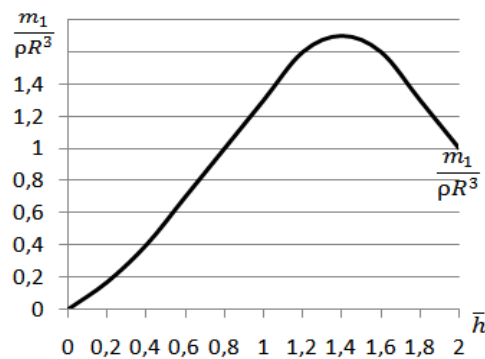
Fig. 5. The dependence of the pendulum mass 1 pitch oscillation on the relative depth of filling

If the mass of the pendulum attributed to the total mass of the fluid filling the tank with an increase in the depth filling the relative mass decreases monotonically.

V. CONCLUSION

Parameters pendulum models are used to study the stability of the longitudinal and lateral movement of the aircraft.

The results of numerical-based parametric studies linearized mathematical model agreed with experimental results only for the case of small lateral oscillations of the liquid in the tanks of rectangular shape within the range of 0.2 – 0.6 relative tank filling. Mathematical model of liquid oscillations are starting to address this problem and create a new mathematical models that take into account the different geometry of the tanks, as well as the properties of fluids and the construction of additional devices within the tank.

Further use of the results pendulum model fluid vibrations in fully filled tanks may be used for optimization problems dampers accommodation [4].

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Ю. М. Кеменяш. Розробка математичної моделі руху рідини в частково-заповненій порожнині
Розглянуто розв'язання задачі про коливання рідини в порожнинах з прямолінійною твірною при різних рівнях заповнення. Отримано результати розрахунків параметрів маятникової моделі для поздовжніх коливань рідини.
Ключові слова: потенціал швидкостей рідини; частота коливань; математичний маятник; ємність.

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Ю. М. Кеменяш. Разработка математической модели движения жидкости в частично-заполненной полости
Рассмотрено решение задачи о колебаниях жидкости в полостях с прямолинейной образующей при различных уровнях заполнения. Получены результаты расчетов параметров маятниковой модели для продольных колебаний жидкости.
Ключевые слова: потенциал скоростей жидкости; частота колебаний; математический маятник; бак.

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