UDC 656.7.073.28 (045)
DOI:10.18372/1990-5548.68.16096

[1]**D. O. Shevchuk,**
[2]**V. A. Kasianov,**
[3]**Yu. V. Shevchenko**

ENSURING FREIGHT DELIVERY IN CONDITIONS OF UNCERTAINTY

[1,2,3]Faculty Transport, Management and Logistics, National Aviation University, Kyiv, Ukraine
E-mails: [1]dmitroshevchuk@gmail.com ORCID 0000-0001-9911-7214, [2]vakasyanov@mail.ru ORCID 0000-0003-1318-9589, [3]yulsashe@gmail.com ORCID 0000-0003-0160-3228

Abstract—*The article discusses the issue of ensuring the delivery of goods in conditions of uncertainty, designed to predict the time of the transport task. The initial information for training the model is the carrier's data on the expected average time to complete the task. The analysis uses the entropy method. The analysis of the obtained results has been carried out. The results show that the use of the entropy method allows us to investigate its sensitivity to changes in the value of preferences. In the work on the application of entropy, three criteria are used: entropy should be minimal for well-defined quantities, be maximal for equiprobable quantities, and universal – applicable for both finite and infinite, discrete and continuous distributions. When changing the values of the parameters, we used cross entropy and quadratic entropy and, as a result, we obtained an estimate of uncertain variables that can be used to solve the transport problem under uncertainty.*

Index Terms—Transport system; freight company; entropy; transport task; intelligent technology.

I. INTRODUCTION

Entropy, introduced by Boltzmann, is a traditional object of research in physics, in information theory (introduced by Chenon), in synergetics, [1] –[3].

In recent decades, entropy has been used as a research tool in biology, economics, learning theory, logistics, and many other fields.

There are a number of definitions of entropy: the entropy of Carnot, Clausewitz, Boltzmann, Chenon, Kolmagorov, Rashevsky, Sinai. The entropies of Renyi and Tsilas should also be mentioned [4].

Entropy is an important tool in synergetics [5], [6], etc., and it is considered not only as a measure of uncertainty, but, at the same time, as a measure of orderliness [Toffler 1986, p. 25].

The founder of synergetics, Haken [6], is not inclined to overestimate the role of entropy, in contrast to Prigogine's Brussels school.

According to Haken: "Although the concept of entropy and related concepts are extremely useful in thermodynamics and in the so-called thermodynamics of irreversible processes, they turn out to be too crude when considering self-organizing structures. In the general case, in such structures, the entropy changes only by a very small amount. Thus, other approaches are needed" [6].

Note that the concepts of "roughness", "small variability" are qualitative and relative. They are necessarily associated with a general analysis of the "problem situation". On the other hand, entropy, as an integral intensive characteristic of uncertainty in a system, including and especially in an active system, is very preferable.

II. PROBLEM STATEMENT

Subjective entropy of preferences and subjective information as a factor in ensuring the delivery of goods in conditions of uncertainty. Below we will touch on other measures of uncertainty that have properties similar to entropy, in particular, in the works of Ivanenko and Labkovsky [7], the form of the "uncertainty criterion" is associated with the form of the "loss function". In the works of Levich [8], the choice of this or that uncertainty exponent is a consequence of the choice of an integral invariant given over the set of morphisms in S_a.

An ambiguous and critical attitude to the entropy paradigm is often associated with the so-called "non-constructiveness" of the corresponding theory. The author, however, thinks that in combination with Jaynes's variational principle [2], [6], Linsker's "Infomax" principle, three variational problems formulated by Stratonovich, this paradigm acquires "constructiveness".

This work is an attempt to demonstrate its “constructiveness” in formalizing the problem of generating and developing preferences in the depths of the psyche. Obviously, the corresponding theory cannot do without a number of additional postulates, and assumptions.

Below, as a fundamental criterion, the entropy in the form of Chanon is used, more precisely, the subjective entropy of preferences. Entropy, together with the postulated principle of optimality, has a number of significant advantages over other criteria.

The use of entropy leads to easily solvable analytic equations (linear with respect to the logarithm of the preference function) for the so-called canonical distributions [9] (Stratonovich). Entropy has an extremely important property of hierarchical additivity.

The widespread use of entropy in physics and information theory allows for a far-reaching analogy. For example, this concerns the use of the concepts of mental and emotional "temperatures", "emotional overheating" and "emotional hypothermia".

The irreversibility of time, the "arrow of time" fit well into the entropy paradigm. Some authors introduce "entropy time" and, moreover, the theory of "entropy mechanics" is being constructed, the law of conservation is introduced for entropy [10].

The author is inclined to consider subjective entropy not only as a measure of "uncertainty" of preferences, a convenient research tool, but also a criterion "organically" inscribed in the psyche, participating in the management of objectively occurring mental processes.

In particular, subjective entropy can be associated with a postulate, the meaning of which is the subject's striving for inner freedom and its projection onto external freedom. It can be assumed that entropy criteria are related to the emergence and development of intrapersonal and interpersonal conflicts.

The idea of using subjective entropy is attractive in that it allows us to formulate in terms of the entropy paradigm a number of additional assumptions and concepts that correspond to "common sense" and make the theory more structured and rich.

In this paper, as in papers [11], [12], subjective entropy and subjective preference information are introduced, defined on the set of alternatives.

III. Problem Solution

Below we will touch on other measures of uncertainty that have properties similar to entropy, in particular, in the works of Ivanenko and Labkovsky [7], the form of the "uncertainty criterion" is associated with the form of the "loss function". In the works of Levich [8], the choice of this or that uncertainty exponent is a consequence of the choice of an integral invariant given over the set of morphisms in S_a.

An ambiguous and critical attitude to the entropy paradigm is often associated with the so-called "non-constructiveness" of the corresponding theory. The author, however, thinks that in combination with Jaynes's variational principle [2], [6], Linsker's "Infomax" principle, three variational problems formulated by Stratonovich, this paradigm acquires "constructiveness".

