UDC 629.735.05 (045)

[1]V. M. Vasyliev,
[2]D. V. Vasyliev,
[3]K. V. Naumenko

MULTILATERATION AIRCRAFT TRACKING USING STOCHASTIC FILTERING TECHNIQUES

[1]Aviation Radio-electronic Complexes Department, National Aviation University, Kyiv, Ukraine,
[2]Ukrainian State Air Traffic Services Enterprise (UkSATSE), Boryspil, Ukraine,
[3]ANTONOV Company, Kyiv, Ukraine,
E-mails: [1]v_vasylyev@ukr.net, [2]dvasyliev@ukr.net, [3]nau3000@ukr.net

***Abstract*—*The use of stochastic filtering techniques for aircraft tracking based on the data of multilateration surveillance system is considered. The recurrent algorithm of aircraft tracking is synthesized, computer simulation and result analysis are performed.*

Index Terms—Navigation; surveillance; multilateration; aircraft tracking; measurement errors; stochastic filtering techniques.

I. INTRODUCTION

Multilateration (MLAT) is a cooperative Air Traffic Services (ATS) surveillance system that calculates an aircraft's position by measuring the time difference between a signals being sent by airborne transponder and when it is received by a minimum of three ground stations.

Multilateration requires no additional avionics equipment, as it uses replies from Mode A, C and S transponders and offers the possibility of providing a surveillance data at a potentially much lower cost, greater reliability and higher levels of accuracy than conventional Secondary Surveillance Radars (SSR).

The application of MLAT for monitoring of aircraft movements on the airport's surface enables to avoid the radar's disadvantages when the radar's line of sight is blocked by the large airport terminal buildings, hangars and other obstacles by properly location of the ground stations. An important application of MLAT is its ability to provide greater safety while significantly increasing landing capacity by precision runway monitoring.

The same for Terminal Area of airports when lower altitude operations are restricted by the presence of high terrain, which can block aircraft interrogations from nearby secondary radars

Wide Area Multilateration (WAM) can be used as the need for traffic surveillance expands over areas not covered by conventional secondary radar, taking into account the advantage of the cost benefits of multilateration versus new radar installations.

In general, there is an opinion that traditional radar technology will not be able to fully support advanced Air Traffic Management (ATM) concept.

Thus, today, multilateration is a vital element of Advanced Surface Movement Guidance and Control Systems (A-SMGCS), which are currently being installed at many major airports, also can be effectively used for control in Terminal Area and for Wide Area Multilateration application.

The concept for surveillance system using multilateration, the role of MLAT in air traffic management and MLAT applications are described in works [1] – [3]. The results of a study on WAM, analyses of the advantages and disadvantages of WAM, recommendations for further study and analysis are given in [3]. Some works [4] – [5] are dedicated to analyses of accuracy of object localization and to search of optimal location of Remote Units (RU) in order to provide required accuracy of localization in controlled zone.

However, for air traffic control systems it is not sufficient have only surveillance data but also need tracking the aircraft movement in real time to provide a reliable prediction of air traffic environment for the controller decision-making support.

To solve this problem, it is necessary to take into account the uncertainties caused by the measurement errors of time difference that have stochastic nature.

The objective of this work is synthesis the aircraft tracking algorithm using MLAT data for automated air traffic control systems, based on recurrent Stochastic Filtering Techniques and research its accuracy.

II. PROBLEM STATEMENT

Stochastic estimators processing the measured parameters of some process require a mathematical measurement model adapted to the given measurements, and an adequate process model, which is derived for our problem from the physical behavior of an aircraft.

When using the time difference of arrival method the three-dimension (3D) position of the object is

© National Aviation University, 2016

defined as the intersection of at least three hyperboloids and it is need at least four receiver stations. In civilian Air Traffic Control (ATC) the height z is often directly derived from the Mode C SSR transponder reply. In this case, only three sites are required for a 3D solution.

To solve the problem on a plane let denote the rectangular 2D coordinate system (x, y). To determine the location of the object it is necessary to have at least three receiver stations located at coordinates respectively (x_1, y_1), (x_2, y_2) and (x_3, y_3). The relative position of aircraft and ground receiver sites in a rectangular coordinate system is shown in Fig. 1.

