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LINES WITH CONTROLLED CURVATURE CONSTRUCTED MEANS OF MATHEMATICAL PACKAGE MAPLE

Abstract: The method of description and construction of curves is considered on a plane, the curvature calculated with a positive degree is included in natural equations of that.

Keywords: curves with the guided curvature, classic curvature, law of curvature change, elastiques of euler, maple.

Statement of the problem. In some calculations to describe curves and images appropriate to apply their natural equation [1]. In other words, these equations, which describe some curvature curve depending on its natural parameter *s,* which determines the length of the curve from a point of it. For example, thanks to the natural curves of the two equations can "manage" the process of their mutual touch during the run-in using a planetary gear.

Analyse existing research and publications. Curves Research in natural equation which includes trigonometric function cosine dedicated work Krause [1]. The equations of plane curves with natural equations are devoted S.F.Pylypaky [2] S.Yu.Badaeva, T.Y.Schoholevoyi [3] V.D.Borysenka, S.A.Ustenka, VE Spitsyna [4] and other authors.

A further generalization curves with controlled curvature will study and implement in practice more complex dependency entry in the classic description of the resulting curvature curves.

The wording of Article goals. Consider the case when the formula describing the resulting curvature lines are calculated with a positive exponent (type $k^w(s)$).

The main part. It is known [5] that the natural equation of the curve in the plane has the form

$$\frac{1}{R} = f(s) \tag{1}$$

where R - radius of the circle tangent to a smooth curve at a certain point it, and the function $f(s)$ determines the variation of curvature of the line (ie, the value $k = 1/R$), depending on the natural parameter *s.* By definition (classical) curvature of the arc of the curve at point M is called the value

$$k = \lim_{\Delta s \to 0} \left| \frac{\Delta \alpha}{\Delta s} \right| \tag{2}$$

where α - the angle between the tangents at the points M and M_Δ, and Δs - arc length MM_Δ.

Using the original formula (2) can be written as

$$k = |\dot{\alpha}| \qquad (3)$$

where $\alpha(s)$ - angle of the tangent at point M, which depends on the path traversed by the curve - Derivative of the function $\alpha(s)$ on the parameter s.

Equation (3) helps to describe the curve that a given natural equation. Indeed, the equation function is evaluated

$$\alpha(s) = \int_0^s f(s)ds + \alpha_0 \qquad (4)$$

whereby the angle of the tangent is calculated based on the length of the arc.

Then according to the known [4] dependencies

$$\frac{dx}{ds} = \cos\alpha \text{ and } \frac{dy}{ds} = \sin\alpha \qquad (5)$$

parameterization of the curve, you can specify as

$$x(s) = \int_0^s \cos\alpha(s)ds + x_0, \quad y(s) = \int_0^s \sin\alpha(s)ds + y_0 \qquad (6)$$

or, taking into account (4), the desired equation of the curve has the form

$$x(s) = \int_0^s \cos\left(\int_0^s f(s)ds + \alpha_0\right)ds + x_0 \ ,$$

$$y(s) = \int_0^s \sin\left(\int_0^s f(s)ds + \alpha_0\right)ds + y_0 \qquad (7)$$

It is easy to see that the rotation curve alters the formulas (7) α constant $_0$ and parallel transfer curve associated with a change in parameters x_0 and y_0. Equality of these constants to zero determines the position of the curve so that the starting point of the arc point is located at the origin, and the positive direction of the tangent it coincides with the positive direction of the axis Ox.

In the course of differential geometry proved [4] that every equation of the form (1) where the function $f(s)$ is continuous, can be taken as some natural equation of the curve, which can be described in parametric form. Canvas proof of this statement is as follows. In equation (1) is determined for α first look at the formula (4), and the formulas for x and y are defined by equations (6) - that is, the result is a parametric representation of the curve. If the expression (6) differentiate the variable s, we arrive at the equation (4), where

$$ds^2 = dx^2 + dy^2 \ . \qquad (8)$$

That really is the differential ds arc of the curve, and s - arc curve.

From the relations (5), it follows that α is the angle of contact of the curve with the axis of x. After differentiating (4) we find that the curvature is equal to

$$\frac{d\alpha}{ds} = f(s) \ . \tag{9}$$

That is, we conclude that equation (1) turns out to be really natural curve equation.

A further generalization curves with controlled curvature will introduce the concept of the degree of curvature of the line.

