AN APPROACH TO ROBUST CONTROL OF AIRCRAFT MOTION

The article deals with approaches to designing aircraft control systems based on the robust synthesis. The mathematical model of the aircraft control system both for deterministic and stochastic cases is considered. The Dryden filter models are represented. The state-space conception is applied. The concept of robust designing based on H-infinity synthesis, function of the mixed sensitivity, and loop shaping is represented. The features of Robust Control Toolbox necessary for automated designing of aircraft control systems are studied. The weighting transfer functions are proposed. Results of simulation of the synthesized robust control system are shown in the form of transient processes in lateral and longitudinal motions. The proposed approach is directed on providing the possibility of the aircraft to function in conditions of influence of disturbances. The possible applications of obtained results is control of aircraft motion in civil aviation.

Index Terms—Aircraft control system; robust synthesis; weighting coefficients; function of mixed sensitivity; aerodynamic disturbances; lateral and longitudinal motions.

I. INTRODUCTION AND PROBLEM STATEMENT

The design of modern aircraft control systems is characterized by the progress of technology and the growth of competition that leads to higher requirements for the level of safety and economy of flight. The development of civil aviation industry makes it necessary to take into account global design trends. Under such conditions, the design of robust aircraft motion control systems becomes relevant [1]–[3].

The main feature of robust control systems is the possibility to keep performances during influence of the perturbations. This is very important for aircraft control systems as they are used in real operating conditions accompanied by sufficient aerodynamic perturbations. Hence, the advantages of robust aircraft motion control systems are caused by the fact that they provide acceptable accuracy of control processes in difficult conditions due to the effect of coordinate and parametric disturbances real operation [4]–[6].

There is a technique for developing robust systems, using singular values determined for the system’s transfer function and minimization of H-infinity norms of sensitivity functions [7]–[9].

The H-infinity synthesis purpose is designing such a controller that delivers minimum H-infinity norm for the closed circuit transfer function. Searching optimal controller is realized on the quantity of controllers that make closed circuit internally stable, that is, on the set of admissible controllers [10]–[12].

It is noteworthy that there are multitudinous and contradictory requirements for aircraft control systems. The most significant contradiction is the need to provide performance and robustness at one time that requires searching a compromise solution.

The new direction in the automatic control theory is connected with creation of the control systems, which are characterized by low sensibility to variations of the model’s parameters and presence of deviation of the plant’s model from the real mathematical descriptions. The control systems designed in accordance with this approach are called robust. Designing robust systems is founded on minimizing H-infinity-norm of the transfer function for the closed circuit.

In the article, we propose to use robust approaches to designing aircraft control systems. The basic essence of the used approach lies in creating the mathematical model of the aircraft control system and designing the robust control system. The model is augmented by the weight transfer functions for the possibility to use loop-shaping. This allows us to ensure the desired frequency characteristics of the system. The applied robust approach is based on H-infinity synthesis including the method of the mixed sensitivity.
II. MATHEMATICAL DESCRIPTION OF AIRCRAFT CONTROL SYSTEM

We will consider that the studied aircraft control system includes a plant, which can be described by the linearized model in space of states [13], [14]

\[
\begin{align*}
\dot{x} &= Ax + Bu, \\
y &= Cx, \\
z &= C_0x,
\end{align*}
\]

(1)

where \( x \) is the state vector; \( A, B \) are matrices of appropriate aerodynamic derivatives; \( u \) is the vector of controls; \( y \) is the vector of observations for the real system; \( C, C_0 \) are observation matrices; \( z \) is the observation vector for calculating quality index.

For example, the state vector for aircraft lateral motion looks like \( x = [\beta, p, r, \varphi, \psi]^T \), where \( \beta \) is the slide angle, \( p, r \) are angular rates of roll and yaw, \( \varphi, \psi \) are angles of roll and yaw. The control vector is determined as \( u = [\delta a, \delta e]^T \), where \( \delta a, \delta e \) are angles of deviations of ailerons and direction angle.