This work is an attempt to demonstrate its "constructiveness" in formalizing the problem of generating and developing preferences in the depths of the psyche. Obviously, the corresponding theory cannot do without a number of additional postulates, and assumptions.

Below, as a fundamental criterion, the entropy in the form of Chanon is used, more precisely, the subjective entropy of preferences. Entropy, together with the postulated principle of optimality, has a number of significant advantages over other criteria.

The use of entropy leads to easily solvable analytic equations (linear with respect to the logarithm of the preference function) for the so-called canonical distributions [9] (Stratonovich). Entropy has an extremely important property of hierarchical additivity.

The widespread use of entropy in physics and information theory allows for a far-reaching analogy. For example, this concerns the use of the concepts of mental and emotional "temperatures", "emotional overheating" and "emotional hypothermia".

The irreversibility of time, the "arrow of time" fit well into the entropy paradigm. Some authors introduce "entropy time" and, moreover, the theory of "entropy mechanics" is being constructed, the law of conservation is introduced for entropy [10].

The author is inclined to consider subjective entropy not only as a measure of "uncertainty" of preferences, a convenient research tool, but also a criterion "organically" inscribed in the psyche, participating in the management of objectively occurring mental processes.

In particular, subjective entropy can be associated with a postulate, the meaning of which is the subject's striving for inner freedom and its projection onto external freedom. It can be assumed that entropy criteria are related to the emergence and development of intrapersonal and interpersonal conflicts.

The idea of using subjective entropy is attractive in that it allows us to formulate in terms of the entropy paradigm a number of additional assumptions and concepts that correspond to "common sense" and make the theory more structured and rich.

In this paper, as in papers [11], [12], subjective entropy and subjective preference information are introduced, defined on the set of alternatives.

Below we consider two types of preferences: "subject" and "rating" preferences, denoted by π and ξ, respectively. Subjective entropy is naturally associated with subjective information. In the general case, it is not a probabilistic quantity (and the value of information, which is discussed, for example, in [5]), since it always refers to a specific individual "carrier" – a subject, to a given moment in time and can be considered as a quantity with random properties only if:

a) in determining the values of the subject's preferences $\pi(\sigma_i)$ (measurements of preferences), random errors are allowed.

b) preferences "organically" contain remnant (random) components.

In works on information theory [13], [14], [3], entropy and information are expressed through the probability distribution. In our case, the assumption of the existence of the general population is optional. In particular, the ergodicity problem does not arise. It is replaced by the assumption of the existence of an extreme principle of formation of preferences "built in" into each "individual psyche" and, accordingly, that for all individuals this principle has common features and a common structure. At the same time, the structural parameters can be different, and in this the individuality of the subjects can be manifested. Their psyches are "structurally" similar, but "parametrically" different. Differences arise in certain individual "cognitive functions" (see below).

Subjective entropy and subjective information are expressed through the distribution of preferences on a set of alternatives or a set of subjects in a group, the number and essence of which are the result of subjective preferences, preliminary analysis of quantitative and qualitative characteristics, as well as subconscious, intuitive assessments of a virtual object – a "problem situation". While not probability distributions, preference distributions have formal similarities with them, which leads to far-reaching analogies.

The similarity, however, is incomplete, which manifests itself in fewer a priori constraints imposed in the form of axioms.

Let $S_a|_{\sigma_e}$ is the set of dimension alternatives N, $\sigma_e \overline{\in} S_a|_{\sigma_0}$ is the initial state, and $\pi(\sigma_i)$ is the distribution of preferences for $S_a|_{\sigma_e}$. We introduce the entropy of the distribution $\pi(\sigma_i)$ in Chanon form:

$$H_\pi = -\sum_{i=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i), \quad \sigma_i \in S_a|_{\sigma_0}. \tag{1}$$

In information theory, Chenon defined in this form the average information per message, expressed in terms of partial probabilities p_i [10].

Entropy in the form (1) has the following properties.

1) When all values of the function $\pi(\sigma_i)$ are the same, that is, the alternatives σ_i are equally preferred, the entropy has a maximum value $H_{\max}$. Provided that there is a unit normalization

$$\sum_{i=1}^{N} \pi(\sigma_i) = 1. \tag{2}$$

$\pi(\sigma_i) = \dfrac{1}{N}$, and $H_{\max} = \ln N$.

2) With a singular distribution, when the preferences of all alternatives are equal to zero, except for the preference for one alternative:

$$\pi(\sigma_i) = \begin{cases} 0, & i \neq k, \\ 1, & i = k, \end{cases} \quad (i \in \overline{1,N}),$$

entropy is minimal and equal to zero ($H_{\min} = 0$). Thus, the entropy H_π enclosed within:

$$0 \leq H_\pi \leq \ln N$$

and therefore.

3) Under the chosen normalizing condition, the entropy is non-negative.

The choice of unit normalization (2) is largely arbitrary. Conditionally, the normalization can be assigned a psychological meaning: if the preference of some alternative increases, then the preference of some other alternative decreases.

Let the set $S_a|_{\sigma_0}$ contains k equivalence classes and L_s – number of alternatives in s-class, and π_{Ls} – the value of the preference function for elements (alternatives) belonging to this class. Then the entropy will take the form:

$$H_\pi^k = -\sum_{s=1}^{k} L_s \pi_{Ls} \ln \pi_{Ls}, \quad (k \in \overline{1,N}). \tag{3}$$

Writing $\pi_s = L_s\pi_{Ls}$ – "class preference", we get:

$$H_\pi^k = -\sum_{s=1}^{k} \pi_s \ln \pi_s + \sum_{s=1}^{k} \pi_s \ln L_s. \tag{4}$$

Both terms are always positive (non-negative). The second term is the preference-weighted entropy of the size (cardinality) of the classes.