Definitions. Let curvature along the curve varies according to the law $k^w(s)$, where $w > 0$ and s - a natural setting. With Set expression as

$$\alpha_w(s) = \int_0^s k^w(u)du \tag{10}$$

Then w is a parameter exponent curvature line $\{x\ (s),\ y\ (s)\}$, obtained as a result of solving the differential equations

$$\frac{d}{ds}x(s) = \cos\big(\alpha_w(s)\big), \qquad \frac{d}{ds}y(s) = \sin\big(\alpha_w(s)\big) \ . \tag{11}$$

Specify that the curvature of the line of the second degree is used in constructing the elastic Euler [6], and curvature lines more high powers are used, for example, the construction of the tracks of elementary particles in magnetic fields.

To build a positive degree of curvature curves were drawn Maple-program fragment in which a procedure function is:

```
profil: = proc (kap, a, b)
local sys, p, ics, p1:
sys: = diff (alpha (s), s) = kap (s) ^ w,
diff (x (s), s) = cos (alpha (s)),
diff (y (s), s) = sin (alpha (s)):
krai: = x (0) = 0, y (0) = 0, alpha (0) = 0;
ans: = dsolve ({sys, krai}, {x (s), y (s), alpha
(s)},
type = numeric);
pic: = odeplot (ans, [x (s), y (s)], s = a .. b,
numpoints = 12000, thickness = 2, color = black):
display (pic, scaling = constrained);
end:
```

Here through **kap** is (classical) curvature of the curve, because **a** and **b** - the border of the parameter of arc length, the system of differential equations Freinet marked through **sys,** the solution of this system indicated through **ans,** and prepared to render information marked as **pic.**
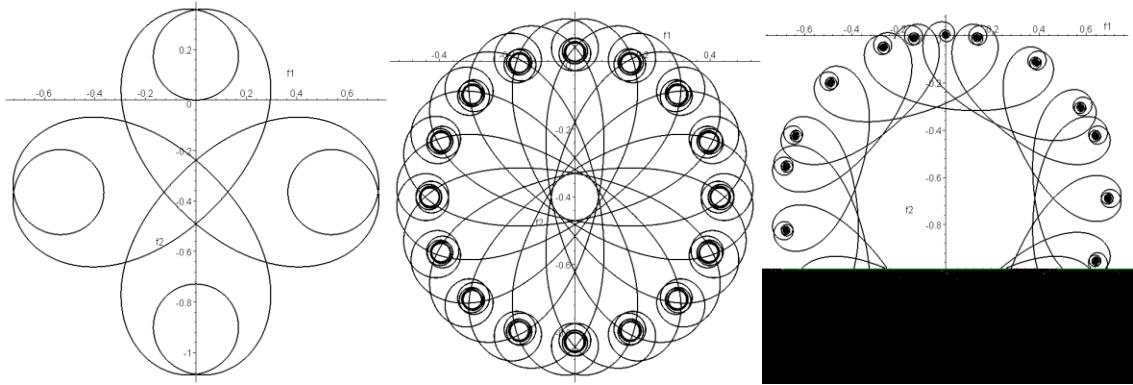
Apply the procedure should function as follows:

```
w: = 2.:
kap: = s -> 1 + 5 * cos (s) ^ 2;
profil (kap, 0, 6 * Pi);
```

As a result, the system of Cartesian coordinates $\{x,\ y\}$ is constructed lines that match the parameters **kap** and **w.**

Example *1*. Suppose $(5\cos^2(s) + 1)^w$ is a description of the law of change of curvature along the *line*. Fig. 1 shows the relevant lines of different degrees of curvature.
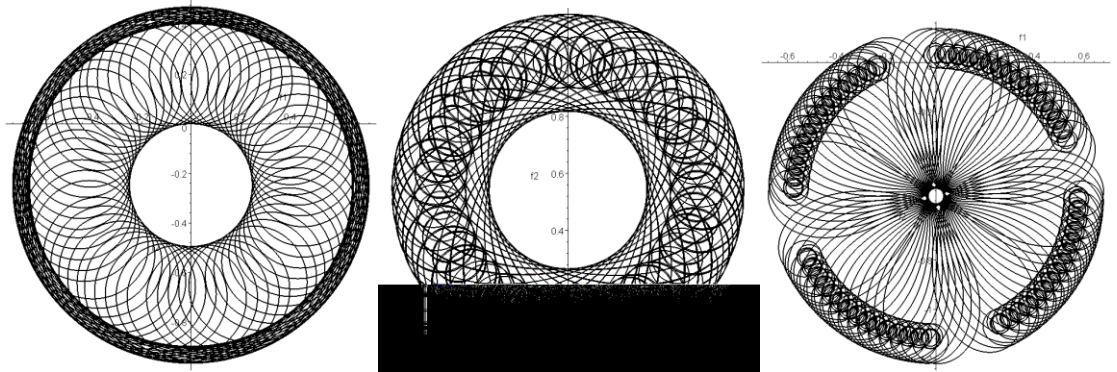

$$w = 1, 0 \le s \le 6\pi \qquad w = 2, 0 \le s \le 6\pi \qquad w = 3, 0 \le s \le 60\pi$$