Matrices of aerodynamic derivatives \( A \) and \( B \) can be represented in the following way [2], [15], [16]

\[
A = \begin{bmatrix}
Y_\beta & Y_p & Y_r & Y_\varphi & Y_\psi \\
L_\beta & L_p & L_r & L_\varphi & L_\psi \\
N_\beta & N_p & N_r & N_\varphi & N_\psi \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix},
\quad
B = \begin{bmatrix}
Y_{\beta_0} & Y_{\beta_0} \\
L_{\beta_0} & L_{\beta_0} \\
N_{\beta_0} & N_{\beta_0} \\
0 & 0 \\
0 & 0
\end{bmatrix}.
\]

(2)

The expression (2) uses the following notations: \( Y \) means observation signal of the lateral system, \( L \) and \( M \) are moments of the roll and yaw, respectively.

Observation matrix \( C \) is used for developing the model of the aircraft control system. It is determined by features of the real navigation system. Observation matrix \( C_0 \) is determined by state variables. The slide angle \( \beta \) is rare enough measured in real systems. At the same time, restriction of the angle \( \beta \) is of great importance for flight safety. The same conclusion could be made relative to the angle \( \alpha \) for the longitudinal motion. Therefore, the variable \( y \) in the state-space mathematical model \( y \) is used for creating the controller. The variable \( z \) is used for calculating quality index [17] - [19].

In civil aviation, the rigid requirements are given for bounds of short-periodic and long-periodic accelerations [20] – [22]. Therefore, it is necessary to choose a method, which allows us to take into consideration given requirements for quality indices in important practical situations. Such an approach can be illustrated on the example of the lateral motion. The lateral acceleration

The lateral acceleration except of gravity is measured by the accelerometer. This acceleration can be described by the expression [2]

\[
a_y = Y_\beta \cdot \beta + Y_\delta \cdot \delta \cdot a_{11}x_1 + b_{12}u_2.
\]

(3)

As follows from the expression (3), the squared acceleration will include the component \( 2a_{11} \cdot b_{12} \cdot x_1 \cdot u_2 = 2Y_\beta \cdot Y_\delta \cdot \beta \cdot \delta \).

The plant equation (3) must be supplemented by the controller equations taking into consideration the negative feedback [23]

\[
\begin{align*}
\dot{x}_u &= A_u x_u + B_u (\bar{r} - Cx), \\
u &= C_u x_u + D_u (\bar{r} - Cx),
\end{align*}
\]

(4)

where \( \bar{r} \) is the reference signal, \( A_u, B_u, C_u, D_u \) is the quadruple of matrices, which represents the mathematical description of the controller.

The diagram of the closed-loop system is represented in Fig. 1.

The state-space model of the closed-loop system for the vector of states, \( x_0 = [x, x_u]^T \), reference signal \( \bar{r} \), and observation vectors \( y_0 = [y, u]^T \) and \( z = C_0^T x_0 \) can be obtained from equations (1) and (4) in the following form

\[
\begin{align*}
x_0 &= A_0 x_0 + B_0 \bar{r}, \\
y_0 &= C_0 x_0 + D_0 \bar{r}, \\
z &= C_0^T x_0,
\end{align*}
\]

(5)

where the quadruple of matrices in the state-space mathematical description can be determined in the following way

\[
\begin{align*}
A_0 &= \begin{bmatrix}
A - BD_u C & BC_u \\
-\bar{B}_u & A_u
\end{bmatrix}, \\
B_0 &= \begin{bmatrix}
BD_u \\
B_u
\end{bmatrix}, \\
C_0 &= \begin{bmatrix}
C & 0 \\
-D_\bar{r} C & C_u
\end{bmatrix}, \\
D_0 &= \begin{bmatrix}
0 \\
D_u
\end{bmatrix}.
\end{align*}
\]

(6)

here \( \bar{r} \) is the input (reference) signal; \( z \) is the observation vector, which forms the quality index.

