THEORY AND METHODS OF SIGNAL PROCESSING

UDC 621.396.969.1 (045)

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WIDEBAND COPULA AMBIGUITY FUNCTION IN RADAR AND NAVIGATION SYSTEMS

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Abstract—*This paper discusses the notion of the wideband radar ambiguity function, which was introduced earlier, and particularly its nonparametric variant. Consideration is focused on application to radar and navigation systems. The new nonparametric variant of the ambiguity function, which is based on the notion of copula, is suggested. This version of the ambiguity function can be effectively used for the signal synthesis and detection in different electronic systems in case of signals with an unknown spectrum and probability density function that is normally typical for radar and sometimes for navigation systems.*

Index Terms—Ambiguity function; copula; non-parametric algorithm; wideband signal; radar; navigation..

I. INTRODUCTION

The first wireless communications experiments were based on using sparks as a source of high frequency oscillations, which are actually random signals and have wide spectral band. However that was just because of absence of alternative generators. Later majority of radio engineers tried to use narrowband signals with harmonic carrier; classical theory of information transmission and optimal signal detection was established in details especially for this case. Nevertheless, advantages of random signals are well known nowadays, and random signals are also used in radio engineering already many years. Moreover, during last years the use of wideband signals, including noise-type and random signals is becoming very popular, they are used in communications, radio navigation, radar and sonar systems.

The use of wideband signals in navigation and radar allows us to increase accuracy of spatial coordinate measurements together with a space resolution [3]. For a wideband signal generation many principles and methods are used, among them the random signal generation, when the signal can be represented as a random stochastic process [4]. In order to estimate potential of a signal, or sounding waveform, designed for measurement of range (time delay) and velocity (Doppler shift) as well as for resolving signals in time and frequency domains, the ambiguity function is used.

The notion of the radar ambiguity function was suggested by Woodward [1] for narrow band signals and then was extended for wideband and random signals [2].

In this paper we are using the traditional definition of the radar wideband ambiguity function, in which we are defining the ambiguity function as a mathematical expectation of the product of two random processes, the first–in the natural time scale and the second– with a time scale coefficient.

The cross-ambiguity function for two random processes $X(t)$ and $Y(t)$ can be defined as an average

$$\chi(\tau,\alpha)=\sqrt{|\alpha|}E\left\{X(t)Y^{*}(\alpha(t-\tau))\right\},$$

where $\alpha=\dfrac{c-v}{c+v}$ is a scale coefficient, c is a velocity of the wave, v is a velocity of the target.

For the stationary ergodical process we can consider, that the cross-ambiguity function can be calculated as an average in a time space

$$\chi(\tau,\alpha)=\lim_{T\to\infty}\frac{\sqrt{|\alpha|}}{T}\int_{-T/2}^{T/2}x(t)y^{*}(\alpha(t-\tau))dt.$$

With the help of the ambiguity function we can simultaneously measure not only the space coordinate but also the moving target velocity. For this purpose the signals with a drawing pinform ambiguity function are preferable. The best variant is the form of the delta function. But such signal – a white noise is physically not realizable. But a fair good approximation can be found among different random and pseudorandom signals as is shown in Fig. 1.

This approach can be used for determining the coordinates in the hyperbolic navigation systems with pseudo random signals for moving aircrafts. For these purposes the ambiguity function should be projected to the area where lines with the same signal difference of arrival are mapped (Fig. 2).

The intention of our work is to design using nonparametric principle of invariance the mathematical instrument which gives us a possibility for synthesis of proper radio signals and their signal processing in order to obtain smaller ambiguity of the measured coordinates.For this purpose we are suggesting to use a new statistical notion – a copula, which gives us the possibility to design a measure of the signal space and velocity ambiguity free from distribution function.