Rice. 1. The curvature along the line varies according to the law $(5\cos^2(s) + 1)^{w.}$

Example 2. Let $(5\cos^2(s) / 4 + 1)^w$ is a description of the law of change of curvature along the *line*. Fig. 2 shows the corresponding lines of different degrees of curvature.
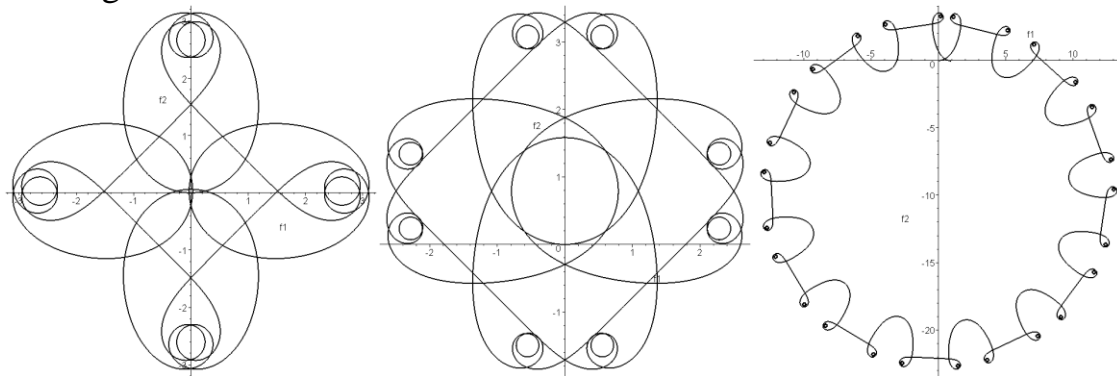

$$w = 2, 0 \le s \le 60\pi \qquad w = 3, 0 \le s \le 60\pi \qquad w = 4, 0 \le s \le 60\pi$$

Rice. 2. Curvature along the line changes
by law $(5\cos^2(s) + 1)^{w.}$

Example 3. Let $(\cos^2(s/2) + \sin^2(s/3))^w$ is a description of the law of change of curvature along the *line*. Fig. 3 shows the corresponding lines of different degrees of curvature.

$$w = 2, 0 \le s \le 14\pi \qquad w = 3, 0 \le s \le 60\pi \qquad w = 4, 0 \le s \le 78\pi$$

Rice. 3. Curvature along the line changes
by law $(\cos^2(s/2) + \sin^2(s/3))^{w.}$

Example 4. Let $(3\cos^2(s/2) + 2\sin^2(s/3))^w$ is a description of the law of change of curvature along the *line*. Fig. 4 shows the corresponding lines of different degrees of curvature.
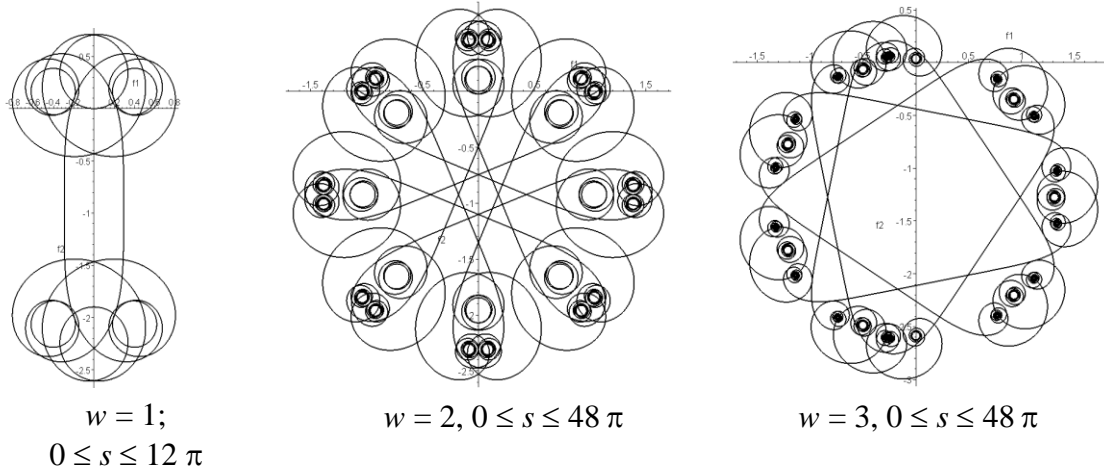