Entropy $H_{k\pi}$ for a given number $k < N$ reaches a maximum if all π_s are the same and equal $\pi_s = \dfrac{1}{k}$.

$$\sum_{i=1}^{N} \pi(\sigma_i) = \sum_{s=1}^{k} \pi_{Ls} L_s = 1 \Rightarrow \sum_{s=1}^{k} \pi_s = 1,$$

and provided $\pi_s = \dfrac{1}{k}$ for $\forall s \in \overline{1,k}$ find from (4):

$$H_\pi^k = \ln k + \ln \sqrt[k]{L_1 L_2 \cdot \ldots \cdot L_k}.$$

We can see from the formula that, H_π^k reaches its maximum when $k = N$. In this case $L_s = 1 \;\; \forall s \in \overline{1, N}$ and the second term vanishes. Consequently

$$H_\pi^k (k < N) < H_\pi^N \text{ and } H_\pi^k (k = N) = H_\pi^N = \ln N.$$

As we can see, there are equivalence classes such that for at least one of $L_s > 1$ leads to a decrease in entropy.

In information theory, the quantity $H_\pi = k\ln N$, called Hartley information, where N is the the number of equally probable experimental results. When $x = 1$, then the Hartley information, measured in the natural units–natas, if $x = (\ln 2)^{-1}$, so H_π expressed in binary units – bits. If the states are unequal, then each state has its own information:

$$H_i = -\ln p(\sigma_i),$$

where $p(\sigma_i)$ is the probability of "occurrence" of the state σ_i. In our case, the place of probability is taken by the preference function $\pi(\sigma_i)$, and the private entropy $H_\pi(\sigma_i)$ reflects the uncertainty associated with the alternative σ_i and can be interpreted as "frozen" subjective information that is released if σ_i is chosen as the target. Entropy $H_\pi(\sigma_i)$ after selection σ_i vanishes as a target and, accordingly, information is released $I(\sigma_i) = H_\pi(\sigma_i)$.

Entropy (1) is the result of averaging the subjective partial entropies over preferences.

Subjective entropy characterizes the mental state of a subject in a problem situation.

It seems natural to assume that the level of mental tension is the higher, the higher the entropy. In turn, entropy depends on the type of preference distribution and on the number of alternatives. It is convenient in some cases to use the normalized entropy:

$$H_\pi = (\ln N) - 1 - H_\pi$$

which, as is easy to see, always lies within $0 \le H_\pi \le 1$ (if the normalization $\pi(\sigma_i)$ identity), if the normalization $\pi(\sigma_i)$ non-identity, the normalized entropy should be taken in the form:

$$H_\pi = \frac{H_\pi - \varphi \ln(\varphi)^{-1}}{\varphi \ln \frac{N}{\varphi} - \varphi \ln \frac{1}{\varphi}}.$$

Range $\varphi \ln \varphi, -\varphi \ln \frac{\varphi}{N} (\varphi \ln N)$ depends from the variable φ.

Let us find out within what limits the entropy changes if the normalization is not unitary: $\sum_{i=1}^{N} \pi_i = \varphi$. With a uniform distribution $\pi_i = \varphi N^{-1}$. Decide as $\pi_i = \varphi N^{-1} + \sigma_i$, where δ_i is the deviation from a uniform distribution that satisfy the conditions:

$$\sum_{i=1}^{N} \delta_i = 0, \; |\delta_i| << 1.$$

Let us change the entropy

$$H_\pi = -\sum_{i=1}^{N} \left(\frac{\varphi}{N} + \delta_i \right) \ln \left(\frac{\varphi}{N} + \delta_i \right) \approx -\sum_{i=1}^{N} \left(\frac{\varphi}{N} + \delta_i \right) \left(\ln \frac{\varphi}{N} + \frac{N}{\varphi} \delta_i \right) = -\sum_{i=1}^{N} \frac{\varphi}{N} \ln \frac{\varphi}{N} - \frac{N}{\varphi} \sum \delta_i^2.$$

Thus, any small deviations from a uniform distribution lead to a decrease in entropy.

Let us determine what values depending on φ takes on the minimum entropy. We put $\pi_i = \varphi \cdot \pi_i^0$, $\sum_{i=1}^{N} \pi_i^0 = 1$ then

$$H_\pi = -\sum_{i=1}^{N} \varphi \pi_i^0 (\ln \varphi + \ln \pi_i^0) = -\varphi \ln \varphi + \varphi H_\pi^0,$$

$$H_\pi^0 = -\sum_{i=1}^{N} \pi_i^0 \ln \pi_i^0.$$

As the minimum value $H_{\pi \min}^0 = 0$, then

$$H_{\pi \min} = \frac{\min H_\pi}{\pi_i^0 \in \Pi^0} = -\varphi \ln \varphi.$$

The graphic of function $-\varphi \ln \varphi$ is pictured on Fig. 1.

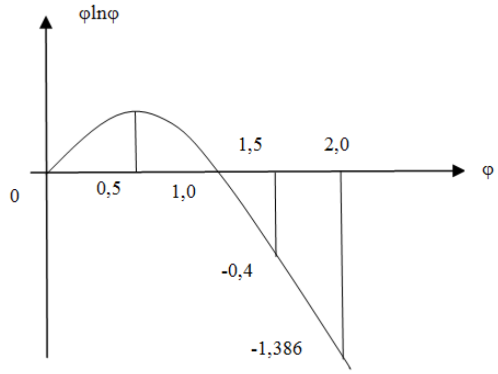

Fig. 1. The graphic of function $-\varphi \ln \varphi$ the quantity of absolute minimum $H_{\pi\min}$ depends on φ and when $\varphi \to \infty$ $H_{\pi\min} \to -\infty$, $\max_\varphi (H_{\pi\min}) = e^{-1} \cong 0.3679\ldots$

If you require that H_π was nonnegative, then we find that the condition – ln φ + $H_\pi^0 \geq 0$.

Conditionally, the normalization can be attributed to a psychological meaning: if the preference of any alternative σ_i increases, the preference for some other alternative decreases.