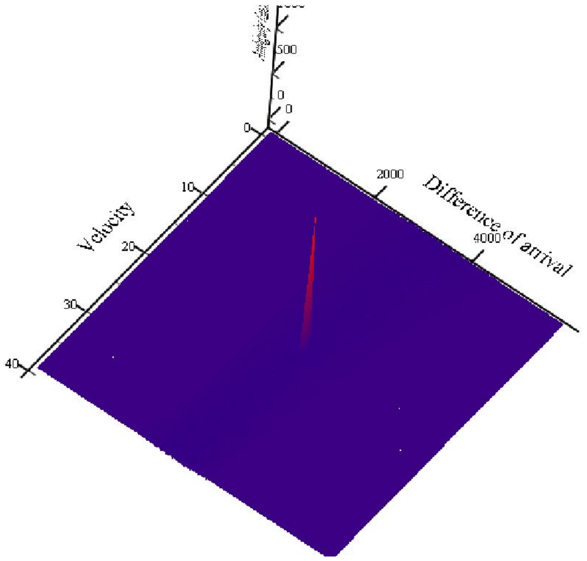

Fig. 1. The ambiguity function for noise signal. Result of simulation

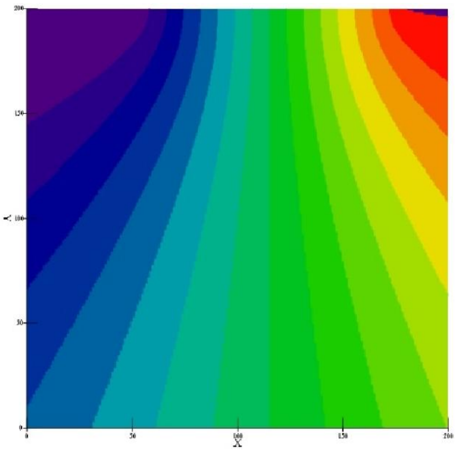

Fig. 2. The map of the region with equal difference of arival distances

II. NONPARAMETRIC TRANSFORM

A. Copula transform

We can transform the vector of random variables (x,y) to a new a random variable (x_T,y_T), using two marginal cumulative distribution functions $x_T=F_x(x)$, $y_T=F_y(y)$ as functional transforms. It is easy to prove that vector (x_T,y_T) has uniform distribution if random variables x and y are independent. The bivariate cumulative distribution function of the transformed variables (x_T,y_T) is called a copula of these variables and according to the Sklar's theorem is equal

$$F(x,y)=C(F_x(x),F_y(y)).$$

where $F(x,y)$ is a bivariate cumulative distribution function of (x,y).The density function, corresponding to the copula $C(x_T,y_T)$ is

$$c(x_T,y_T)=\frac{\partial C(x_T,y_T)}{\partial x_T \partial x_T}.$$

If a useful signal is absent the copula density function has a uniform distribution on $[0, 1]^2$.If a useful signal is present a copula density function has some other distribution on $[0, 1]^2$.

A copula density function can be estimated using kernel estimates.

The cumulative distribution functions $F_x(x)$ and $F_y(y)$ can be replaced by their estimates $\hat{F}_x(x)$ and $\hat{F}_y(y)$. It is assumed, that if the size of a sample is increased, the estimate converges to a cumulative distribution function. Transformations of the sounding and reflected signals

$$x_{Ti}=\hat{F}_x(x_i)\,,\ y_{Ti}=\hat{F}_y(y_i),$$

will be used later.

B. Copula Estimates

The kernel estimates of the cumulative distribution functions will be used as the nonparametric transform.

Thus the estimate of a bivariate copula density function $c(x_T,y_T)$ will look like the sum of the kernels $K_i(x_T,y_T)$

$$\hat{c}(x_T,y_T)=\sum_{i=1}^{N}K_i(x_T,y_T).$$

where N is the sample size, which is the basis for finding an estimate. We suppose, that the kernels can be calculated, using the formula

$$K_i(x_T,y_T)=\frac{1}{N}w(x_T-x_{Ti},y_T-y_{Ti}),$$

where $w(x_T,y_T)$ is a function, which equal to some probability density, for example, normal, (x_{Ti}, y_{Ti}) is the sample unit i, which is the basis for a kernel estimate.

For the estimate $\hat{F}_x(x)$ of one-dimensional cumulative distribution function the kernels look as follows

$$Q_i(x) = \frac{1}{N}\int_{-\infty}^{x}\int_{-\infty}^{\infty} w(u - x_i, v - y_i)\,dud\tilde{v}$$