The main stochastic disturbance that influences on an aircraft is the turbulent wind [2]. Therefore, firstly, it is necessary to have the mathematical model of the disturbance in the form suitable for aircraft models in the state space.
The simplest description of the turbulent wind can be done by means of the Dryden model [2], which represents the short periodic $u_g$, long periodic $w_g$, and lateral $v_g$ components of the vector of the wind’s instantaneous speed as stationary random processes with the following spectral densities

$$S_u(\omega) = \frac{2\sigma_u^2 L_u}{U_0 \pi} \left(1 + \tau_u^2 \omega^2\right), \quad \tau_u = \frac{L_u}{U_0},$$

$$S_v(\omega) = \frac{\sigma_v^2 L_v}{U_0 \pi} \left(1 + \tau_v^2 \omega^2\right), \quad \tau_v = \frac{L_v}{U_0},$$

$$S_w(\omega) = \frac{\sigma_w^2 L_w}{U_0 \pi} \left(1 + \tau_w^2 \omega^2\right), \quad \tau_w = \frac{L_w}{U_0},$$

where $L_u$, $L_v$, $L_w$ are appropriate scale coefficients of turbulence; $U_0$ is the constant speed of the aircraft; $\sigma_u$, $\sigma_v$, $\sigma_w$ are root mean squares of speed components.

These random processes can be represented as outputs of forming filters. Inputs of these filters represent non-correlated white noise $\eta_x$, $\eta_y$, $\eta_z$. The long-periodic speed $w_g$ can be easily calculated in the turbulent angle of attack $\alpha_x = w_g / U_0$. The longitudinal motion can be characterized by input $\eta = [\eta_x, \eta_z]^T$ and output $g = [u_g, \alpha_x, q_g]^T$ vectors, where $q_g$ can be represented as $q_g = -\dot{\alpha}_g$ [2]. In this case, the equation of the turbulent wind motion can be written as

$$\dot{x}_g = A_g \cdot x_g + B_g \cdot \eta,$$

$$g = C_g \cdot x_g + D_g \cdot \eta,$$

where

$$A_g = \begin{bmatrix} -\lambda_u & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -\lambda_w^2 - 2\lambda_w \end{bmatrix}, \quad B_g = \begin{bmatrix} K_u & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & K_u \beta_w & K_\alpha \\ 0 & K_w \lambda_w^2 & K_a (2\lambda_w - \beta_w) \end{bmatrix}, \quad D_g = \begin{bmatrix} 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$K_u = \frac{\sigma_u \sqrt{2U_0}}{\pi L_u}, \quad K_w = \frac{\sigma_w \sqrt{3 \pi L_W}}{L_w}, \quad \lambda_u = \frac{U_0}{L_u}, \quad \lambda_w = \frac{U_0}{L_w}, \quad \beta_w = \frac{\lambda_w}{\sqrt{3}}.$$

In the similar way, it is possible to represent the forming filter for the lateral motion. In this case, the input signal represents the white noise $\eta_y$, and output signals are the turbulent lateral speed $v_g$, and turbulent angular rates $\rho_g$, $\tau_g$. The Dryden model is standardized [2]. It should be noted that coefficients in expressions (7) for various modes (cruising flight, landing and so on) are determined in experimental way. These models can be used for checking accuracy of stabilization during flights in the turbulent atmosphere.

The stochastic model of longitudinal motion presented in Fig. 2 represents a sequential connection of the forming filter in the form (9), (10) and the previously considered closed-loop control system.
The diagram of the closed-loop system presented in Fig. 2, slightly differs from the one shown in Fig. 1 by the input point of the external signal. For the input signal $u$, the expressions for the quadruple of block matrices describing the closed-loop system in the state space can be represented in the following form

$$
\begin{align*}
A_u &= \begin{bmatrix} A - BD_g C & -B_C A_s \\ B_C & A_s \end{bmatrix}, \\
B_0 &= \begin{bmatrix} B \\ 0 \end{bmatrix}, \\
C_0 &= \begin{bmatrix} 0 \\ D_C C_s \end{bmatrix},
\end{align*}
$$