$w = 1;$
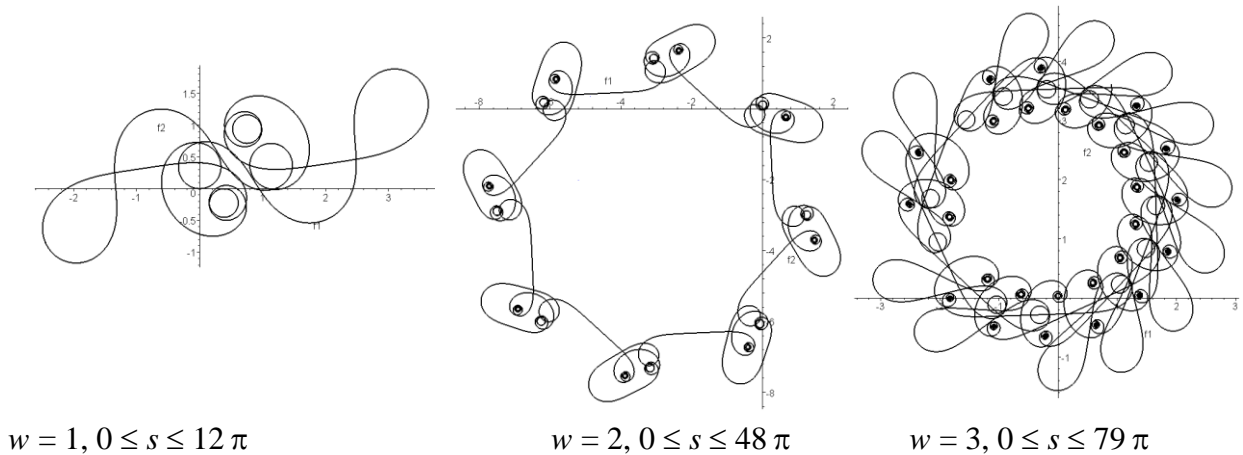$0 \le s \le 12\pi$ $w = 2, 0 \le s \le 48\pi$ $w = 3, 0 \le s \le 48\pi$

Rice. 4. Curvature along the line changes
by law $(3\cos^2(s/2) + 2\sin^2(s/3))^w.$

Example 5. Let $(3\cos^2(s/2) + 2\sin^3(s/3))^w$ is a description of the law of change of curvature along the *line*. Fig. 5 shows the corresponding lines of different degrees of curvature.

$w = 1, 0 \le s \le 12\pi$ $w = 2, 0 \le s \le 48\pi$ $w = 3, 0 \le s \le 79\pi$

Rice. 5. Curvature along the line changes
by law $(3\cos^2(s/2) + 2\sin^3(s/3))^w.$

Despite the "ornamentaly" character lines of different degrees of curvature can be used in various fields of science. For example, in the Yu.L.Sachkova [6] lists the following use cases elastic $(w = 2)$:

- theory of elasticity and strength of materials (modeling form columns, beams or elastic rod);
- Size and shape in biology (curvature of the spine, mechanical insect wings);
- analogues splines and approximation (H.Birkhof, CR de Boer, 1964);
- restore damaged images and restore hidden images in computer vision robots (D.Mamford, 1994);

- modeling of thin optical fibers and flexible connections in microelectronics (V.Dzheirezbhoy, 2008);
- axis vortex dynamics and the Schrödinger equation (H.Hasimoto, 1971);
- Modeling DNA (R.S.Manninh, 1996).

Summary: Euler elastic can be used in road transport to calculate the optimal path of motion of the vehicle (or mobile robot) on the plane, and as for the development of control car train with two or three trailers, which is especially important in the case of a combination of movement in reverse.

Prospects for further research. Refers to the curves described used to design object design. Specification of the direction of their use - further objective of the study.

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Аннотация

Куценко Л.Н., Челомбитько В.Ф. Линии с управляемыми кривизнами, построенные средствами математического пакета maple. В статье рассмотрен способ описания и построения кривых на плоскости, в натуральные уравнения которых входит кривизна, заранее вычисленная с положительной степенью.

Ключевые слова: кривые с управляемой кривизной, классическая кривизна, закон изменения кривизны, эластики эйлера, maple.

Анотація

Куценко Л.М., Челомбітько В.Ф. Лінії з керованими кривизнами, побудовані засобами математичного пакету *maple*: розглянуто спосіб опису й побудови кривих на площині, у натуральні рівняння яких входить кривизна, завчасно обчислена з позитивним степенем.

Ключові слова: криві з керованою кривизною, класична кривизна, закон зміни кривизни, еластики ейлера, *maple*.