The estimate is calculated using the following expression

$$\widehat{F}(x_T, y_T) = \sum_{i=1}^{N} Q_i(x_T, y_T).$$

III. COPULA AMBIGUITY FUNCTION

Using the copula density function we can define its copula ambiguity function [5] as a second mixed central moment of the copula density

$$\chi(\tau,\alpha) = \sqrt{|\alpha|}E\left\{(F_x(X(t)) - m_x)(F_y(Y^*(\alpha(t-\tau))) - m_y)\right\}$$

or for the ergodic process

$$\chi(\tau,\alpha) = \lim_{T\to\infty}\frac{\sqrt{|\alpha|}}{T}\int_{0}^{T}(F_x(x(t)) - m_u)(F_y^*(y(\alpha(t-\tau))) - m_v)dt.$$

Using the kernel estimates of the cumulative density function [6] we can obtain the copula ambiguity function kernel estimate in some finite time interval

$$\chi(\tau,\alpha) = \sqrt{|\alpha|}\int_{t_1}^{t_2}(\hat{F}_x(x(t)) - m_u)(\hat{F}_y^*(y(\alpha(t-\tau))) - m_v)dt.$$

The authors also are suggesting in another variant of the copula ambiguity function. In this formula we are using an estimate of the moment of the second order for the uniform distribution. For obtaining the statistics, which depends from two parameters, we will use an additional functional transform, transforming the copula statistic to a normal distribution [8]

$$\chi(\tau,\alpha) = \sqrt{|\alpha|}\int_{t_1}^{t_2}F_N^{-1}(\hat{F}_x(x(t)))F_N^{-1}(\hat{F}_y^*(y(\alpha(t-\tau))))dt.$$

where F_N^{-1} is an inverse cumulative function of a normal distribution.

IV. EXPERIMENT

With the help of the noise acoustic radar, designed and constructed by authors (Fig. 3), the copula ambiguity function was measured for real signals.

The experimental radar consists ofa computer, which is used for the signal generation and processing, two acoustic aerials with a sounder and a microphone and a power amplifier. The sounding signal is obtained from the random numbers generating program (Figs 4 and 7) and has the normal distribution function (Fig. 6). The spectrum of the sounding signal is presented in Figs 5 and 8.

Fig. 3. The noise acoustic radar

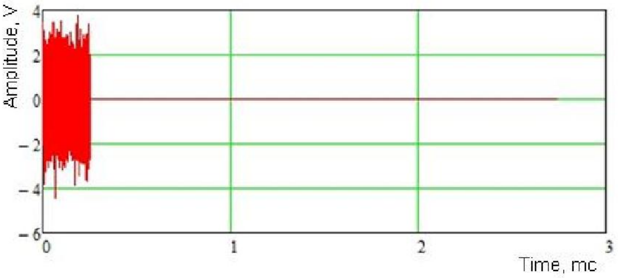

Fig. 4. The sounding signal, generated by the computer. Signalamplitude is in *V*, time in ms

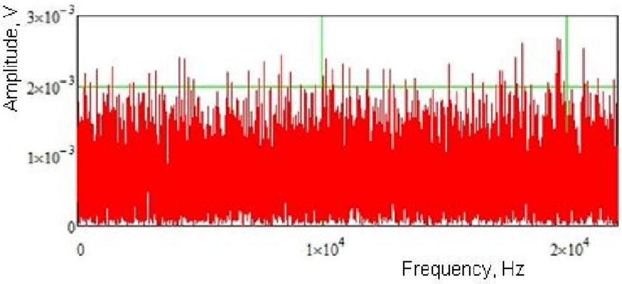

Fig. 5. Spectrum of the sounding signal. Amplitude in *V*, frequency in Hz

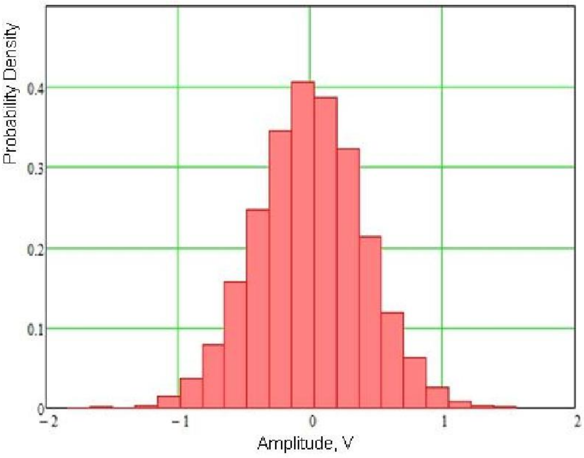

Fig. 6. The noise acoustic radar sounding signal histogram. Dependence of the probability density on the signal amplitude in *V*

The acoustic radar sounding signal is a wideband random signal with a normal distribution [9]. The signal reflected from the solid object at the distance equal to 70 m from the radar. For this signals the copula ambiguity functions were calculated. The results are presented in Figs 9 and 10.

