

UDC 004.925.3 (045)

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## A VISUALIZATION OF 3D IMAGES BY BACK RAY TRACING: THE GEOMETRIC APPROACH

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**Abstract**—In this paper is considering the problem of visualization synthetic images that is one of the most widely known problem in computer graphic. A general object consists of primitives and a camera used as viewer. Time solving this task is determine by time of scene visualization. For solving, this problem proposed a mathematical approach that based on geometric representation the simplest figures such as plane, circle, polygon and sphere in Cartesian coordinates. The main aim this task consist in estimation of position objects on the scene and its illumination with respect to viewer. Implementation proposals to algorithm program render are proposed. The results of the studying different scenes with some objects are given.

**Index Terms**—Rendering; Cartesian coordinates; spherical coordinate; plane; circle; polygon and sphere.

### I. INTRODUCTION

The main task of visualizing of volumetric figures in computer graphics is determination the some point intersection a surface of the object with the ray from camera. Ray of light source that incoming to the camera is mirrored from the different points of complex object surface. A form of complex object consist of set or network with triangles or other primitives that are touching one to other. As primitives (the simplest) geometrical figures of complex object are using circle, triangle, square, polygons and sphere. In most problems of computer graphics, widely used networks that based on polygons that have the number of parties more than three sides, NURSBs surface, based on B-spline, or parts of their surfaces. Presentation of object as a different surfaces is make the development of software for rendering very difficult, which need used a lot of different type geometrical figure in real object. Therefore rendering problem is transformed to finding ray point intersection with complex object a consisting of primitives.

For space figure by these points we can find face side and back side of the object, as well as to set a background of the synthetic image that allow solving the task illumination of the object for obtaining realistic image. For creating software, we want have universally calculation procedure that we can repeat for any surface and that adequate passed by ray through real object. As universal figure for presentation 3D figures is use a set of triangles. However, presentation the simplest object as a sphere for example by of 1000 or 10000 triangles leads only to increasing the number of calculations and, respectively, it increases of rendering time for this figure.

Therefore when we make rendering by the algorithm of ray propagation, desirable to use approaches that have less quantity operations for calculations.

For calculations, we can use both accurate and approximate methods. The accurate methods assume presentation of object as a figure for which we can look for analytical expression for it description. Calculations that made on accurate expression of surface or flat figure need less time than in case presentation complex object as set triangles or polygons and that really mirrored on general time performance mission. Therefore, this approach can use in general for decreasing the quantity calculations.

In spite of polygonal representation an object is not considered as accurate method it has a more accurate displayed and this approach is can used for obtaining coordinates texture that are located on the surface of the object. Approximate methods has been using for objects which have descriptions in form rational expression or not it explicit. The basic problem that looking in this paper is building an effective rendering means consists in setting general method for performance calculations.

### II. PUBLICATIONS REVIEW

A visualization synthetic scene relate to computer vision tasks, where it main problem is define image sense for decisions making about a future actions. Problems, methods and algorithms of computer graphics has been described in [1], [2]. The scene considered as motionless and running only camera, which by image skeleton and lighting allows us receive presentation about scene. Stereofotometric approach for solving this problem from [1] based on data from the three lighting sources. Although this

approach has used in practice, nevertheless is not seeing reliable, it not given satisfactory results for wide set of scenes.

The approach using a few cameras is proposed in [2]. This approach and other that based on stereovision have problems related with image superposition as a result the body loses rigid boundaries.

For obtaining quality images, the approach based on compositions different filtration methods that proposed in [3] is perspective. This approach can required to apply on finish stage of image processing. Most often in computer graphic is applies approaches that based on the creation of synthetic pictures [4]. In this case 3D-images are creates as a skeleton of polygons that pull on space form. Finish image scene is gets after rendering operation, when is performs calculation of illumination all scene objects. However, analysis real software that conducted in [5], allows us not to made conclusion about final solution this problem. To do this, the camera needs to scan scene, by this data calculated background image, face side of the object and to determine its color. At the core problem is definitions ray point range intersection that is coming to camera and from grid element. Case of elementary polygon presentation such as triangle in barycentric coordinates has been gave in [6]. Despite some significant benefits of this presentation, a significant gain in the calculation is absent.

For forming one approach to task definition of ray position on object in the scene advisable to use Cartesian coordinates and to develop calculating algorithms that allow decreasing computing the volume when rendering the scene.

### III. PROBLEM FORMULATION

A camera ray in direction of the scene we can describe as equation in parametric form:

$$P = O + tR, \tag{1}$$

where  $P$  is the ray point that outputting from origin point  $O$  in direction, setting direction  $R = (r_x, r_y, r_z)$ , in space  $(x, y, z) \in \mathbb{R}^3$ ;  $t$  is the parameter that receiving value in the interval  $t \in [0, T]$ . Primary value of parameter  $t = 0$  corresponds camera position collecting rays of light from the source, terminal value  $t = T$  corresponds position point  $P$  on the object surface.

