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MATHEMATICAL MODEL FOR THE INVESTIGATION OF HUMAN ORGANISM FUNCTIONAL SELF-ORGANISATION

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Abstract**—***Mathematical modeling of processes occurring in living organism is convenient and reliable tool for the understanding of mechanisms of human organism self-organization, interaction and inter-influence of its functional systems. The simulations of processes occurring in organism during various extreme perturbations at mathematical models allow us to study the parameters of self-organization in these perturbations at the level unavailable currently for modern invasive methods as well as to predict the organism steady state at given level of perturbing effects. The objects of study were the reactions of respiratory and blood circulatory systems, because these systems, according to the theory of adaptation by F. Meerson, are the most sensitive to the disturbing effects of environment. The paper provides a brief overview of mathematical models of respiratory and blood circulatory system; in the construction of these models rather complex mathematical apparatus was used and, accordingly, the implementation of which requires significant computational resources. The mathematical model of the functional respiratory system was proposed; it is based on the principle of the main function of respiratory system realization and takes into account conflict situations that occur in organism during this function fulfillment. This conflict happens between the governing and executive self-regulatory organism organs as well as between the different tissues groups in their fight for the oxygen. Mathematically, the model is a system of ordinary nonlinear differential equations that describe the transport and mass transfer of respiratory gases in all structural parts of respiratory system. The task of control of gases dynamics in organism was solved using the principle of Pontryagin maximum.*

Index Terms—Functional respiratory system; controlled dynamic system; self-organization of respiratory system; operators of continuous interaction system; disturbing influence of environment.

I. INTRODUCTION

Mathematical and simulation models are widely used nowadays for the investigation of patterns of physiological processes. The advantage of their use to gain new knowledge is the ability to obtain information at the level inaccessible to modern invasive methods. This direction is based primarily on the works of P. K. Anokhin, whose main ideas were the theory of functional systems and the application of systems approach to the study of physiological functions.

In articles[1], [2], a coherent theory of organism adaptation was presented, in which the respiratory and blood circulatory system were distinguished among other functional systems, as ones that the most noticeably reacted to changes in human living conditions. It was shown too [3] that if human organism was presented as a chain with "weak link" (in terms of the theory of reliability), then such "weak link" will be the respiratory and blood circulatory systems.

Mathematical modeling is an effective tool that allows us to simulate extreme disturbances influences on the human organism and to predict its reactions to disturbances of the internal and external environment, while modern diagnostic methods characterize only the current state of organism. It should be noted that there are a large number of works related to the mathematical modeling of certain subsystems of organism and organism as a whole [5]. Among the models of respiratory system, the model of F. Grodins [6] become a real breakthrough in which the respiratory system is considered as a dynamic system, i.e. it makes it possible to use the appropriate mathematical apparatus.

II. PROBLEM STATEMENT

There are a large number of models of respiratory and blood circulatory systems, which use rather complex mathematical apparatus. Without touching "black box" models, which also have the right to exist because they allow to identify causal relationships and significant dependencies at the population level, but do not allow to analyze the processes occurring within the system, we will pay more attention to structural models, which are developed on the basis of laws and hypotheses on the structuring and functioning of biosystem.

Mathematical models of the respiratory system differ between themselves depending on the purpose of the study. The models of respiratory mechanics, in which the lungs are represented by elastic shells connected to the atmosphere by a tube with some hydraulic resistance [5], which allow to obtain the simplest relationships between physical parameters that characterize lung functions, but do not take into account the spatial heterogeneity of respiration in the human lung are widespread. [6] provides a brief overview of human lung models that vary in complexity, from the simplest, which are presented as a rigid container combined with the atmosphere, to a model in which volume and pressure change under the influence of muscle work, taking into account gas exchange with blood and blood perfusion. In article [7] the two-chamber model of lungs which consists of alveolar space through which the blood perfusion is carried out and the anatomic dead space is presented. This model is used to estimate the minute volume of blood circulation.

In article [8], a one-dimensional model of air transfer from the trachea to the alveoli is considered, taking into account the respiratory gases exchange with blood and blood perfusion. The paper assumes the correct dichotomy of airways and laminar airflows; it explains the reasons for the existence of exactly 23 generations of airways, although the Weibel model by itself was proposed much earlier [9]. Although we have to note that the assumption about the laminar style of flow in the airways was substantiated by domestic scientists on mathematical model [10] much earlier. Other authors came to the similar results too [11].