With a fixed right-hand side in the normalization condition, the preference values play the role of comparative preferences of alternatives on S_a at the moment, and do not in any way reflect the "absolute" power of "desirability." You can, for example, imagine a situation when all desires are dulled, their intensity drops to zero and, consequently, $\pi(\sigma_i)$ should in this case decrease to zero.

Thus, in order to take into account not only the comparative, but also the absolute preference, the case of an arbitrary, including non-stationary, normalization is considered:

$$\sum_{i=1}^{N} \pi(\sigma_i) = \varphi, \quad \varphi \geq 0. \tag{5}$$

We will consider that φ = φ (t).

If in the theory of probability the choice of a unit normalization is the result of an agreement and is mainly due to considerations of convenience, then in this case, we want to give preferences also the function of being characteristics not only of comparing alternatives, but also of the intensity of their desirability.

In this sense, the choice of the normalizing condition ceases to be a trivial task. Let us consider the consequences of using the normalization condition in the form (5).

We put $\pi(\sigma_i) = \varphi \cdot \pi^0(\sigma_i)$, where $\sum_{i=1}^{N} \pi_i^0(\sigma_i) = 1$.

Then the entropy

$$H_\pi = -\varphi \ln \varphi + \varphi \cdot H_\pi^0, \tag{6}$$

where $H_\pi^0 = -\sum_{i=1}^{N} \pi_i^0 \ln \pi_i^0 \geq 0$, with the condition that ln φ > H_π^0, $H_\pi < 0$.

In condition with ln φ = H_π^0, we find that $H_\pi = 0$.

Let in (6) H_π^0 given, we determine at what φ H_π reaches the highest value.

$$\frac{\partial H_\pi}{\partial \varphi} = -\ln \varphi - 1 + H_\pi^0 = 0.$$

Therefore, $\varphi_{\text{opt}} = e^{H_\pi^0 - 1}$. This is equal to maximum H_π, as the

$$\left.\frac{d^2 H_\pi(\varphi)}{d\varphi^2}\right|_{\varphi_{\text{opt}}} = -\frac{e}{H_\pi^0} < 0.$$

We find that

$$H_\pi\Big|_{\varphi_{\text{opt}}} = -e^{H_\pi^0 - 1}\left(\ln\left(e^{H_\pi^0}\right) - 1\right) + e^{H_\pi^0 - 1} \cdot H_\pi^0 = e^{H_\pi^0 - 1}. \tag{7}$$

That means that $H_{\pi\max} = \frac{N}{e}$, as the $H_{\pi\max}^0 = \ln N$.

Let us oversee the criteria

$$\Phi = H_\pi + \gamma\left(\sum_{i=1}^{N} \pi_i - \varphi\right) \qquad 8)$$

and define such a distribution for which the criterion Φ takes the maximum value. From (8) we find:

$$-\lg \pi_i - 1 + \gamma = 0. \qquad 9)$$

So, $\pi_{i_{\text{opt}}} = e^{-1+\gamma} = \text{const}, \quad \forall i \in \overline{1, N}$.

Writing as $e^{-1+\gamma} = c$, we get $c = \frac{\varphi}{N} = \varphi \cdot \pi_{i_{\text{opt}}}^0$ ($\pi_{i_{\text{opt}}}^0 = N^{-1}$).

As the absolute maximum H_π is obtained when $H_\pi^0 = \ln N$, and $\pi_{i_{\text{opt}}}^0 = N^{-1}\varphi_{\text{opt}}$, where $\varphi_{\text{opt}} = \frac{N}{e}$, so $\pi_{i_{\text{opt}}} = \frac{1}{e}$.

Let's see how entropy changes over time. If φ = φ(t). We can see that

$\frac{d}{dt}\left(\sum_{i=1}^{N} \pi_i(t)\right) = \dot{\varphi}(t)$ and if $\dot{\varphi}(t) \neq 0$, so $\sum_{i=1}^{N} \dot{\pi}_i(t) \neq 0$.

It can be shown that the production of entropy in the considered case $q = \frac{dH_\pi}{dt}$ is given by the formula

$$\frac{dH_\pi}{dt} = \frac{dH_\varphi}{dt} + \frac{d\left(\varphi H_\pi^0\right)}{dt}, \tag{10}$$

where $H_\varphi = -\varphi \ln \varphi$, and H_π^0 is found by the formula (6).

Thus, the entropy does not decrease: $\frac{dH_{\sigma\pi}}{dt} \geq 0$, if

$$\dot{\varphi}\left(\ln \varphi + 1\right) \leq \frac{d\left(\varphi H_\pi^0\right)}{dt}, \quad (\varphi \geq 0).$$

In the case of non-unit normalization (5), we define such a value $\varphi*$, when the maximum entropy for an arbitrary $\varphi > 0$ takes the highest possible value $H_{\max,\max}$. We get

$$H_{\max}(\varphi) = -\varphi \ln \frac{\varphi}{N}, \tag{11}$$

where $\pi_i = \frac{\varphi}{N}, \ \forall i \in \overline{1,N}$. let us find $\varphi*$ from the issue that $\frac{dH_{\max}(\varphi)}{d\varphi} = 0$, we find $\varphi* = e^{-1}.\cdot N$, then $H_{\max}(\varphi*) = \varphi*$.

As the $\left.\frac{d^2 H_{\max}(\varphi)}{d\varphi^2}\right|_{\varphi=\varphi^*} = -\frac{1}{\varphi*} < 0$, then the value $\varphi = \varphi*$, deliver the absolute maximum entropy $H_{\max,\max}$, and this maximum is equal to $\varphi*$.

The Table I shows the comparative values $H_{\max,\max}$ and $H_{\max}(\varphi = 1)$.