Camera's ray scans graphic image that consist of primitives. To primitives we relate a plane, a circle, a polygon, a sphere and a cub. A plane in spatial coordinates represented by the expression [7]

$$AX + D = 0, \tag{2}$$

where  $A$  is a normal to plane,  $X$  is a vector Cartesian coordinates,  $D$  is a plane shift relatively origin.

A circle in Cartesian coordinates has been wrote by formula

$$(x - s_x)^2 + (y - s_y)^2 = r^2, \tag{3}$$

where  $s_x, s_y$  is a coordinates of the circle center and  $r$  is its radius. Polygon in Cartesian coordinates has no formula presentation, it we can write as a set point in coordinate space that is connected segments.

A sphere in the Cartesian coordinate system represented by equation

$$(x - s_x)^2 + (y - s_y)^2 + (z - s_z)^2 = r^2. \tag{4}$$

In equation (4)  $s_x, s_y, s_z$  is a coordinates the center of the sphere, and  $r$  is a radius.

The task is to determine the point of intersections camera rays of lighting source with the primitives specified by (2) – (4).

### IV. PROBLEM SOLUTION

Let consider the problem of definitions the point of ray intersection with plane, circle, polygon and sphere. For solving used the formulas from previous section. Flat figure have single point with the camera ray. This point calculated by decision linear equation about a parameter  $t$  in equation (1). In cases of 3D figure, we have the two points ray intersection as minimum and the second point is a rear side this object.

#### A) Plane

In problem ray-plane intersection (Fig. 1), the last subject conveniently represented by coordinate point  $Q_0$  that remote from origin coordinates system and by normal  $n$ . We assume that ray intersection the plane  $Q$  in the some point  $P$ , then this point lies in the same plane as point  $Q_0$ . This means that for any vector located in plane  $Q$  is a true relation

$$\overrightarrow{PQ_0} \cdot \vec{n} = 0. \tag{5}$$

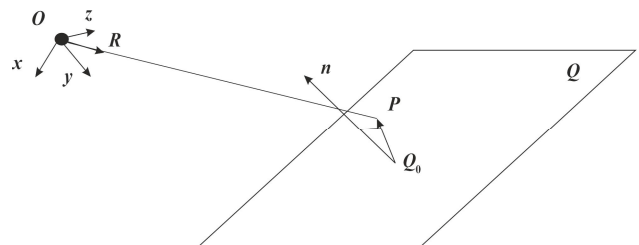


Fig. 1. Intersection ray and plane

In order that to determine point intersection we use description formula (1) for ray. Be noted that in (1) point  $O$  does not match with of ray origin there-

fore this vector not zero. In this case, parameter  $t$  value matching point intersection is determined relations scalar product

$$t = \frac{(O - Q_0) \cdot n}{D \cdot n} \tag{6}$$

By calculation, we must follow that denominator should not zero in (6) what forbids not only operations a divide zero, but also it except a case parallelism ray and plane  $Q$ . In last case, the general point of ray and plane intersection is absent and calculation in accordance to (6) are gave error result as a remote infinite point intersection.

*B) Circle*

Approach proposed in previous task we can also use when rendering circular surface. However, if plane has infinite size, a circle is a flat figure, which have bounded radius  $r_{cir}$ , and it location on a plane determined center coordinates  $S_{cir}$ . Therefore determination task of radius circle  $r_{cir}$  on the some surface consist in definition of ray intersection point with plane where lays disk, and definition belonging it to circle that is done compared distances between point intersection  $P$  and the center of the circle  $S_{cir}$  with radius  $r_{cir}$  (Fig. 2).

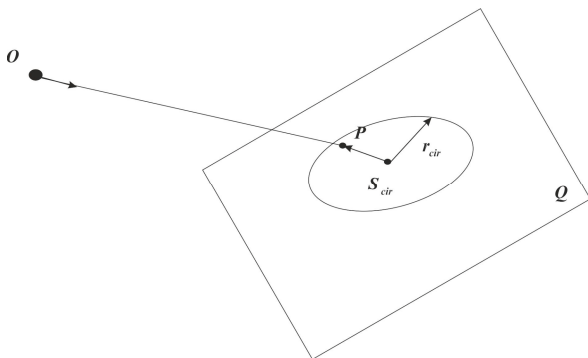


Fig. 2. Intersection ray and circle

Thus, operation of rendering circle by except (7) includes operations

$$|PS_{cir}| \leq r_{cir} \tag{7}$$

which easier to realize (7) if it square

$$|PS_{cir}|^2 \leq r_{cir}^2 \tag{8}$$

If the condition (8) not performs, the ray missed. Results (6), (8) confirm estimates that are gave in [8].