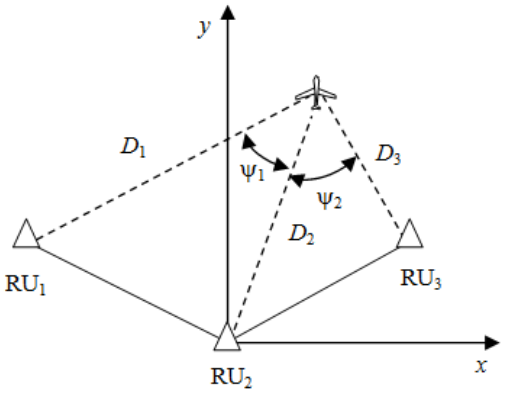

Fig. 1. Relative position of aircraft and ground Remote Units

Consider the site 2 is taken to be at the coordinate system origin, then $x_2 = 0, y_2 = 0$.

The distance from the aircraft to the ith site is determined by the expression

$$D_i(x,y) = \sqrt{(x-x_i)^2 + (y-y_i)^2}\,, \qquad (1)$$

where (x, y) are unknown coordinates of aircraft location; (x_i, y_i) are known coordinates of ith receiver site location.

The pulses from the aircraft with unknown coordinates (x, y) arrives to each of the receiver site at different time t_1, t_2 and t_3 respectively. Then the time difference of arrival between pulses arriving directly at the central site and those coming via the other sites are respectively $\Delta t_{12} = t_1 - t_2$, $\Delta t_{32} = t_3 - t_2$.

Thus the differences of corresponding distances are equal to $\Delta D_{12} = D_1 - D_2 = c \cdot \Delta t_{12}$ and $\Delta D_{32} = D_3 - D_2 = c \cdot \Delta t_{32}$, where c is the pulse propagation rate, often the speed of light.

To determine the aircraft coordinates (x, y) based on the time difference of arrival measurements the system of nonlinear equations

$$\begin{aligned} D_1(x,y) - D_2(x,y) &= c(t_1 - t_2), \\ D_3(x,y) - D_2(x,y) &= c(t_3 - t_2) \end{aligned} \qquad (2)$$

or from (1)

$$\sqrt{(x-x_1)^2 + (y-y_1)^2} - \sqrt{(x-x_2)^2 + (y-y_2)^2} = c(t_1 - t_2),$$

$$\sqrt{(x-x_3)^2 + (y-y_3)^2} - \sqrt{(x-x_2)^2 + (y-y_2)^2} = c(t_3 - t_2)$$

must then be solved.

If limited only to the location problem, one can use known methods for solving systems of nonlinear equations, for example, a least squares method or Newton–Raphson method.

But to solve the assigned task to use the data of multilateration surveillance system for aircraft tracking it is necessary to apply the methods of stochastic optimal measurement estimation. This requires to synthesize mathematical models that satisfy recurrence method of statistical estimation of aircraft trajectory parameters using MLAT.

III. Mathematical Model of Measurements

For synthesis the algorithm of trajectory estimation and aircraft tracking it is required at first to define a functional connection between the estimated and measured parameters.

The moments of signals arrivals t_1, t_2, t_3 are measured in the presence of noise with errors that are random variables v_{t1}, v_{t2}, v_{t3} respectively. Assume that the measurement noise is additive, so a result of the measurement can be written as

$$t_1^* = t_1 + v_{t1}; \quad t_2^* = t_2 + v_{t2}; \quad t_3^* = t_3 + v_{t3}\,. \qquad (3)$$

Also assume that they are normally distributed with zero mean value and variances $\sigma_{t1}^2, \sigma_{t2}^2, \sigma_{t3}^2$ respectively.

Denoting the difference of the measured times (3) of the pulses arrival as

$$\Delta t_{12}^* = t_1^* - t_2^*; \quad \Delta t_{32}^* = t_3^* - t_2^*$$

and the differences between the measurement errors

$$\Delta v_{12} = v_{t1} - v_{t2}; \quad \Delta v_{32} = v_{t3} - v_{t2}$$

we get

$$\Delta t_{12}^* = (t_1 - t_2) + \Delta v_{12}; \quad \Delta t_{32}^* = (t_3 - t_2) + \Delta v_{32}\,. \qquad (4)$$

Using the expression (2) let denote the time difference as

$$\begin{aligned} h_1 &= \frac{1}{c}\big(D_1(x,y) - D_2(x,y)\big); \\ h_2 &= \frac{1}{c}\big(D_3(x,y) - D_2(x,y)\big). \end{aligned} \qquad (5)$$

As a result, after (4) and (5), we obtain the expression connecting the measured parameters with the coordinates of aircraft location

$$\Delta t_{12}^{*} = h_1(x,y) + \Delta v_{12}; \qquad \Delta t_{32}^{*} = h_2(x,y) + \Delta v_{32}. \quad (6)$$

Obviously, the relationship of the measured and estimated parameters is nonlinear.