With the intensive progress of computational methods of gas dynamics and the means of their implementation the three-dimensional models of air flow began to develop, the air by itself is considered as multicomponent mixture of gases. An overview of these models is given in [12]. The same paper proposes a mathematical model of human respiratory system, which consist on three related sub-models that describe the respiratory process as a set of synchronized processes of gas dynamics in the bronchial system, gas movement in a deformable saturated sparse medium and diffusion. This model is positioned by the authors as sub-model of multilevel model of the whole human organism.

As for the mathematical models of blood circulatory system, there are also a large number of developments related to this topic. This is primarily due to the fact that the study of physiological and pathophysiological processes in the cardiovascular system is a topical issue in many modern studies [13]. Over the last thirty years, several key approaches have been developed to describe local and systemic processes related to blood flow, which have varying degrees of spatial detail, depending on the applied problem to be solved. Usually a mathematical apparatus is used, which includes algebraic and differential equations [14]. Averaged models of this type are not demanding on computing resources and contain a small number of parameters that are easily determined for a particular organism, but, unfortunately, reflect only the general physiological patterns [13].

More complex models require the use of more complex mathematical apparatus. Thus, a detailed description of blood flow in large vessels is carried out using the Navier–Stokes equation in two or three dimensional approximation [15]. This approach uses methods for solving nonlinear equations in partial derivatives in three-dimensional domains of complex shape [16], [17]. In this case, there is a problem of constructing three-dimensional geometry, which corresponds to the shape of the vessel or vascular bed. The use of two-dimensional or three-dimensional models also requires the setting of boundary conditions at the boundaries through which the blood flows, setting the rheological properties of blood, taking into account the mobility of the vascular wall, elastic properties of the wall, pressure of surrounding tissues, and etc. [13]. All this makes the use of such models quite inefficient; in addition, it requires the use of significant amount of computing resources. Although there is a suitable area for the application of such models: three-dimensional analysis of blood circulation parameters in the aorta [18], in the main cerebral vessels [19], in the aneurysm [20].

Summarizing all the above, it can be argued that the proposed models require the use of rather laborious mathematical apparatus and significant computing resources. In addition, they are not always justified from a mathematical point of view, based on a number of significant limitations. The questions of checking of adequacy of such models are present as well. Therefore, the scope of their application seems to be quite limited, there are some difficulties in the practical application of such models associated with input data obtaining. In addition, such models (at least some of them) are the parts of more complex formations, but it is not clear how they take into account the interaction and inter-influence with other functional systems of organism.

At the same time, there is a specific demand for mathematical models that could investigate the processes occurring in human organism at the level of predicting organism steady state in disturbances of various etiologies, the input data for which would be experimental data available for obtaining.

The purpose of the work was to develop mathematical model of the functional respiratory system to study its current state and to predict the parameters of self-organization of the human organism at a given level of disturbing effects.

III. Problem Solution

A. Model description

The main function of the respiratory system is the adequate delivery of oxygen in time to the tissues of the working organs and removal of carbon dioxide. The partial variables used for the estimation of the state of functional respiratory system are the partial pressures of respiratory gases in the airways and alveolar space and the tensions of respiratory gases in arterial and mixed venous blood and blood of tissues capillaries. Depending on the purpose of modeling, the apparatus of the theory of differential equations with concentrated or distributed parameters are used usually for the estimation of the functional state of respiratory and blood circulatory system [21], [22]. The dynamics of partial pressures and tensions of respiratory gases are described by a system of ordinary differential equations. The principle of material balance and continuity of flow was used for their construction.

Structurally, the respiratory system consists on the upper respiratory tracts; the alveolar space from which respiratory gases with the blood of the pulmonary capillaries enter the arteries; the arterial channels with which the gases are transferred to the tissues where the metabolism takes place (with consumption of oxygen, energy release for the vital functions, as well as release of carbon dioxide and water). Oxygen-depleted and saturated with carbon dioxide blood in venous channels returns to the alveolar space, where gas exchange takes place again and it is saturated with oxygen and gives off carbon dioxide.