*C) Polygon*

As previous cases, we will considered polygon as a flat figure that located in plane  $Q$ , it bounded line segments and it end fixed corner points, Fig. 3.

Then problem of definition ray and polygon a point intersection perhaps to consider as a solution of

two interrelated problems. The first problem is a definition of ray point intersection with plane, in which lays polygon and the second is a decision problem belonging of point intersection to polygon.

As known [9] for three points  $P, A$  and  $B$ , that are located on one line will be true equality

$$\det \begin{vmatrix} P_x & P_y & P_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = 0 \tag{9}$$

If the point located right or left in relation to this line, the equality (10) not performed. If the point is on right side in relation to line or it belong line segment will be satisfactory inequalities

$$\det \begin{vmatrix} P_x & P_y & P_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \leq 0 \tag{10}$$

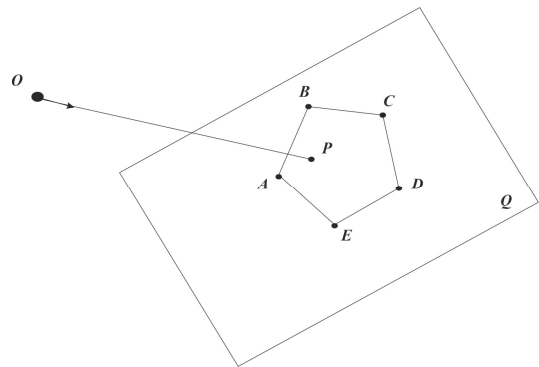


Fig. 3. Intersection ray and polygon

We imagine polygon as a convex figure with corner points that connected by line segments minimum length. If we have direction to bypass clockwise of corner points, we notes if the ray point  $P$  all time located on right side about line segments of polygon, then this means the point ray intersection located intra polygon, in another case the ray passes by. Further calculation for  $t$  we can do as for plane by (6).

*D) Sphere*

Let we have a spherical coordinate system  $(\rho, \theta, \varphi)$  [7], where  $\theta$  is a corner in horizontal plane (azimuth),  $0 \leq \theta < 2\pi$ , and  $\varphi$  is a corner in vertical plane (corner place),  $0 \leq \varphi \leq \pi$  (Fig. 4).

In this coordinate system at  $\rho = 1$  the coordinates of visible point  $P$  will written in form

$$P_x = \sin \varphi \cos \theta, P_y = \cos \varphi, P_z = \sin \varphi \sin \theta \tag{11}$$

Analysis placement the sphere and ray one near another allows us to say about there are three situations:

a) ray passes across sphere and we have two general points, as a result of intersection: close (incoming) and far (out coming);

b) the ray is touching sphere and we have only one general point;

c) the ray and sphere have not general points and ray passes by.

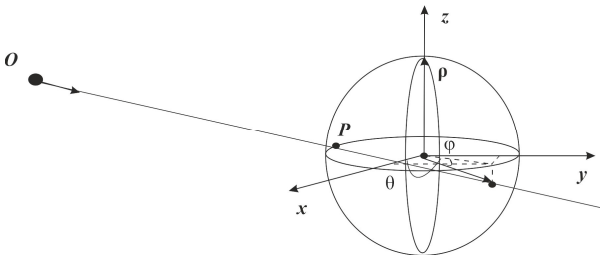


Fig. 4. Intersection ray and sphere

As base for definitions this situations is equations of ray (1) and sphere (4). Using (1) and (4) allows us find general points from square equation so their solutions are expression

$$t = -\frac{O-S}{D} \pm \frac{1}{2D} \sqrt{\Delta}, \quad (12)$$

where  $\Delta$  is a discriminate, which equal is magnitude

$$\Delta = 4r^2 - 3(O-S)^2. \quad (13)$$

These three situations, which describes early, depends from value  $\Delta$ . If value  $\Delta$  is a positive, parameter  $t$  accept two meanings, which corresponds situation, when the ray passes across sphere. If value  $\Delta$  equal zero the ray is touching sphere. Finally, when value  $\Delta$  is a negative the ray passes by sphere.

It should be noted that sign of parameter  $t$  determines the position source lighting with respect to sphere about [8]. If both roots (12) in (4) are positive, the sphere located before the source of light. Vice versa, when both roots are negative the sphere behind the source of light. Situation, when both roots have a different signs the source of light is located inside the sphere. Last case of the solutions is undesirable.

#### V. RENDERING ALGORITHM FOR PRIMITIVE

Calculations performed in accordance to basic and additional functions. Basic computational procedure is building on calculation of ray parameter  $t$  to plane (6), circle (8), polygon (10) and sphere (13) when setting scan algorithm for camera. The point intersection of ray with plane is determined, farther determined intersection ray with object and them it color and position with respect to camera. In another case, when point intersection is not found its value accepted as background. Further the picture of syn-

thetic scene is building by setting render. Fig. 5 shows test results rendering procedure.