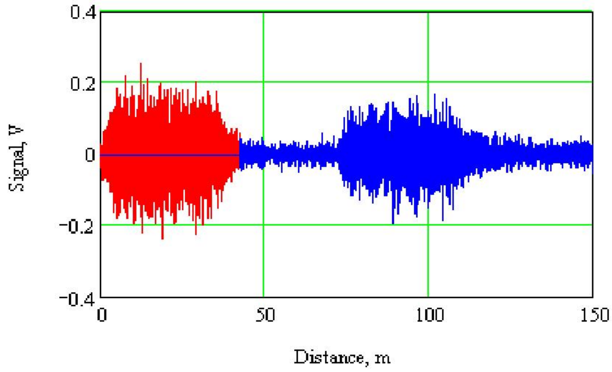

Fig. 7. The noiuse acoustic radar sounding and reflected signals

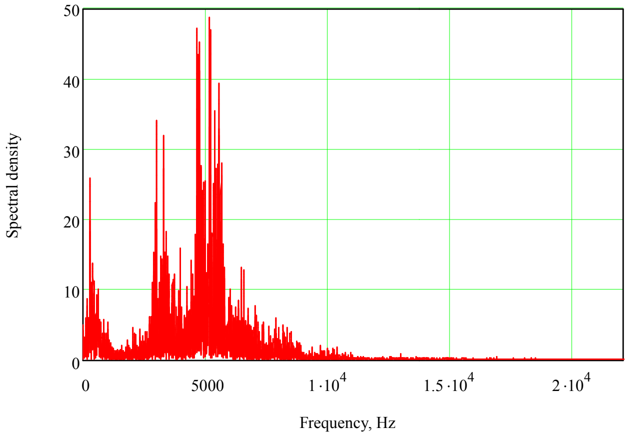

Fig. 8. Spectrum of the sounding signal, recorded by a microphone

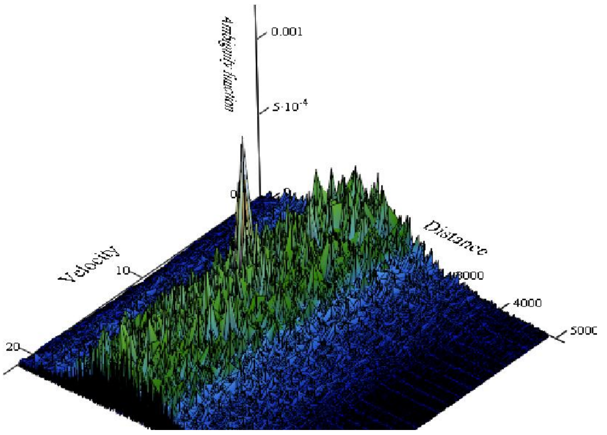

Fig. 9. Estimate of the copula cross-ambiguity function for the acoustic radar. Range in distance samples and velocity in ADC digits. One digit for velocity is 1 m/s, 11 corresponds to zero velocity, one digit for distance is equal to 0.0038820862 m

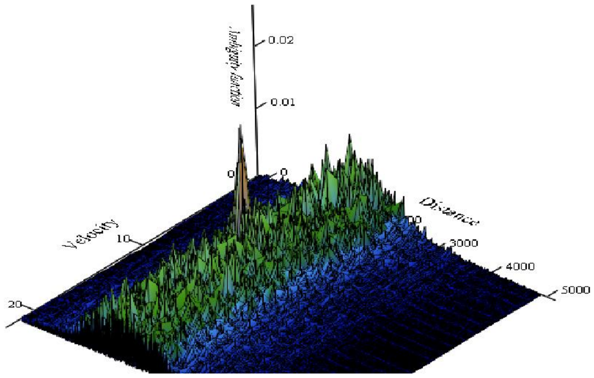

Fig. 10. Estimate of the copula cross-ambiguity function (with an aditional functional transform) for the acoustic radar

The cross section of the ambiguity function in time area (or in distance area) for zero velocity of the target propagation is the correlation function. The result of the calculations is presented in Fig. 11. The same calculations were done for the cross section of the copula ambiguity function. The result is presented in Fig. 12.