Using the method of statistical linearization expands the expression (6) in a Taylor series at the point (x_0, y_0) in the vicinity of $\Delta x = x - x_0$, $\Delta y = y - y_0$ and limited to two terms of the expansion get the result

$$\Delta t_{12}^{*} = h_1(x_0,y_0) + \left.\frac{\partial h_1(x,y)}{\partial x}\right|_{\substack{x=x_0\\y=y_0}} \Delta x + \left.\frac{\partial h_1(x,y)}{\partial y}\right|_{\substack{x=x_0\\y=y_0}} \Delta y + \Delta v_{12};$$

$$\Delta t_{32}^{*} = h_2(x_0,y_0) + \left.\frac{\partial h_2(x,y)}{\partial x}\right|_{\substack{x=x_0\\y=y_0}} \Delta x + \left.\frac{\partial h_2(x,y)}{\partial y}\right|_{\substack{x=x_0\\y=y_0}} \Delta y + \Delta v_{32} \quad (7)$$

where

$$\left.\frac{\partial h_1(x,y)}{\partial x}\right|_{\substack{x=x_0\\y=y_0}} = \frac{1}{c}\left(\frac{x_0 - x_1}{D_1(x_0,y_0)} - \frac{x_0 - x_2}{D_2(x_0,y_0)}\right);$$

$$\left.\frac{\partial h_1(x,y)}{\partial y}\right|_{\substack{x=x_0\\y=y_0}} = \frac{1}{c}\left(\frac{y_0 - y_1}{D_1(x_0,y_0)} - \frac{y_0 - y_2}{D_2(x_0,y_0)}\right);$$

$$\left.\frac{\partial h_2(x,y)}{\partial x}\right|_{\substack{x=x_0\\y=y_0}} = \frac{1}{c}\left(\frac{x_0 - x_3}{D_3(x_0,y_0)} - \frac{x_0 - x_2}{D_2(x_0,y_0)}\right);$$

$$\left.\frac{\partial h_2(x,y)}{\partial y}\right|_{\substack{x=x_0\\y=y_0}} = \frac{1}{c}\left(\frac{y_0 - y_3}{D_3(x_0,y_0)} - \frac{y_0 - y_2}{D_2(x_0,y_0)}\right).$$

Let use this result of linearization for synthesis of the algorithm of optimal estimation of flight trajectory parameters based on linear Kalman filter.

At first let define the vector of the estimated (recovered) state. Mandatory elements of the state vector are the coordinates of aircraft location (x, y). Let us assume the following structure of the state vector $\mathbf{X} = [\underbrace{x, y, \ldots}_{n}]^{\mathrm{T}}$.

Now represent the measurement model in a form satisfying the Kalman filter. For this denote the vector of measurements $\mathbf{Z} = [z_1, z_2]^{\mathrm{T}}$, where $z_1 = \Delta t_{12}^{*}$; $z_2 = \Delta t_{32}^{*}$. And denote the vector of measurement errors $\mathbf{v} = [v_1, v_2]^{\mathrm{T}}$, where $v_1 = \Delta v_{12}$, $v_2 = \Delta v_{32}$.

Denote the Jacobian

$$H = \underbrace{\begin{bmatrix} \dfrac{\partial h_1(x_0,y_0)}{\partial x} & \dfrac{\partial h_1(x_0,y_0)}{\partial y} & 0 & \cdots & 0 \\ \dfrac{\partial h_2(x_0,y_0)}{\partial x} & \dfrac{\partial h_2(x_0,y_0)}{\partial y} & 0 & \cdots & 0 \end{bmatrix}}_{n}$$

and finally write down the measurement model in vector-matrix linear form of

$$\mathbf{Z} = \mathbf{H}\Delta\mathbf{X} + \mathbf{v}, \quad (8)$$

where $\Delta\mathbf{X} = \mathbf{X} - \mathbf{X_0}$, $\mathbf{X_0}$ is vector containing the parameters of the trajectory relative to which the linearization was done.