(11)

here $D_0$ is the zero matrix of the appropriate dimension. The serial connection of the closed-loop control system and Dryden forming filter (9) allows to take into consideration the turbulent atmosphere. The matrix of controls represents the augmented block matrix

$$
B_{0g} = \begin{bmatrix} B_{0g}^0 \\ B_0 \end{bmatrix},
$$

(12)

here the matrix $B_{0g}^0$ defines the influence of the external disturbance $g$ on the plant. It consists of columns of the matrix $A$ with opposite signs. These components correspond to turbulent disturbed variables of the state. For short-periodical motion, these components represent an increment of the short-periodic speed, angle of the attack, and angle rate of the pitch. If the mathematical description of the forming filter in the space of states is defined by the quadruple of matrices $[A_s, B_s, C_s, D_s]$, the serial connection of the closed-loop system and forming filter taking into consideration expressions (11), (12) can be represented in the following form

$$
\begin{align*}
\dot{x}_s &= A_s x_s + B_s \eta, \\
y &= C_s x_s, \\
z &= C_{s0} x_s,
\end{align*}
$$

(13)

where $x_s$ is the vector with components consisting of state variables of the forming filter and closed-loop system; $\eta$ is the vector of the white noise at the input of the forming filter; $y$ is the measured output of the control system; $z$ is the vector of state variables, which form the quality index in the stochastic case, matrices $A_s, B_s, C_s, D_s$ look like

$$
\begin{align*}
A_s &= \begin{bmatrix} A_s \\ B_s C_0 A_s \end{bmatrix}, \\
B_s &= \begin{bmatrix} B_s^0 \\ 0 \end{bmatrix}, \\
C_s &= \begin{bmatrix} 0 \\ C_s \end{bmatrix},
D_s &= D_0 D_s,
\end{align*}
$$

(14)

As matrix $D_s = 0$, the following equation is true $D_s = 0$. Matrix $C_{s0}$ is formed based on those state variables, which define the quality indices of the control system.

During synthesis of aircraft robust control systems, it is necessary to determine a range of changing plant parameters, which have the greatest influence on the performance and stability of the control system. For manned aircraft, modern requirements for robustness that is for keeping stability and performance of control reduces for the following situation. The aircraft control system must keep quality indices in permissible flight modes for following factors:
1) changing the position of the centre of a mass (in the range from 15% to 23% of the average aerodynamic chord in the horizontal plane and from 0% to 21% of the average aerodynamic chord in the vertical plane);

2) variation of an aircraft mass (in 1.5 times);

3) variation of delay of digital controller reaction (from 50 to 100 ms).

We will consider that the studied aircraft control system includes a plant, which can be described by the linearized model in space of states [13], [14] control, and complementary sensitivity function.

III. DESIGNING OF ROBUST CONTROL SYSTEM

The control systems, which provide keeping basic characteristics during changes of plant parameters in sufficiently wide range are robust ones. There are some methods of designing such systems. It should be noted that the quality of designing works depends essentially on the possibility of their automation. One of the best instruments of automated designing robust systems is Robust Control Toolbox (MatLab). Methods realizing in Robust Control Toolbox are based on operations with frequency characteristics of control systems. The basic approach of designing robust control systems by Robust Control Toolbox is based on satisfying sufficient condition of robust stability

\[ \|L\|_\infty \|K(1+G_0K)^{-1}\|_\infty < 1, \]  

(15)

where \( L \) is the upper bound of variation of the transfer function; \( G_0 \) is the transfer function of the plant; \( K \) is the transfer function of the controller.

In other words, the sufficient condition of robust stability (1) is formulated as restriction of \( H_\infty \)-norm, which is weighted as function of the system’s nominal transfer function. Deviations of the plant transfer function from nominal one are caused by both change of parameters and non-modelled dynamics. Influence of disturbances are also taken into consideration. The non-modelled dynamics includes small inertia moments, non-linearities, and other factors.

One of modern approach for designing robust systems is the so called loop-shaping. This approach is based on requirements for frequency characteristics of control systems. It can be explained by the diagram represented in Fig. 3. Here \( G \) is the plant, \( K \) is the controller, \( r \) is the reference signal, \( w \) is the disturbance, \( v \) is the noise; \( e \) is the error of regulation, \( u \) is the control, \( W_1, W_2, W_3 \) are transfer weighting functions depending on frequency.

![Diagram for synthesis robust controller](image)

The matrix transfer function

\[ S = (I + GK)^{-1} \]  

(16)

is the transfer function on the error or the function of the sensitivity. It defines connections between \( r, w, v, \) and the error \( e \).