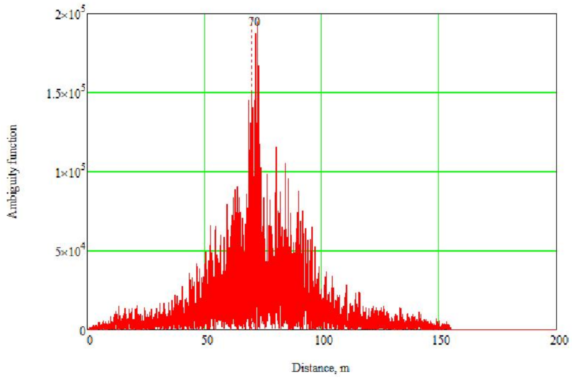

Fig. 11. Estimate of the cross-ambiguity function cross section (cross-correlation function) for the acoustic noise radar

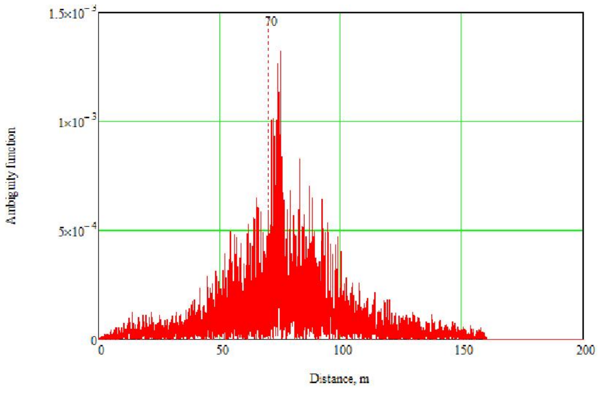

Fig. 12. Estimate of the copula cross-ambiguity function cross section (cross-correlation function) for the acoustic noise radar

The shape of the suggested variant of the ambiguity function does not depend on the probability density functions of the sounding and reflected signals. That is why, signal detection algorithms, which are based on this notion, are distribution free and have a constant level of the false alarm probability. The detection can be done with the help of the simple thresholding of the copula ambiguity function.

We have done the computer and physical simulation of coordinate measurements for the random signal source with the help of the copula ambiguity function. The random signal with a normal probability density function is received with the help of two antennas and we measure the difference of the signal arrival. For this purpose the results of ambiguity function calculation are projected to the map of hyperbolic lines (Fig. 13). The crossing of the curves, which are formed by the maximums of calculated ambiguity functions, gives us the point, where the source of the wave is situated and allows us to measure its coordinates. As we can see from the results of simulation, the usage of the wideband random signal gives us wonderful results.

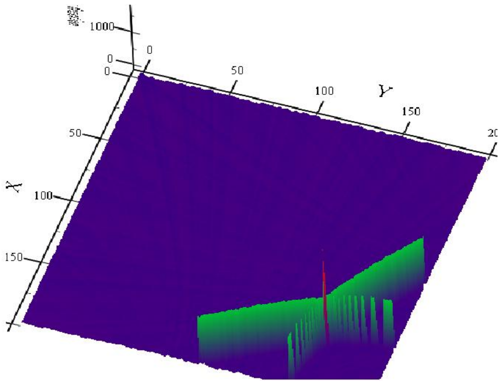

Fig. 13. The measurements of the coordinates of the source of the signal with the help of two antennas. The computer simulation

The physical experiment was made with the help of the acoustic source of the pseudo random signal (the computer with a loudspeaker) the coordinates of which were measured with the help of two acoustical antennas (microphones), which were connected to another computer. The results of experimental measurements are shown in Figs 14 and 15.