Define statistical characteristics of measurement errors vector, and write them in the form of a covariance matrix of the measurement errors

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}. \quad (9)$$

Note that the error of fixing pulse arrival time for each separate receiver are independent, it means that $M\{v_{ti} v_{tj}\} = 0$ for $i \neq j$, but the differences of the errors Δv_{12} and Δv_{32} are mutually dependent, because these differences includes the error of signal arrival to the second receiver v_{t2}.

For the diagonal elements of matrix **R** (9) define

$$r_{11} = M\left\{\Delta v_{12}^2\right\} = M\left\{(v_{t1}^2 - 2v_{t1}v_{t2} + v_{t2}^2)\right\} = \sigma_{t1}^2 + \sigma_{t2}^2,$$

$$r_{22} = M\left\{\Delta v_{32}^2\right\} = M\left\{(v_{t3}^2 - 2v_{t3}v_{t2} + v_{t2}^2)\right\} = \sigma_{t3}^2 + \sigma_{t2}^2.$$

For the non-diagonal elements

$$r_{12} = r_{21} = M\left\{\Delta v_{12}\Delta v_{32}\right\} = M\left\{(v_{t1}v_{t3} - v_{t1}v_{t2} - v_{t2}v_{t3} + v_{t2}v_{t2})\right\} = \sigma_{t2}^2.$$

IV. Mathematical Model of Aircraft Movement

Mathematically describing the motion of the object we take into account one of MLAT system benefit that the data update in MLAT is carried out much more frequently than in radar surveillance system and can be updated with a frequency of up to 1 second.

It can be accepted that behind such time interval the flight direction does not change significantly, and describe the aircraft movement by mathematical model of uniform rectilinear motion, which in discrete vector-matrix form is written as

$$\mathrm{X}(t_i) = \Phi\mathrm{X}(t_{i-1}), \quad (10)$$

for the state vector $\mathbf{X} = [x, y, V_x, V_y]^{\mathrm{T}}$, where V_x, V_y are the components of velocity at the corresponding coordinates, and transition matrix

$$\Phi = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where T is sampling step that is equal to the interval of data updating.

Thus, the measurement model (8), and a model of the object movement (10) are represented in a linear form that satisfy mathematical statement of the Kalman filter.

V. A Priori Stochastic Characteristics of Aircraft Position Errors

To implement the filter it is necessary to define the initial value of covariance matrix $\mathbf{P}$ of estimation errors in the Kaman filter equations. It is a priori information, which is defined in the following way.

Let use the results obtained previously after linearization. Write down the expression (7) in the form

$$a_1\Delta x + b_1\Delta y = s_1, \quad a_2\Delta x + b_2\Delta y = s_2, \tag{11}$$

where

$$a_1 = c\frac{\partial h_1(x,y)}{\partial x}; \quad b_1 = c\frac{\partial h_1(x,y)}{\partial y};$$

$$a_2 = c\frac{\partial h_2(x,y)}{\partial x}; \quad b_2 = c\frac{\partial h_2(x,y)}{\partial y}$$

$$s_1 = \Delta t_{12}^* - h_1(x_0, y_0) - \Delta v_{12};$$

$$s_2 = \Delta t_{32}^* - h_2(x_0, y_0) - \Delta v_{32}.$$

Solving the system of equations (11) for the unknown Δx, Δy find the variances of errors in determining the deviations from the nominal coordinate

$$\sigma_{\Delta x}^2 = M\{\Delta x^2\} = \frac{c^2}{(a_1b_2 - a_2b_1)^2}\left[\sigma_{t1}^2 b_2^2 + \sigma_{t2}^2(b_1 - b_2)^2 + \sigma_{t3}^2 b_1^2\right];$$

$$\sigma_{\Delta y}^2 = M\{\Delta y^2\} = \frac{c^2}{(a_1b_2 - a_2b_1)^2}\left[\sigma_{t1}^2 a_2^2 + \sigma_{t2}^2(a_1 - a_2)^2 + \sigma_{t3}^2 a_1^2\right]. \tag{12}$$

Derived expressions define the diagonal elements of the covariance matrix $\mathbf{P}$: $p_{11} = \sigma_{\Delta x}^2$; $p_{22} = \sigma_{\Delta y}^2$.