Modeling of rendering operation on scene with size 640×480 pixels is show on Fig. 5. Objects of scene are spheres, their centers and radiuses are arbitrary. For modeling is used world coordinate system  $(x, y, z)$ , in which  $x$  and  $y$  is a coordinates horizontal and vertical dimension of scene, and  $z$  is a normal axis to scene. The center of scene have  $(0, 0)$  coordinates.

In this coordinate system the camera is located in center of scene and remote from it on  $z$  coordinate. Light source emitted rays on geometry scene. These rays reflected off of diffuse surfaces and reached camera. Further performed shading that assume is calculating amount of light rays in scene direction.

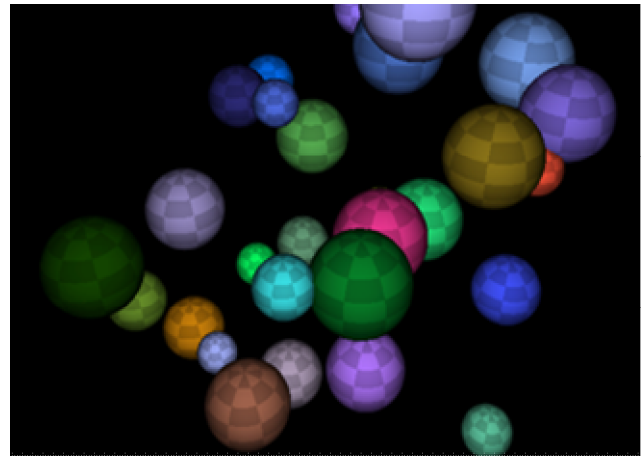


Fig. 5. Ray tracing set of spheres

Our result shows that the solving the task of intersection camera ray and surface on scene is a step to building effective visualization synthetic images.

#### IV. CONCLUSION

In the paper considered one of basic tasks in computer graphics, it is visualization of synthetic object that it represented by network of polygons as three-dimensional figure, where the illumination of scene elements are determinate. Basic result of this work is a rendering algorithm that consist in to scan consequently of scene elements and to definite point intersection with selected primitive that network is presented and color estimation of surface are performing. Assumes what as primitives of element surface will be used plane, circle, polygon and sphere. The less computationally expensive are objects that consist of primitives such as circle and sphere. The represented algorithm is give a foundation for the future research in the optimization of calculations in visualization task of synthetic images.

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Received January 28, 2016

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**Д. П. Кучеров, К. О. Моргун, Т. І. Голенковська. Візуалізація 3D моделей зворотним розповсюдженням променя: геометричний підхід**

Розглянуто проблему візуалізації синтетичних зображень, яка вважається фахівцями в області комп'ютерної графіки однією з найбільш широко відомих проблем. Загальним поданням об'єкта на сцені є набір примітивів, який освітлюється зовнішнім джерелом, потрібно обчислити освітленість по відношенню до глядача. Вирішення цього завдання вимагає значних витрат часу. Запропоновано підхід до вирішення цього завдання, заснований на геометричному поданні найпростіших геометричних фігур, до яких відносять площину, коло, багатокутник і сферу в декартовій системі координат. Мета проведеного дослідження полягає в оцінюванні положення геометричних об'єктів на сцені і визначення освітлення по відношенню до глядача. Запропоновано програмну реалізацію алгоритму рендерінгу. Наведено результати вивчення найпростіших сцен з розглянутими об'єктами.

**Ключові слова:** рендерінг; декартові координати; сферичні координати; площина; коло; багатокутник; сфера.

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**Д. П. Кучеров, К. О. Моргун, Т. И. Голенковская. Визуализация 3D моделей обратным распространением луча: геометрический подход**

Рассмотрена проблема визуализации синтетических изображений, которая считается специалистами в области компьютерной графики одной из самых широко известных проблем. Общим представлением объекта на сцене является набор примитивов, который освещается внешним источником, требуется вычислить освещённость по отношению к зрителю. Решение этой задачи требует значительных временных затрат. Предложен подход к решению этой задачи, основанный на геометрическом представлении простейших геометрических фигур, к которым относятся плоскость, круг, многоугольник и сфера в декартовой системе координат. Цель проводимого исследования заключается в оценивании положения геометрических объектов на сцене и определения освещения по отношению к зрителю. Предложено программную реализацию алгоритма рендеринга. Приведены результаты изучения простейших сцен с рассматриваемыми объектами.

**Ключевые слова:** рендеринг; декартовы координаты; сферические координаты; плоскость; круг; многоугольник; сфера.

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