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OPTIMAL CONTROL IN WATER SUPPLY CONDITION STABILIZATION MODE

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***Abstract*—*In this article describes the method for determining the controller gains in water supply system in the stabilization mode. It is believed that the signal from the pressure sensor is linear. Implementation of the method makes it possible to obtain significant coefficients without performing a lot of computations, which is important for on-board systems with microcontroller core.*

Index Terms—Optimal control; water supply.

I. INTRODUCTION

To ensure optimal control in stabilization mode the state of the water supply system should determine the coefficient $K(t)$. The coefficient $K(t)$ determined from the solution of the stochastic differential equations Riccati type. The solution of this equation system requires a significant amount of calculations in real time, which is associated with a number of deficiencies in the on-board systems. Algebraic Riccati equations appear in many linear optimal and robust control methods such as in Linear Quadratic Regulator (LQR), linear-quadratic-Gaussian (LQG), Kalman filter, H^2 and H^∞. One of the main methods in linear optimal control theory is the LQR in which a state feedback law is designed to minimize a quadratic cost function. In continuous-time domain, the optimal state feedback gain, K, is calculated such that the quadratic cost function. Solving this equation often very difficult or even impossible. Thats why for water supply control systems needs to find a simpler method that provides optimization of cost function.

II. PROBLEM STATEMENT

In traditional LQR theory, it is a standard assumption that the control weighting matrix in the cost functional is strictly positive definite; for example, see Anderson and Moore [5]. In the deterministic case, this is necessary for there to exist a finite optimal cost that is achievable by a unique optimal control. In fact, this assumption means that an energy or penalty cost is associated with the control that tries to drive the system state as close as possible to a desirable position, which is clearly a sensible assumption. Under this assumption, there is a tradeoff between the closeness of the state from the target and the size of the control, and the controller has to carefully balance the two in order to achieve an overall minimum cost. On the other hand, if the control weighting matrix is negative (which means that the control energy is rewarded rather than penalized), then the cost can be made arbitrarily negative by choosing a sufficiently large control input (assuming that there is no restriction on the control size); that is, the larger the control size, the better. Indeed, this is no longer an optimization problem because it does not involve making tradeoffs. The problem is trivial or incorrect. Mathematically, the cost functional becomes concave when the control weighting matrix is negative. Minimizing this cost function over the whole space is meaningless (trivial).The extension of deterministic LQR control to the stochastic case, or the so-called LQG problem, has been a notable and active research area in engineering design and applications (see [4], [3] and the references therein). In the literature on the stochastic LQR problem, however, positive definiteness of the control weight is generally taken for granted. In such a case, there appears to be little difference between the deterministic and the stochastic LQR problems. Indeed, the optimal control for both of these problems is given by a linear state feedback, the feedback gain being identical in both cases and determined by the solution of a backward Riccati equation. The goal of this work to determine the control signal without using Riccati equation.

III. WATER SUPPLY CONTROL SYSTEM DESCRIPTION

Let the water system describes by the matrix differential equation

$$\dot{x}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t), \qquad (1)$$

with $\mathbf{A}$ is the matrix, $n \times n$; $\mathbf{B}$ is the matrix $n \times m$; $\mathbf{x}$ is the vector of the measuring coordinates of water system $n\times1$; is the $\mathbf{u}$ – matrix $m\times1$.

It's need to find a control signal that minimizes the function

$$J(\mathbf{x},\mathbf{u}) = \frac{1}{2}\int_{t_0}^{t_k}[\mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}(t)\mathbf{u}(t)]dt + \frac{1}{2}\mathbf{x}^T(t_k)\mathbf{S}\mathbf{x}(t_k), \qquad (2)$$

with **Q**, **R**, **S** are symetric matricies; **R** is the positively defined matrix; **Q**, **S** is the positively semidefinite matrix.

In [2] was shown that optimal control $u^*(t)$ in stabilization mode with minimal energy consumption determined by the equation of the form

$$\mathbf{u}^*(t) = -\mathbf{R}^{-1}(t)\mathbf{B}^T\mathbf{P}(t)\mathbf{x}(t), \quad (3)$$

where $\mathbf{P}(t)$ must satisfy the solve of matrix differential equation Riccati type. It causes problems when for calculation we use on-board microcontrollers.

Introducing designation in (3)

$$K(t) = -\mathbf{R}^{-1}(t)\mathbf{B}^T\mathbf{P}(t), \quad (4)$$

equation $u^*(t)$ transfer as follows

$$\mathbf{u}^*(t) = K(t)\mathbf{x}(t). \quad (5)$$

The aim of this study is to determine the value of $K(t)$ to defined structure of the input and devoid of these shortcomings.

Let the input of the measuring system signal mixed with additive noise $V(n)$, that is $Z(n) = X(n) + V(n)$, with $V(n)$ is the random variable with normal distribution, correlation function which is

$$P(n) = P_0\delta(n,h).$$

$$\delta(n,h) = \begin{cases} 1, & n = mh, \\ 0, & n \neq mh. \end{cases}$$

Consider the type of signal

$$\varphi(n) = X_0 + X_1h + \ldots + X_mh^m, \quad (6)$$

with h sampling step.

Let us consider the system parameters for linear signal, i.e. $\varphi(n) = X_0 + X_1h$. In this case, the signal can be written as a system of difference equations

$$\begin{aligned} X_0(n) &= X_0(n-1) + X_1(n-1)h, \\ X_1(n) &= X_1(n-1). \end{aligned} \quad (7)$$

with $h = t(n) - t(n-1)$.