Define the non-diagonal elements of the covariance matrix

$$p_{12} = p_{21} = M\{\Delta x \Delta y\}$$

$$= \frac{c^2}{(a_1b_2 - a_2b_1)^2}\left[(b_2 - b_1)(a_1 - a_2)\sigma_{t2}^2 - a_1b_1\sigma_{t3}^2 - a_2b_2\sigma_{t1}^2\right]. \tag{13}$$

VI. Computer Simulation and Analysis

For optimal estimation of aircraft localization based on the three-position MLAT system standard Kalman filter was used. The filter equations applied to this problem statement and accepted conditions are written in discrete form and executed sequentially as follows:

– prediction for the sampling step

$$\mathbf{X}_{ie} = \Phi\hat{\mathbf{X}}_{i-1}, \tag{14}$$

$$\mathbf{P}_{ie} = \Phi\mathbf{P}_{i-1}\Phi^{\mathrm{T}}; \tag{15}$$

– estimation (correction of prediction)

$$\mathbf{K}_i = \mathbf{P}_{ie}\mathbf{H}^{\mathrm{T}}(\mathbf{H}\mathbf{P}_{ie}\mathbf{H}^{\mathrm{T}} + \mathbf{R}_i)^{-1}, \tag{16}$$

$$\hat{\mathbf{X}}_i = \mathbf{X}_{ie} + \mathbf{K}_i(\mathbf{Z}_i - \mathbf{H}\mathbf{X}_{ie}), \tag{17}$$

$$\mathbf{P}_i = \mathbf{P}_{ie} - \mathbf{K}_i\mathbf{H}\mathbf{P}_{ie}, \tag{18}$$

where $\mathbf{P}$ is the matrix of estimation error variances (covariance matrix); $\mathbf{K}$ is the matrix of correction coefficient; i denotes the current instant; e is the index, meaning extrapolated value.

Computer simulation was carried out according the following scenario. The use of three receiving stations, located as shown in Fig. 2, was considered. The second station was accepted as referenced and as origin placed at the runway end. The coordinates of the stations were taken to be:

$x_1 = -20000$ m; $y_1 = 10000$ m; $x_2 = 0$ m; $y_2 = 0$ m; $x_3 = 20000$ m; $y_3 = 10000$ m.

Two situations were considered (Fig. 2). The first situation – when aircraft 1 is approaching to landing. The second situation – when aircraft 2 is flying across the approach line.

The coordinates of aircraft initial position were taken:

– the first situation (a/c 1) $x_0 = 0$; $y_0 = 60000$ m;

– the second situation (a/c 2) $x_0 = -15000$ m; $y_0 = 20000$ m.

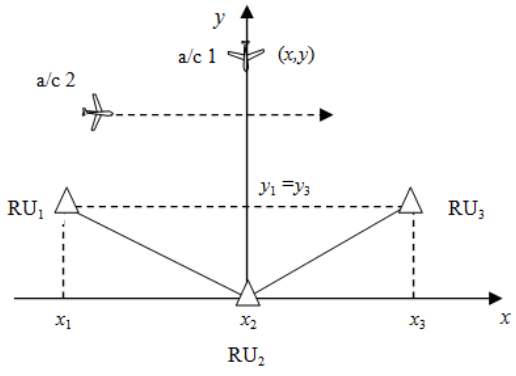

Fig. 2. The scheme of simulated scenario

Data processing for both situation was simulated for 150 s of aircraft flying with speed equal $V = 200$ m/s.

Statistical characteristics of measurement errors of arrival time of the signals were set to:

$$\sigma_{t1}=\sigma_{t2}=\sigma_{t3}= 3.33\cdot 10^{-8}\ \text{s}.$$

The time interval of data updating was set to $T = 3$ s.

For comparative analysis there was calculated the radial root mean square error (RMS) of localization of flying aircraft when three-position range-difference system was using without optimal filtering.

Object location is defined as the point of the intersection of position lines, each of which represents a hyperbola for two pairs of ground stations.