Let suppose that $p_{RV}\,O_2$, $p_{RV}\,CO_2$ – partial pressures of oxygen and carbon dioxide in respiratory tracts, $p_A\,O_2$, $p_A\,CO_2$ – in the alveolar space, $p_a\,O_2, p_a\,CO_2$ is the tension of respiratory gases in the arterial blood, $p_{\bar{v}}\,O_2, p_{\bar{v}}\,CO_2$ – mixed venous blood, $p_{lc}\,O_2, p_{lc}\,CO_2$ – in pulmonary capillary blood, $p_{c_{t_i}}\,O_2, p_{c_{t_i}}\,CO_2$ – in tissue capillaries, $p_{t_i}\,O_2$, $p_{t_i}\,CO_2$ – in tissue fluid.

In general, the equation of the model can be written as follows:

$$\frac{dp_i\,O_2}{d\tau} = \varphi\left(p_i\,O_2, p_i\,CO_2, \eta_i, \dot{V}, Q, Q_{t_i}, G_{t_i}\,O_2, q_{t_i}\,O_2\right), \tag{1}$$

$$\frac{dp_i\,CO_2}{d\tau} = \psi\left(p_i\,O_2, p_i\,CO_2, \eta_i, \dot{V}, Q, Q_{t_i}, G_{t_i}\,CO_2, q_{t_i}\,CO_2\right), \tag{2}$$

where the functions φ and ψ are described in detail in [21] – [24]; $\dot{V}$ is the ventilation; η_i is the degree of saturation of hemoglobin by oxygen; Q is the volumetric rate of systemic and Q_{t_i} local blood circulation $q_{t_i}\,O_2$ is the rate of oxygen consumption by i th tissue reservoir; $q_{t_i}\,CO_2$ is the rate of carbon dioxide release in i th tissue reservoir. The rate $G_{t_i}\,O_2$ of flow of oxygen from the blood into the tissue and $G_{t_i}\,CO_2$ carbon dioxide from the tissue into the blood is determined by the ratio:

$$G_{t_i} = D_{t_i} S_{t_i} (p_{ct_i} - p_{t_i}), \tag{3}$$

where D_{t_i} are the permeability coefficients of gases through the air-hematic barrier, S_{t_i} is the surface area of gas exchange.

Here is the equation that characterizes the changes in the tensions of respiratory gases in the blood tissue capillaries and tissue fluid of organ:

$$\frac{dp_{ct_i}CO_2}{d\tau}=\frac{1}{V_{ct_i}\left(\alpha_{21}+\gamma_{BH}\cdot BH\dfrac{\partial z_{ct_i}}{\partial p_{ct_i}CO_2}\right)}\Big(\alpha_2 Q_{t_i}\left(p_a CO_2-p_{ct_i}CO_2\right)+\gamma_{BH}\cdot BH\cdot Q_{t_i}\cdot Hb\cdot Q_{t_i}\cdot z_a-G_{t_i}CO_2$$

$$+Q_{t_i}\cdot\gamma_{BH}\cdot BH\cdot(z_a-z_{ct_i})+(1-\eta_a)\cdot\gamma_{Hb}\cdot Hb\cdot Q_{t_i}\cdot z_a-(1-\eta_{ct_i})\cdot\gamma_{Hb}\cdot Hb\cdot z_{ct_i}+\gamma_{Hb}\cdot Hb\cdot V_{ct_i}\frac{d\eta_{ct_i}}{d\tau}\Big),$$

$$\frac{dp_{t_i}O_2}{d\tau}=\frac{1}{V_{t_i}\left(\alpha_1+\gamma_{Mb}\cdot Mb\dfrac{\partial\eta_{t_i}}{\partial p_{t_i}O_2}\right)}\cdot\left(G_{t_i}O_2-q_{t_i}O_2\right),$$

$$\frac{dp_{t_i}CO_2}{d\tau}=\frac{1}{V_{t_i}\alpha_2}\left(G_{t_i}CO_2+q_{t_i}CO_2\right),$$

where

$$\eta_{ct_i}=1-1.75\exp(-0.052m_{ct_i}p_{ct_i}O_2)+0.75\exp(-0.12m_{ct_i}p_{ct_i}O_2),$$

$$m_{ct_i}=0.25(pH_{ct_i}-7.4)+1,$$

$$pH_{ct_i}=6.1+\lg\frac{BH}{\alpha_2 p_{ct_i}CO_2}.$$

$$z_{ct_i}=\frac{p_{ct_i}CO_2}{p_{ct_i}CO_2+35}.$$

where $\alpha_1,\alpha_2,\alpha_{1t_i},\alpha_{2t_i}$ are coefficients of solubility of respiratory gases in blood and tissue fluid; Q_{t_i} is the volumetric velocity of blood circulation in the capillary bed of the tissue reservoir t_i; V_{ct_i}, V_{t_i} is the volume of blood and tissue fluid, respectively.