TABLE I. COMPARATIVE VALUES $H_{\max,\max}$ AND $H_{\mathrm{MAX}}(\varphi = 1)$

No	$\varphi* = \frac{N}{e} = H_{\max,\max}$	$H^{\circ}_{\max}(\varphi = 1)$
1	0.367879…	0
2	0.7357588…	0.6931471…
3	1.1036832…	1.0986122…
4	1.4716776…	1.3862943…
5	1.8393972…	1.60943791…
…	…	…

In order to determine at what "point" the difference $H_{\max}(\varphi)$ and $H_{\max,\max} = H_{\max}(\varphi*)$ reaches a minimum, consider the function $f(x) = \frac{x}{e} - \ln x$ on the semi-axis [1, +∞]. on the semi-axis $f'(x) = 0$ we find $x* = e$ and $f''(x*) > 0$, herein

$$\frac{x*}{e} - \ln x* = \frac{e}{e} - \ln e = 0.$$

Returning to the variable N, we find that

$$f(N = 3) = \frac{3}{e} - \ln 3 = 0.005023...$$

$$f(N = 2) = \frac{2}{e} - \ln 2 = 0.045061...$$

Thus, the entropy reaches the value $H_{\max\max}$, if

$$\sum_{i=1}^{N} \pi(\sigma_i) = H_{\max,\max}(N), \tag{12}$$

that is, if the distribution of preferences is normalized to the maximum possible entropy

$$\varphi* = \frac{N}{e} = H_{\max,\max}(N). \tag{13}$$

We see that there is a "distinguished" normalization and, in addition, the number of alternatives N is equal to the maximum possible entropy multiplied by the base of the natural logarithm e (Fig. 2). If in the case of such an extreme normalization all particular values of the preference function are the same, then they are equal $\pi(\sigma_i) = 1/e$.

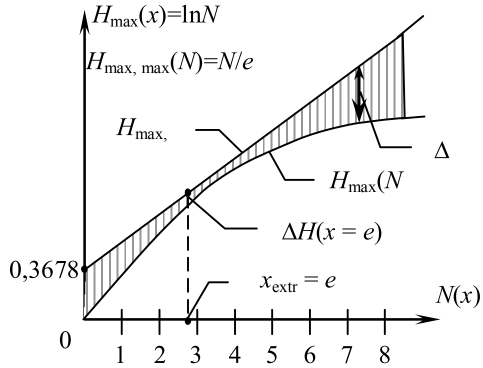

Fig. 2. Particular values of the function

Consider the same problem if is a real variable defined on the interval [0, a] and $\pi(\sigma)$ normalized per unit. Homogeneity condition $\pi(\sigma)$ is

$$\pi(\sigma) = \begin{cases} \frac{1}{a}, & \sigma \in [0, a], \\ \sigma, & \sigma \in [0, +\infty], \end{cases}$$

where $\pi(\sigma)$ - density of distribution of preferences. Let the normalization have the form:

$$\int_0^a \pi(\sigma)\, d\sigma = \varphi, \tag{14}$$

then under homogeneous conditions $\pi(\sigma) = \frac{\varphi}{a}$ and

$$H_{\max}(\varphi) = -\int_0^a \frac{\varphi}{a} \ln \frac{\varphi}{a}\, d\sigma = \varphi \ln \frac{\varphi}{a}.$$

Hence from the equation $\frac{dH_{\max}(\varphi)}{d\varphi} = 0$ we find $\varphi* = \frac{a}{e}$, then $H_{\max\max} = -\frac{a}{e} \ln \frac{a}{e} = \frac{a}{e}$.

As much information as possible when changing the area [0,a] on Δa or area [0, N] on value ΔN is

$$I(\Delta a) = \pm \frac{\Delta a}{e}, \qquad I(\Delta N) = \pm \frac{\Delta N}{e}.$$

When the number of alternatives changes by one $I(\Delta N = 1) = \pm(1/e)$.

Other indicators of preference uncertainty.

In this section, we consider some of the uncertainty indicators that have properties that make them closer to the entropy in the Chanon form. These properties include the following:

- for a singular distribution, the corresponding exponent vanishes,
- for an even distribution, it should be maximum.

Consider the following functions:

$$H_A = \sum_{i=1}^{N} (1-\pi_i)\pi_i, \qquad (15)$$

$$H_B = -\sum_{i=1}^{N} \left(1-e^{1-\pi_i}\right)\pi_i. \qquad (16)$$

In the case of a singular distribution H_A and H_B turns into zero: $H_A = 0$, $H_B = 0$.

When all alternatives are equally preferable (set Sa coincides with its unique equivalence class) from the normalization condition $\pi_i = \frac{1}{N}$ and $H_A = 1 - \frac{1}{N}$. If $N \to \infty$ $H_A = 1$.

We get $\Phi = \sum_{i=1}^{N}(1-\pi_i)\pi_i + \gamma\sum_{i=1}^{N}\pi_i$, where γ is Lagrange coefficient from the condition $\frac{\partial\Phi}{\partial\pi_i} = 0$ taking into account the normalization condition $\sum_{i=1}^{N}\pi_i = 1$, we find $\pi_{i_{\text{opt}}} = \frac{1}{N}$.

This value delivers the maximum entropy H_A.

If the normalization is not unit, then if $\sum_{i=1}^{N}\pi_i = \varphi$, so $\pi_{i_{\text{opt}}} = 1/N$, and maximal entropy gets the value

$$H_{A\max} = \varphi^2\left(1-\frac{1}{N}\right) \text{ or } \varphi = \sqrt{\frac{N \cdot H_{A\max}}{N-1}}.$$

We can see that in the first case $\left(\sum_{i=1}^{N}\pi_i = 1\right)$ when $N \to \infty$ $H_A = 1$, in the second case $\left(\sum_{i=1}^{N}\pi_i = \varphi\right)$ $H_A \to \varphi^2$.