When errors of the position lines determination are independent the value of radial RMS of the location is calculated using the expression [6]

$$\sigma_r = \frac{c\sqrt{\sigma_{\tau 1}^2 \sin^2(\psi_1/2) + \sigma_{\tau 2}^2 \sin^2(\psi_2/2)}}{2\sin\gamma \sin(\psi_1/2)\sin(\psi_2/2)}, \quad (19)$$

where $\sigma_{\tau 1}, \sigma_{\tau 2}$ are the RMS of measurement of τ_1, τ_2 that are the differences of the measured time of the pulses arrival respectively for the first and for the second pair of ground stations; ψ_1, ψ_2 are the angles subtended by the first and second base of the ground stations (see Fig. 1). For accepted scheme of relative position of the stations and the object the angle γ in formula (19) is calculated as $\gamma = (\psi_1 + \psi_2)/2$.

The results of the computer simulation are shown in Figs 3 and 4 in a form of graphics of radial mean square error of aircraft localization while flying.

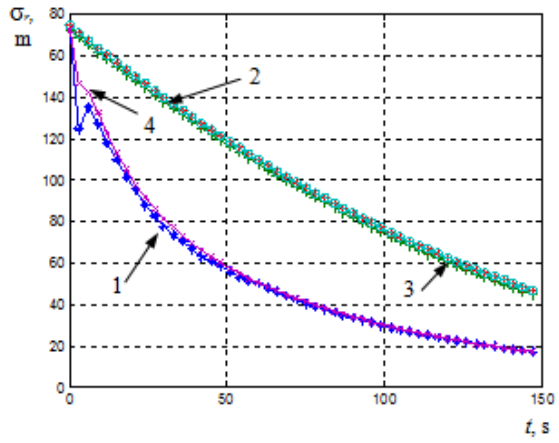

Fig. 3. Comparison of radial RMS of aircraft location – first situation

Curve 1 is the result of statistical processing of aircraft position estimation made by Kalman filtering (14) – (18). Curve 2 is the result of calculating the radial mean square error of localization using formula (19), i.e. error of range-difference method without optimal filtering. Curve 3 is the radial mean square error of localization calculated using results of linearization nonlinear measurement model (12).

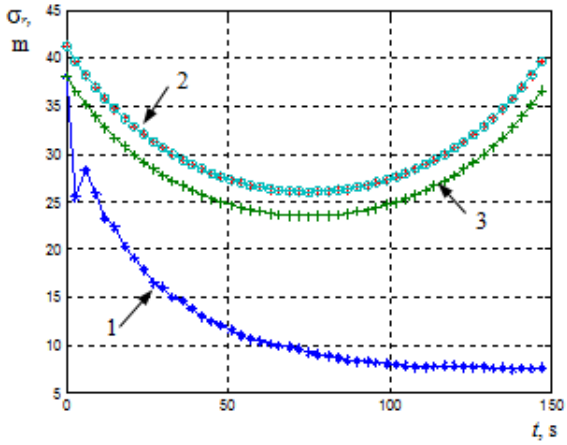

Fig. 4. Comparison of radial RMS of aircraft location – second situation

Comparing obtained results the following conclusions can be done. Applying the proposed algorithm can significantly improve the accuracy of aircraft position estimation and simultaneously carry out aircraft tracking. Practical coincidence of the curves 2 and 3 in Fig. 2 shows the correctness of problem solving and mathematical transformations. The discrepancy between the curves 2 and 3 in Fig. 3, which you can see through scale reduction and reaches only several meters, is likely due to linearization errors because of accepted geometry in the second situation.

It was also interesting to compare the statistical estimation of filtering (curve 1 in Fig. 3) with the value of some elements of covariance matrix **P**, which characterize Kalman filter accuracy. In Fig. 3 curve 4 is the result of calculation of radial RMS by solution equations of Kalman filter for covariance matrix (15), (16), (18) and calculation radial RMS using two diagonal elements from matrix **P** concerning coordinates x and y, so that $\sigma_r = \sqrt{p_{11} + p_{22}}$. The comparison shows practically identical results that confirm the correctness of solution of the task.

VII. Conclusion

To provide aircraft tracking based on measurement data of Multilateration surveillance system the algorithm of optimal stochastic filtering was synthesized.

The algorithm presented in this article is based on the recurrent linearized Kalman filter, and it can be used for aircraft tracking in automated air traffic