The participation of biochemical structures – hemoglobin, myoglobin and buffer bases in the processes of mass transfer of gases add significant nonlinearity into the system of differential equations. Naturally, this makes serious difficulties for the mathematical analysis of the dynamic system, but is a very powerful mechanism for maintaining of organism gas homeostasis, and from the standpoint of control theory – the control mechanism.

Using the above model, the local and systemic blood circulations during exercise and hypoxic hypoxia were calculated. The calculated data differed from those obtained experimentally, and they did not answer a number of theoretical and applied questions, for example, they did not explain the causes of tissue hypoxia during low-intensity muscle work in hypoxic environment, when there are still reserves for the growth of systemic circulation, the role of the hypercapnic stimulus of regulation known in physiology, at hypoxia of loading, hypoxic hypoxia, and etc. That is why a more general model of gas dynamics control in the organism was proposed using Pontryagin maximum principle [25]. This model is based on the following principles.

The system which is modeled is considered as self-organized. Respectively, the model was also formulated as a model of self-organization of blood circulation. Self-organization means the ability of the model when the perturbation to change the parameters of the system is so, that the effect of perturbations was insignificant. At the same time, certain quality criteria should be minimized. The control in such systems should be carried out with the resolution of arising conflict situations of various nature.

Let's formulate the problem of system control (1) and (2) as follows:

- the initial state of the system is set by the partial pressures and tensions of respiratory gases in all parts of the system;
- area of change of control parameters is:

$$\left.\begin{array}{l}\dot V_{\min}\le\dot V\le\dot V_{\max},\\ Q_{\min}\le Q\le Q_{\max},\\ Q_{t_i\min}\le Q_{t_i}\le Q_{t_i\max},\quad i=\overline{1,m}\\ \sum\limits_{i=1}^{m}Q_{t_i}=Q,\end{array}\right\}; \qquad (4)$$

- terminal set of the states due to the relations:

$$\left.\begin{array}{l}\left|G_{t_i}O_2-q_{t_i}O_2\right|\le\varepsilon_{t_i}O_2,\quad i=\overline{1,m},\\ \left|G_{t_i}CO_2+q_{t_i}CO_2\right|\le\varepsilon_{t_i}CO_2,\quad i=\overline{1,m},\end{array}\right\} \qquad (5)$$

where $\varepsilon_{t_i}O_2$, $\varepsilon_{t_i}CO_2$, $i=\overline{1,m}$ are rather small positive values. The solution of the formulated problem will be any set of values of the controlling $\dot V$, Q, Q_{t_i}, $i=\overline{1,m}$ from (4). Let's assume that the optimal control parameters $\dot V$, Q, Q_{t_i}, $i=\overline{1,m}$ from (4), which provide a minimum of functional:

$$I = \min_{\substack{0 \le \dot{V} \le \dot{V}_{\max} \\ 0 \le Q_{t_i} \le Q_{\max}}} \int_{\tau_0}^{T} \left[\rho_1 \sum_{t_i} \lambda_{t_i} \left(G_{t_i} O_2 - q_{t_i} O_2 \right)^2 + \rho_2 \sum_{t_i} \lambda_{t_i} \left(G_{t_i} CO_2 + q_{t_i} CO_2 \right)^2 \right] d\tau, \quad i = \overline{1, m}, \tag{6}$$

on the trajectories of the perturbed dynamic system, ρ_1 and ρ_2 are coefficients of sensitivity of the organism to lack of oxygen and carbon dioxide excess. Coefficients λ_{t_i} are formed during evolution.

It is known that damage of the heart muscle, brain tissues, liver, kidneys and some others leads to the death, and perhaps this is why the density of capillaries in them is quite high. In mathematical modeling, the dependence is accepted

$$\lambda_{t_i} = \phi\left(V_{ct_i} / V_{t_i} \right).$$

The quadratic function ϕ characterizes the degree of filling by the blood of unit volume of tissue reservoir.