The normalization condition is

$$\sum_{i=1}^{N}\pi_i = \sqrt{\frac{N \cdot H_{A\max}}{N-1}}. \qquad (17)$$

Consider the quasi-entropy H_B. In the case of a singular distribution H_B vanishes. With an increase in the number of alternatives N H_B growing monotonously. For uniform distribution $\pi_i = N^{-1}$, $\forall i \in \overline{1,N}$.

$$H_B^* = -\left(1-e^{1-\frac{1}{\pi}}\right),$$

whence it can be seen that $\lim_{N\to\infty} H_{B\max} = -(1-e) = 1.718281\ldots$

Function $H_C = 0$ for singular distribution

$$\pi(\sigma_i) = \begin{cases} 0, & \forall i \neq k, \\ 1, & i = k, \end{cases} \quad i,k \in \overline{1,N}.$$

In the case of a uniform distribution $\pi_i = N^{-1}$, ($\forall i \in \overline{1,N}$) $H_C = \left(1-\frac{1}{N}\right)^{\frac{1}{N}}$ when $N \to \infty$, when the function $H_C' = \sum_{i=1}^{N}\left(1-\pi_i^{\pi_i}\right)$ is also zero in the case of a singular distribution, but tends to ∞, if $N \to \infty$.

Sometimes it is convenient to use the normalized entropy, which is obtained by dividing by the maximum value:

$$\left.\begin{aligned} \bar{H}_\pi &= -\frac{1}{\ln N}\sum_{i=1}^{N}\pi_i \ln \pi_i, \\ \bar{H}_A &= \frac{N}{N-1}\sum_{i=1}^{N}\pi_i(1-\pi_i), \\ \bar{H}_B &= -\frac{1}{1-e^{1-N^{-1}}}\sum_{i=1}^{N}\left(1-e^{1-\pi_i}\right)\pi_i, \\ \bar{H}_C &= \frac{1}{1-\left(N^{-1}\right)^{N-1}}\sum_{i=1}^{N}\left(1+\pi_i^{\pi_i}\right)\pi_i. \end{aligned}\right\} \qquad (18)$$

One of the characteristics of entropy as a criterion of uncertainty is its sensitivity to changes in the value of preferences. In Figure 3 shows the sensitivity functions of three criteria H_π, H_A and H_B:

$$S_{H_\pi}^{\pi_i} = \frac{\partial \bar{H}_\pi}{\partial \pi_i}, \quad S_{H_A}^{\pi_i} = \frac{\partial \bar{H}_A}{\partial \pi_i}, \quad S_{H_B}^{\pi_i} = \frac{\partial \bar{H}_B}{\partial \pi_i}.$$

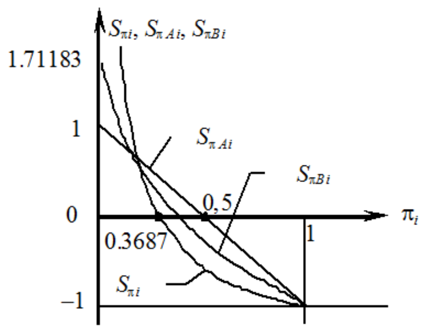

Fig. 3. Sensitivity functions of three criteria H_π, H_A and H_B

Along with the sensitivity functions, the uncertainty criteria can be characterized by the elasticity value $\varepsilon_H^{\pi_i}$. In this case, for three "entropies" elasticities are calculated by the formula:

$$\varepsilon_{H_\pi}^{\pi_i} = \frac{S_{H_\pi}^{\pi_i}}{H_\pi} \cdot \pi_i, \quad \varepsilon_{H_A}^{\pi_i} = \frac{S_{H_k}^{\pi_i}}{H_A} \cdot \pi_i, \quad \varepsilon_{H_A}^{\pi_i} = \frac{S_{H_A}^{\pi_i}}{H_A} \cdot \pi_i. \quad (19)$$

These functions are analogous to elasticities that are used in economic applications.

The entropy of Renyi and Tsallis. It is necessary to mention two more types of entropy: the Renyi entropy and the Tsallis entropy.

We will write down the appropriate formulas for the distribution of preferences. The Renyi entropy is determined by the following formula:

$$H_\alpha(\sigma) = \frac{1}{1-\alpha} \ln\left(\sum_{i=1}^{N} \pi(\sigma_i)^\alpha\right). \quad 20)$$

For uniform distribution $\pi(\sigma_i) = \frac{1}{N}$ we find $H_\alpha(\sigma) = \ln N$ with the normalization condition $\sum_{i=1}^{N} \pi(\sigma_i) = 1.$.

Entropy $H_\alpha(\sigma)$ in the case of a singular distribution $\pi(\sigma_i) = \begin{cases} 1, & i = q \\ 0, & \forall i \neq q \end{cases}$ $q \in \overline{1,N}$, is zero.

In case of even distribution $\pi(\sigma_i) = N^{-1}$, $\forall i \in \overline{1,N}$ is

$$\frac{1}{1-\alpha} \ln(N^{1-\alpha}) = \ln N. \quad (21)$$

If there are two distributions σ_i η_i, that "variety" is characterized by the Renyi divergence:

$$D_\alpha(\sigma,\eta) = \frac{1}{1-\alpha} \ln \sum_{i=1}^{N} \pi(\sigma_i)^\alpha \eta(\sigma_i)^{1-\alpha}, \quad (22)$$

or

$$D_\alpha(\eta,\sigma) = \frac{1}{1-\alpha} \ln \sum_{i=1}^{N} \pi(\sigma_i)^{1-\alpha} \eta(\sigma_i)^\alpha. \quad (23)$$

The entropy of Tsalissa is determined by the formula

$$H_q(\pi) = K \frac{1 - \sum_{q=1}^{N} P_i^q}{q-1}, \quad (24)$$

where $\sum \pi(\sigma_i) = 1, \quad q \in R, \quad K > 0.$

We see that for a singular distribution $H_q(\pi) = 0$, for even distribution

$$H_q(\pi) = \frac{1 - N^{1-q}}{q-1}. \quad (25)$$

If q tends to 1: $q \to 1$ the Tsallis distribution turns into the Shannon distribution:

$$\lim_{q\to1} H_q = K \lim_{q\to1} \frac{1 - \sum_{i=1}^{N} \pi(\sigma_i)\pi(\sigma_i)^{q-1}}{q-1}$$

$$= K \lim_{q\to1} \frac{1 - \sum_{i=1}^{N} \pi(\sigma_i) e^{(q-1)\ln \pi(\sigma_i)}}{q-1}$$

$$\approx K \lim_{q\to1} \frac{1 - \sum_{i=1}^{N} \pi(\sigma_i)\left(1 + (q-1)\ln \pi(\sigma_i)\right)}{q-1}.$$