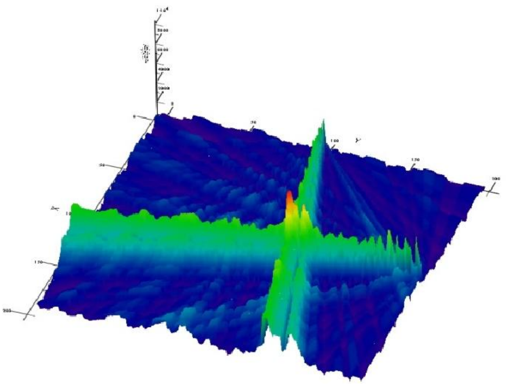

Fig. 14. The measurements of the coordinates of the source of the signal with the help of two antennas. The physical simulation using acoustic waves

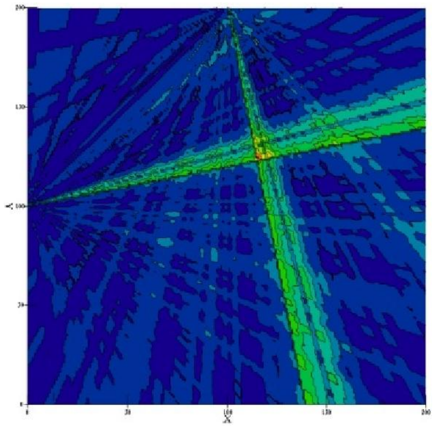

Fig. 15. The measurements of the coordinates of the source of the signal with the help of two antennas. The physical simulation using acoustic waves. View from the top

CONCLUSION

In this paper, different aspects of the signal processing algorithms for wideband random signal radars and radio navigation systems have been discussed.

We believe that the random signal radar is one of the most interesting types of radar. It combines properties of UWB radar with some additional features, based on random nature of the sounding waveform. These new properties allow us to simplify signal detection algorithms and measure a distance, an azimuth and a target velocity simultaneously with high resolution and accuracy, implementing the noise sounding waveform and adequate signal processing. For the signal processing and analysis we suggest to use the wideband ambiguity function, which in the case of the random signal radar allows obtaining a high resolution results for distance and velocity measurements.

The nonparametric generalization of the radar ambiguity function has been suggested. In contrast to classically defined ambiguity function, new one does not dependent on the signal PDF. It can be used as a pure measure of the relation between sounding and reflected signals as well as for the analysis [9], [10].

The signal detection algorithms, which are based on using the copula transform, have a stable false alarm error probability. We suppose that suggested approach of coordinate measurements can be used in navigation systems, which are based on using the signals of different sources of wideband electromagnetic waves, the coordinate of which are known.

The signal detection algorithms which are used in Global Navigation Satellite System (GNSS) very often use the likelihood ratio which can be rather simply transformed to calculation of the ambiguity function and further comparison it with a threshold.

We consider that a copula variant of the ambiguity function will be very useful for GNSS signal detection in a condition of a signal prior uncertainty, high level of noise and a small signal level. The suggested method can provide the stable level of errors and a constant dynamic range of the processed signals, improving the speed of signal processing.

Also we think that the ambiguity function is useful for the syntheses of new forms of signals, which can be used in navigation systems.

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Received 06 Oktober 2014

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Р. Б. Сініцин, Ф. Й. Яновський. Широкосмугова копулярна функція невизначеності в радіолокаційних та навігаційних системах

Обговорюється поняття функції невизначеності, яку було введено раніше у широкосмуговій радіолокації, зокрема її непараметричний варіант. Розгляд орієнтовано на застосування в радіолокації та навігації. Запропонована нова непараметрична версія функції невизначеності заснована на понятті копули. Така функція невизначеності може бути ефективно використана для синтезу сигналів та їх виявлення в різних електронних системах, особливо у разі сигналів з невідомими спектром і щільністю розподілу ймовірності, що є типовим для радіолокаційних систем, а у багатьох випадках актуально і важливо для систем навігації.

Ключові слова: функція невизначеності; копула; непараметричний алгоритм; широкосмуговий сигнал; радіолокація; навігація.

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Р. Б. Синицын, Ф. И. Яновский. Широкополосная копулярная функция неопределенности в радиолокационных и навигационных системах

Обсуждается понятие функции неопределенности, которая была введена ранее в широкополосной радиолокации, в частности ее непараметрический вариант. Рассмотрение ориентировано на применение в радиолокации и навигации. Предложенная новая непараметрическая версия функции неопределенности основана на понятии копулы. Такая функция неопределенности может быть эффективно использована для синтеза сигналов и их обнаружения в различных электронных системах, особенно в случае сигналов с неизвестными спектром и плотностью распределения вероятности, что является типичным для радиолокационных систем, а во многих случаях актуально и важно для систем навигации.

Ключевые слова: функция неопределенности; копула; непараметрический алгоритм; широкополосный сигнал; радиолокация; навигация.

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