Let's emphasize that the formulation of the optimal control problem (1) – (6) is so that gas homeostasis is understood as the relative constancy of oxygen and carbon dioxide tensions, which is in the compromise formation of disturbance-appropriate levels of homeostasis in resolving conflicts of both regional and systemic nature. The Fick ratio can be used to calculate how much it is necessary to increase the volume of blood circulation through the working skeletal muscles in order to maintain the oxygen tension in them at a constant level. When comparing the calculated data with the experimental ones, it is appeared that the first ones exceed significantly the experimental values. According to the proposed model, this is due to ignoring the nature of the conflict that arises in organism between the groups of working tissues and the heart muscle, which provides the necessary cardiac output. In fact, such situations occur every time with the changes of organism's living conditions. An increase in muscle work intensity requires a corresponding increase in muscle circulation (otherwise there will be oxygen deficiency in the muscles) and can be achieved by changing the systemic circulation or its redistribution. In the first case, the intensity of the heart muscle increases (oxygen deficiency appear in it), in the second - a decrease in blood circulation in the tissue reservoirs of other organs, which at a constant rate of oxygen consumption leads to the development of tissue hypoxia. Thus, changing the conditions of the external or internal environment to maintain gas homeostasis in one muscle group requires an increase in blood circulation, which is contrary to the interests of other tissues, because it leads to oxygen deficiency. The solution to the conflict is to find a compromise in which all tissues, on average, sense the lack oxygen and their average oxygen tensions decreases. In the model, this is represented by the introduction of the dependence of the rate of oxygen consumption by the heart muscle on the volumetric rate of systemic circulation

In order to take into account the conflict of situations between the executive organs of self-regulation (respiratory muscles, heart muscle, vascular smooth muscle), which are also consumers of oxygen, and other tissues and organs, the following ratios were introduced

$$q_{\text{resp. m}} O_2 = f(V), \quad q_{\text{card. m}} O_2 = \varphi(Q),$$
$$q_{\text{smooth. m}} O_2 = \psi(Q).$$

In this formulation of the problem of mass transfer of gases process regulating, we can talk about the optimal in relation to criterion (6) choice of the volumetric velocity of blood circulation in organism. The accepted form of setting of problem of control is consistent with the conceptual models that currently exist in modern respiratory physiology. It is only important to make sure that the set of solutions to the problem that is formulated is not empty.

It should be noted that the model is used to study the current state and to predict the parameters of self-organization of the main functional systems of organism for various perturbations, and as initial data were used the data obtained by normal physiological examination, namely – minute volume of respiration, gas composition of alveolar and exhaled air, frequency of respiration, minute blood volume, data on blood acidity and hemoglobin, parameters of the environment in which the examination takes place, and etc. [26]. At the output of the model we obtain the data on the hypoxic state in all parts of respiratory system, which allows us to make the conclusions about the nature of adaptation of particular organism to perturbations. An example of such applications were given in [27, 28] the tasks for determining the optimal parameters of self-organization of operators of continuous interaction systems in conditions of increased situational stress in decision-making. In particular, in [29] the dependence of blood circulation in the brain of operator of continuous interaction with increasing intensity of professional activity was studied, in [27], [28] the role of separate stimuli of respiratory regulation in decision makers under the stressed situational circumstances was studied; the leading role of hypercapnic regulatory stimulus was demonstrated.

B. Results of the simulation

An example of the application of the model are the tasks described in [29], [30] to determine the optimal parameters of self-organization of operators of continuous interaction systems in conditions of high situational stress in decision making. For example, in [31] the dependence of blood circulation in the brain of operator of continuous interaction system was studied; i.e. with the increasing of intensity of professional activity. A number of computational experiments were carried out. Thus, for the organism of average person, the conditions of increased situational stress were simulated by introducing an increased rate of oxygen consumption by brain tissues. It was demonstrated that maintaining of given level of pO_2 in brain tissue with increasing of the rate of oxygen consumption by the brain to 20% of the resting level is possible without the connection of compensatory reactions from the respiratory and circulatory systems. An increase in the rate of oxygen consumption in operator's brain by 30–70% requires a corresponding linear increase in blood circulation in brain tissues, further increase in oxygen demand requires significant nonlinearity of blood circulation growth in brain tissues. With the connection of the compensatory response of the external respiratory system, the growth of blood circulation in brain tissues slows down, but the overall dependence does not change. Even more interesting were the results of the study of the role of individual stimuli of respiratory regulation in decision makers in stressful circumstances [29], [30]; the decisive role of hypercapnic stimulus of respiratory regulation was demonstrated (its contribution is 93%). The optimal values of the control parameters obtained during the simulation were in good agreement with the results of experiments.