Using Lonital's rule, and calculating the derivatives with respect to $t = q - 1$, we find

$$\lim_{q\to1} H_q = -K \sum_{i=1}^{N} \pi(\sigma_i) \ln \pi(\sigma_i). \quad (26)$$

IV. CONCLUSIONS

Among the considered analogs of entropy from a purely "technical" point of view, the HA function is the most convenient, since it leads to easily solvable linear relations when constructing models of preference functions based on variational principles.

However, considerations of a "technical" nature do not prevail in this case. We need more compelling reasons every time when it comes to choosing a criterion (or criteria) with which we are going to associate external manifestations of the psyche, such as, in particular, the process of forming preferences prior to making decisions.

Why are we highlighting the formation of preferences as the main stage in the decision-making process? If the preferences are formed, then the problem of choice is actually solved.

To realize the choice, a volitional effort is also necessary, and here the question naturally arises whether it is possible, within the framework of the developed formalism, to formalize the functioning of “Will” and, in general, what is “Will” from the standpoint of the entropic paradigm. In the roughest approximation, "Will" is interpreted as the ability to make decisions with a high degree of uncertainty, that is, with a high value subjective entropy. Of course, such a definition is not complete and contradicts, for example, factors such as imprudence, caution, irresponsibility when sending goods. The process of forming preferences is hierarchical, "heterogeneous" and includes many factors, stages, actions: experience, intuition, ethical considerations, solving various optimization problems, taking into account constraints, etc.

It is a priority clear that a choice is possible when the preference of alternatives has visible, clearly perceptible differences. Entropy or its analogs are criteria signaling this difference.

There is reason to believe that the role of entropy is not exhausted by this and that it is a value that reflects the state of the psyche and the dynamics of mental and behavioral phenomena.

REFERENCES

[1] A. A. Zhdanovsky, V. A. Shapovalov, G. A. Melnik, D. M. Zhirov, M. S. Prikhodko, and L.G. Odintsova, "Investigation of plasma-slag processes in special and ladle electrometallurgy of steel," *Modern electrometallurgy*. no. 1 (90), pp. 17–23, 2008.

[2] I.M. Makarov, Novoe v sinergetike. Zagadki mira neravnovesnykh struktur, [New in synergy. Mysteries of the world of nonequilibrium structures]. Moscow: Nauka, 1996, 263 p. [in Russian]

[3] D. Middleton, Introduction to the statistical theory of communication; New York, Toronto, and London: McGraw Hill, 1960, 1140 p.

[4] O. L. Korolev and M. Yu. Kussy, "Comparative Characteristics of Economy's Social Attributes of the System Levels of Hierarchy from the Standpoint of Economic and Mathematical Modeling," *Journal of Volgograd. State University. Economics*, 2019, vol. 21, no. 4, pp. 22–35. [in Russian]. https://doi.org/10.15688/ek.jvolsu.2019.4.3

[5] Ilya Prigogine, (2002). Advances in Chemical Physics. New York: Wiley InterScience. ISBN 978-0-471-26431-6. Archived from the original on 17 December 2012. Retrieved 29 July 2008.

[6] G. Haken, Synergetics; Naturwissenschaften, vol. 67, pp. 121–128, 1980.

[7] V. I. Ivanenko and V. Lobkovsky, *The problem of uncertainty in decision-making problems*. Kyiv: Naukova Dumka, 1990, 136 p.

[8] A. P. Levich and A. V. Solov'yov, "Cetegory-funcor modeling of natural systems," *Cybernetics and Systems*, vol. 30, no.6, pp. 571–585, 1999.

[9] R. L. Stratonovich, Izvest. VUZ Radiofiz., 8, 1965, pp. 116–141.

[10] Ingo Müller and Wolf Weiss, *Entropy and Energy: A Universal Competition* (Interaction of Mechanics and Mathematics) ISBN 10: 3540242813 ISBN 13: 9783540242819 Publisher: Springer, 2005

[11] V. Kasianov, *Subjective entropy of preferences. Subjective analysis: monograph*, Warsaw, Poland: Institute of aviation, 2013, 644 p.

[12] V. A. Kasianov, *Subjective analysis: monograph*, Kyiv: NAU, 2007, 512 p.

[13] V. A. Kasyanov, *Elements of subjective analysis: monograph*, Kyiv: NAU, 2003, 224 p.

[14] Leon Brillouin, *Science and Information Theory: Second Edition* (Dover Books on Physics) Second Edition ISBN-13: 978-0486497556 ISBN-10: 0486497550

Received March 26, 2021

Shevchuk Dmytro. ORCID 0000-0001-9911-7214. Doctor of Engineering Science.
Head of the Air Transportation Management Department. Professor.
Faculty Transport, Management and Logistics, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine, (2003).
Research area: methods of restoring the aircraft controllability in special situations in flight.
Publications: 129.
E-mail: dmitroshevchuk@gmail.com

Kasianov Volodymyr. ORCID 0000-0003-1318-9589. Doctor of Engineering Science. Professor.
Honored Professor of the National Aviation University.
Professor of Mechanics Department, National Aviation University, Kyiv, Ukraine.
Education: Nikolaev Shipbuilding Institute, Nikolaev, Ukraine, (1959).
Research area: hydro-gas dynamics, mathematical methods for modeling flight dynamics, identification of models of dynamic objects, methods for processing experimental information, analytical mechanics of dissipative systems.
Publications: 280.
E-mail: vakasyanov@mail.ru