Introducing designation

$$X_0 = \begin{pmatrix} X_0(n) \\ X_1(n) \end{pmatrix};$$

$$F(n,n-1) = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix},$$

resulting system of equations (6) considering measurement errors can be written as $X(n) = F(n,n-1)X(n-1) + \Gamma(n,n-1)W(n-1)$ with $n = 1, 2$ is the discrete points of measurement data; $X(n)$ are sensor readings vector; $\mathbf{F}(n, n-1)$ is the state transition matrix; $\mathbf{\Gamma}(n, n-1)$ is the perturbation transition matrix; $W(n, n-1)$ are the sequence of sensor errors.

If the surveillance system we identify so that the input value $X(n)$ satisfies

$$M\left\{\left[\hat{X}(n) - X(n)\right]\left[\hat{X}(n) - X(n)\right]^T\right\} = \min,$$

then optimal signal evaluation as written

$$\begin{aligned} \hat{X}_0(n) &= \hat{X}_0(n-1) + h\hat{X}_1(n-1) \\ &\quad + K_1(n)[Z(n) - (\hat{X}_0(n) + h\hat{X}_1(n-1))], \\ \hat{X}_1(n) &= \hat{X}_1(n-1) + K_2(n) \\ &\quad \cdot[Z(n) - (\hat{X}_0(n) + h\hat{X}_1(n-1))]. \end{aligned} \quad (8)$$

For us it is important conditions under which the error signal between the assessment $\hat{X}(n)$, and measured signal $Z(n)$ will be minimal. The equation for errors $\Delta X(n)$ gets by subtracting from the system (7) the system (8)

$$\begin{aligned} \Delta X_0(n) &= \Delta X_0(n-1) + h\Delta X_1(n-1) + K_1(n) \\ &\quad \cdot[V - (\Delta X_0(n-1) + h\Delta X_1(n-1))]; \\ \Delta X_1(n) &= \Delta X_1(n-1) + K_2(n)[V - (\Delta X_0(n-1) \\ &\quad + h\Delta X_1(n-1))]. \end{aligned} \quad (9)$$

Powering the first equation of (9) in the square and using transaction expectation, we get

$$\begin{aligned} R_{11}(n) &= R(n-1) + K_1^2(n)[R_v + R(n-1)] \\ &\quad - 2K_1(n)R(n-1), \end{aligned} \quad (10)$$

with

$$R_{11}(n) = M\{\Delta x_0(n)\Delta x_0(n)\};$$
$$R_{11}(n-1) = M\{\Delta x_0(n-1)\Delta x_0(n-1)\};$$
$$R_{12}(n-1) = M\{\Delta x_0(n-1)\Delta x_1(n-1)\};$$
$$R_{22}(n-1) = M\{\Delta x_1(n-1)\Delta x_1(n-1)\};$$
$$R(n-1) = R_{11}(n-1) + 2hR_{12}(n-1) + h^2R_{22}(n-1).$$

It is determines the value K_1, where the error variance $R_{11}(n)$ will be minimal at this step. Differentiating (10) by K_1 and equating to zero is obtains product

$$-2R(n-1) + 2K_1(R_v + R(n-1)) = 0,$$

obtain the required value $K_1(n)$

$$K_1(n) = \frac{R(n-1)}{\left[R_v + R(n-1)\right]}.$$

Similarly, $K_2(n)$ defined as

$$K_2(n) = \frac{[R_{12}(n-1) + hR_{22}(n-1)]}{[R_v + R(n-1)]}.$$

We can significantly reduce the number of possible calculations, if we assume that the signal from the output of the pressure sensor is linear, that is described by the equation

$$x(t) = a_1 + a_2 t.$$

In this case, the optimal filter described by the relation [1]

$$\hat{a}_1 = \hat{a}_2 + K_1(t)[z(t) - \hat{a}_1], \quad \hat{a}_1(0) = 0,$$

$$\hat{a}_2 = K_2(t)[z(t) - \hat{a}_1], \quad \hat{a}_2(0) = 0.$$

And parameters $K_1(t)$, $K_2(t)$ defined as

$$K_1(t) = \frac{t^2}{\frac{N}{P} + \frac{t^3}{3}};$$

$$K_2(t) = \frac{t}{\frac{N}{P} + \frac{t^3}{3}}.$$

IV. Conclusions

Considered by the example of the theory of optimal filtering for derivation coefficient. Important is that the computational complexity of the algorithm is much less than the solving Riccati equation. It should be noted that for this example the input signal is linear with adition white noise. It is possible to reduce the number of calculations to determine the coefficients gain $K(t)$ in water supply system perhaps, if we assume that the signal from the pressure sensor is linear. Then the data rates possible to obtain analytically without having to solve the equation of Riccati type that has significant advantages for implementation in on-board systems.

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В. В. Сидоренко, К. О. Буравченко. Оптимальне керування в режимі стабілізації стану водопостачання

Розглянуто метод визначення коефіцієнтів підсилення регулятора системи водопостачання у режимі стабілізації стану, якщо вважати сигнал з датчика тиску лінійним. Реалізація методу дозволяє отримати коефіцієнти без виконання значних обчислень, що важливо для бортових систем з мікропроцесорними пристроями.

Ключові слова: оптимальне керування; системи водопостачання.

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В. В. Сидоренко, К. О. Буравченко. Оптимальное управление в режиме стабилизации состояния водоснабжения

Рассмотрен метод определения коэффициентов усиления регулятора системы водоснабжения в режиме стабилизации состояния, если считать сигнал с датчика давления является линейным. Реализация метода позволяет получить коэффициенты без выполнения значительных вычислений, что важно для бортовых систем с микропропоцесорнимы устройствами.

Ключевые слова: оптимальное управление; системы водоснабжения.

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