IV. CONCLUSIONS

The mathematical model of the functional respiratory system for the investigation of the current state and prediction of the parameters of self-organization for operators of continuously interacting systems in difficult situational decision-making conditions were suggested in present article. The general model consists on the models of transport and mass exchange of respiratory gases and self-regulation of respiratory and blood circulatory systems. The executive mechanisms of self-regulation were the respiratory muscles, heart muscle and vascular smooth muscle. Respectively, the parameters of self-regulation were alveolar ventilation, systemic circulation and organ blood circulations. The general description of the model and fragment of the model for the area of blood tissue capillaries were suggested. The examination of conflict situations between the executive bodies of self-regulation and the studying of rates dependences for the oxygen consumption in these organs were done.

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Н. І. Аралова, О. М. Ключко, В. Й. Машкін, І. В. Машкіна. Математична модель для дослідження функціональної самоорганізації организму людини

Математичне моделювання процесів, що відбуваються в живому організмі, є простим і надійним інструментів для пізнання механізмів самоорганізації організму людини, взаємодії і взаємовпливу його функціональних систем. Крім того, імітація на математичної моделі процесів, що відбуваються в організмі при різних екстремальних впливах, надає можливість досліджувати параметри самоорганізації при цих впливах на тому рівні, який у в даний час є недоступним для сучасних інвазивних методів та прогнозувати стаціонарний стан організму при заданому рівні збурювальних впливів. Об'єктом даного дослідження були обрані функціональні системи дихання і кровообігу тому, що відповідно до теорії адаптації Ф. Меєрсона саме ці системи найбільш помітно реагують на збурюючі впливи зовнішнього середовища. У роботі представлена математична модель функціональної системи дихання, що ґрунтується на принципі здійснення основної функції системи дихання і враховує конфліктні ситуації, що виникають в організмі при реалізації цієї функції: між керуючими і виконавчими органами саморегуляції і між усіма органами та тканинами в боротьбі за кисень. Запропонована математична модель є системою звичайних нелінійних диференціальних рівнянь, що описують транспорт і масообмін респіраторних газів у всіх структурних ланках системи дихання. Задача керування динамікою газів розв'язувалася із застосуванням принципу максимуму Понтрягіна.

Ключові слова: функціональна система дихання; керована динамічна система; самоорганізація системи дихання; оператори системи неперервної взаємодії; збурюючий вплив середовища.

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Н. И. Аралова, Е. М. Ключко, В. И. Машкин, И. В. Машкина. Математическая модель для исследования функциональной самоорганизации организма человека

Математическое моделирование процессов, происходящих в живом организме, является простым и надежным инструментов для познания механизмов самоорганизации организма человека, взаимодействия и взаимовлияния его функциональных систем. Кроме того, имитация на математической модели процессов, происходящих в организме при различных экстремальных воздействиях предоставляет возможность исследовать параметры самоорганизации при этих воздействиях на уровне, в настоящее время недоступном современным инвазивным методам и прогнозировать стационарное состояние организма при заданном уровне возмущающих воздействий. В качестве объекта данного исследования выбраны функциональные системы дыхания и кровообращения потому, что в соответствии с теорией адаптации Ф. Меерсона именно эти системы наиболее заметно реагируют на возмущающее воздействие окружающей среды. В работе представлена математическая модель функциональной системы дыхания, базирующаяся на принципе выполнения основной функции системы дыхания и учитывающая конфликтные ситуации, возникающие в организме при осуществлении этой функции: между управляющими и исполнительными органами саморегуляции и между всеми органами и тканями в борьбе за кислород. Предложенная математическая модель является системой обыкновенных нелинейных дифференциальных уравнений, описывающих транспорт и массообмен респираторных газов во всех структурных звеньях системы дыхания. Задача управления динамикой газов решалась с применением принципа максимума Понтрягина.

Ключевые слова: функциональная система дыхания; управляемая динамическая система; самоорганизация системы дыхания; операторы системы непрерывного взаимодействия; возмущающее воздействие среды.

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