Shevchenko Yuliya. ORCID 0000-0003-0160-3228. PhD in Economics. Associate Professor.
Air Transportation Management Department, Faculty Transport, Management and Logistics, National Aviation University, Kyiv, Ukraine.
Education: Dneprodzerzhinsk State Technical University, Dneprodzerzhinsk, Ukraine, (2001).
Research area: research of operations in transport systems, passenger and cargo air transport, sale of air transport.
Publications: 80.
E-mail: yulsashe@gmail.com

Д. О. Шевчук, В. О. Касьянов, Ю. В. Шевченко. Забезпечення доставки вантажу в умовах невизначеності
У статті розглянуто питання забезпечення доставки вантажів в умовах невизначеності, призначених для прогнозування часу виконання транспортного завдання. Вихідною інформацією для навчання моделі є дані носія про очікуваний середній час виконання завдання. Аналіз використовує ентропійний метод. Проведено аналіз отриманих результатів. Результати показують, що використання ентропійного методу дозволяє дослідити його чутливість до зміни величини переваг. У роботі над застосуванням ентропії використовуються 3 критерії: ентропія повинна бути мінімальною для чітко визначених величин, максимальною для рівноймовірних величин та універсальною – застосовною як для кінцевих, так і для нескінченних, дискретних і безперервних розподілів. При зміні значень параметрів ми використовували перехресну ентропію та квадратичну ентропію і в результаті отримали оцінку невизначених змінних, які можуть бути використані для вирішення транспортної задачі в умовах невизначеності.
Ключові слова: транспортна система; вантажна компанія; ентропія; транспортне завдання; розумна технологія.

Шевчук Дмитро Олегович. ORCID 0000-0001-9911-7214. Доктор технічних наук. Завідувач кафедри організації авіаційних перевезень. Професор.
Факультет транспорту, менеджменту та логістики, , Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна, (2003).
Напрям наукових досліджень: методи відновлення керованості літака в особливих ситуаціях в польоті.
Публікацій: 129.
E-mail: dmitroshevchuk@gmail.com

Касьянов Володимир Олександрович. ORCID 0000-0003-1318-9589. Доктор технічних наук. Професор.
Заслужений професор Національного авіаційного університету.
Кафедра механіки, Національний авіаційний університет, Київ, Україна.
Освіта: Миколаївський суднобудівний інститут, м. Миколаїв, Україна, (1959).
Напрям наукових досліджень: гідрогазодинаміка, математичні методи моделювання динаміки польоту, ідентифікація моделей динамічних об'єктів, методи обробки експериментальної інформації, аналітична механіка дисипативних систем.
Публікацій: 280.
E-mail: vakasyanov@mail.ru

Шевченко Юлія Вікторівна. ORCID 0000-0003-0160-3228. Кандидат економічних наук. Доцент.
Кафедра організації авіаційних перевезень, Факультет транспорту, менеджменту та логістики, Національний авіаційний університет, Київ, Україна.
Освіта: Дніпродзержинський державний технічний університет, Дніпродзержинськ, Україна, (2001).
Напрям наукових досліджень: дослідження операцій у транспортних системах, пасажирському та вантажному повітряному транспорті, продажі повітряного транспорту.
Публікацій: 80.
E-mail: yulsashe@gmail.com

Д. А. Шевчук, В. А. Касьянов, Ю. В. Шевченко. Обеспечение доставки груза в условиях неопределенности
В статье рассмотрен вопрос обеспечения доставки грузов в условиях неопределенности, предназначенных для прогнозирования времени выполнения транспортного задания. Исходной информацией для обучения модели являются данные носителя об ожидаемом среднем времени выполнения задания. Анализ употребляет энтропийный способ. Проведен анализ полученных результатов. Результаты показывают, что использование энтропийного метода позволяет изучить его чувствительность к изменению величины преимуществ. В работе над применением энтропии используются три критерия: энтропия должна быть минимальной для четко определенных величин, максимальной для равновероятных величин и универсальной – применимой как для конечных, так и для бесконечных, дискретных и непрерывных распределений. При изменении значений параметров мы использовали перекрестную и квадратическую энтропию и в результате получили оценку неопределенных переменных, которые могут быть использованы для решения транспортной задачи в условиях неопределенности.
Ключевые слова: транспортная система; грузовая компания; энтропия; транспортное задание; разумная разработка.

Шевчук Дмитрий Олегович. ORCID 0000-0001-9911-7214. Доктор технических наук.
Заведующий кафедрой организации авиаперевозок. Профессор.
Факультет транспорта, менеджмента и логистики, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина (2003).
Направление научных исследований: методы восстановления управляемости самолета в особых условиях полета.

Публикации: 129.
E-mail: dmitroshevchuk@gmail.com

Касьянов Владимир Александрович. ORCID 0000-0003-1318-9589. Доктор технических наук. Профессор.
Заслуженный профессор Национального авиационного университета.
Кафедра механики, Национальный авиационный университет, Киев, Украина.
Образование: Николаевский кораблестроительный институт, Николаев, Украина, (1959).
Направление научных исследований: гидрогазодинамика, математические методы моделирования динамики полета, идентификация моделей динамических объектов, методы обработки экспериментальной информации, аналитическая механика диссипативных систем.
Публикации: 280.
E-mail: vakasyanov@mail.ru

Шевченко Юлия Викторовна. ORCID 0000-0003-0160-3228. Кандидат экономических наук. Доцент.
Кафедра организации авиаперевозок, Факультет транспорта, менеджмента и логистики, Национальный авиационный университет, Киев, Украина.
Образование: Днепродзержинский государственный технический университет, Днепродзержинск, Украина, (2001).
Направление научных исследований: исследование операций в транспортных системах, пассажирский и грузовой авиатранспорт, продажа авиатранспорта.
Публикации: 80.
E-mail: yulsashe@gmail.com