The transfer function

\[ T = GK(I + GK)^{-1} \]  

(17)

is the transfer function of the closed-loop control system or supplemented function of the sensitivity.

The transfer function

\[ R = K(I + GK)^{-1} \]  

(18)

is the function of sensitivity by control.

The requirements for the system on decreasing disturbances and providing margin of stability can be reduced minimization of the \( H_\infty \)-norm of the function of the mixed sensitivity based on functions (16) – (18)

\[ T = \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \]  

(19)

Minimization of \( H_\infty \)-norm of the function of the mixed sensitivity (19) in Robust Control Toolbox can be realized by operators \texttt{hinf}opt (for continuous system) and \texttt{dhinf}opt (for discrete system). To realize synthesis of the robust controller, it is necessary to create the augmented plant. Application of the operator \texttt{hinf}opt allows us to determine the optimal solution in the form of the transfer function of the closed-loop augmented system \texttt{sclf} and the robust controller \texttt{sscp}. The system \texttt{ss_ft} characterized the transfer function of the closed-loop system, which consists of the serial connection of the controller \texttt{sscp} and the plant \texttt{ssg} closed by the unit feedback by every channel; \texttt{ss_ft} is the transfer function of the sensitivity.

The great emphasis in this procedure is given to the choice of the weighting transfer functions, which
provide creation of the augmented plant and forming the necessary frequency characteristics of the synthesized system. The weighting functions represent the diagonal matrices in the form

\[
W_i = \begin{bmatrix}
\frac{as + b}{cs + d} & 0 \\
0 & \frac{a's + b'}{c's + d'}
\end{bmatrix}
\]

The choice of the weighting characteristics is, mainly, the heuristic procedure.

IV. SIMULATION RESULTS

The described algorithm of was realized for the light airplane of the civil aviation. The mathematical model was represented in the space of states.

The weighting coefficients were chosen in the following form

\[
W_1 = \begin{bmatrix}
\frac{1}{s + 0.01} & 0 \\
0 & \frac{1}{s + 0.01}
\end{bmatrix},
\]

\[
W_2 = 0,
\]

\[
W_3 = \begin{bmatrix}
s^2 / 1000 & 0 \\
0 & (\tau s + 1)s^2 / 1000
\end{bmatrix}.
\]

The coefficients of weighting transfer functions are functions of aerodynamic parameters and can be essentially changed. Transient processes by step signals for lateral motion are represented in Fig. 4. They demonstrate transient processes on the roll, yaw, velocity and path angle correspondingly.

V. CONCLUSIONS

The mathematical models of the aircraft control system are developed and represented. The procedure of designing the robust control system taking into consideration introducing weighting transfer functions is described. The expressions for weight transfer functions have been obtained. The appropriate software has been developed. The results of designing robust systems in the form of the

Fig. 4. Transient processes: on roll angle (a), on yaw angle (b), on air velocity (c), on path angle (d)
transient processes are represented. The obtained results prove the high robustness of the control system.

REFERENCES


Received August 26, 2023
О. А. Сущенко, В. М. Безкоровайний. Підхід до робастного керування рухом літака
У статті розглянуто підходи до проектування систем керування літальними апаратами на основі робастного синтезу. Розглянуто математичну модель системи керування літальним апаратом як для детермінованого, так і для стохастичного випадків. Представлені моделі фільтрів Драйдена. Застосовано концепцію простору станів. Представлено концепцію робастного проектування на основі Н∞, функції змішаної чутливості та формування контурів керування. Досліджено особливості пакету прикладних програм Robust Control Toolbox, необхідні для автоматизованого проектування систем керування літаками. Запропоновано вагові передавальні функції. Показано результати моделювання синтезованої робастної системи керування у вигляді переходних процесів при поперечному та поздовжньому рухах. Запропонований підхід спрямований на забезпечення можливості функціонування літака в умовах впливу завад. Можливе застосування отриманих результатів – управління рухом повітряних суден у цивільній авіації.

Ключові слова: система керування літальним апаратом; робастний синтез; вагові коефіцієнти; функція змішаної чутливості; аеродинамічні збурення; поперечні та поздовжні рухи.