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GAIN-SCHEDULING OPTIMISATION OF PITCH HOLD MODE BASED ON GENETIC ALGORITHMS

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Abstract. This paper presents an algorithm that was developed to select the best controller gains. The method is based on solving robust optimization problem for family of gain-scheduled PID-controllers. Parametric robust optimization provided by using genetic algorithm approach. This technique is useful to obtain flexibility and robustness in the controller automatic design.

Keywords: gain-scheduled controller; genetic algorithm; PID-controller; robustness; unmanned aerial vehicle

1. Introduction

The aircraft studied in the paper is the Unmanned aerial vehicle (UAV) Aerosonde for which the Aerosim MATLAB toolbox was developed. Such UAVs are subjected to various disturbances within the flight envelope. Due to gain scheduling control significantly depends on the flight regime and PID-controller can effectively provide pitch hold mode. To satisfy the UAV usage range, it is proposed to use gain-scheduled PID-controller instead of singular one.

The genetic algorithms (GAs) are powerful widely applicable tool in many control engineering problems. According to the GA-based approach, the task of optimization design of the dynamic PID-controller is reduced to the problem of the research for global minimum of a multivariable function. The GA optimization procedure is adopted to ensure the control system has the best disturbance rejection performance and robustness for any perturbed model of the closed loop system.

2. Analysis of recent research and publications

The gain-scheduled (GS) control problem has been widely discovered both from theoretical and practical viewpoint, see, for example, [1]. An effective approach to solve the nonlinear control problem is using gain scheduling with linear controllers. For example in an aircraft flight control system, the altitude and Mach number might be the scheduling variables with different linear controller parameters available for various combinations of these two variables [2,3]. In the present paper proposed alternative scheduling parameter, which is the dynamic pressure (DP) \( \bar{q} \). The main advantage of DP, that this parameter is related with both flight altitude and speed values.

3. Problem statement

As the above mentioned the DP \( \bar{q} = \frac{\rho v^2}{2} \) is chosen as scheduling value. To prove the efficiency of the proposed technique, the pitch hold mode is considered as a case study. For the reason to find the compromise between the disturbance rejection performance and robustness, it is used a multi-objectives optimization problem, based on insert several objectives in one cost function and try to satisfy them at the same time [4,5]. The GA algorithm provides a multi-directional random search and an informational exchange among best solutions. Mentioned properties ensure generation of search directions in order to avoid local minima.

4. Gain-scheduled PID-controller design based on GA

A PID-controller which is one of the most powerful but complex controller mode operations combines the proportional, integral, and derivative modes. The analytical expression of PID-controller [6] is given as,

\[
W_{C_{\text{PID}}}(z) = P_{\text{pid}}(\bar{q}) + I_{\text{pid}}(\bar{q}) \cdot T \cdot \frac{z}{z-1} + D_{\text{pid}}(\bar{q}) \cdot \frac{z-1}{z}
\]

The scheme of a pitch rate feedback loop is shown on fig.1. Accordingly, the controller gain matrix has the following form \( K = [P_{\text{pid}}, I_{\text{pid}}, D_{\text{pid}}, K_q, K_b] \).
A gain scheduling control system design takes following steps:

1. Obtain a plant model for each operating region. It needs to linearize the plant at several equilibrium operating points.
2. Design a family of linear PID-controllers for the obtained plant models.
3. Implement a scheduling mechanism. It means that the controller coefficients are changed based on the values of the scheduling variables.
4. Provide optimization procedure to family of controllers.
5. Assess control performance with simulation.

5. GA optimization approach

The space of all possible solutions are sets of the controller parameters (chromosomes). In other words we have subspace with five dimensions:

\[
(P_{\text{min}}, P_{\text{max}}) \times (I_{\text{min}}, I_{\text{max}}) \times (D_{\text{min}}, D_{\text{max}}) \times (K_{\text{min}}, K_{\text{max}}) \times (K_{\theta_{\text{min}}, \theta_{\text{max}}})
\]

First of all, it is necessary to generate the start parent population of chromosomes and their fitness function calculation. Each chromosome represents a string of five controller parameters: proportional, integral, derivative, pitch and pitch rate gains. The cost function (fitness function) values are calculated for each string. If any of chromosomes achieves the best value, than without any change it moved to the next generation. Next a new „reproduction” group of chromosomes selected either according to their fitness values, or randomly selected, or selected combining both methods, etc. are used for crossover and mutation operations. After each step we have a new completed parent population. The algorithm exit condition can be following: the fitness function of the best string in some population fulfills the predefined condition or until the predefined number of populations is put into life.

6. Parametric robust optimization

The used method is based on $H_\infty / H_2$ -robust optimization [7, 8].

1. $H_2$ -norms of a model in deterministic case:

\[
J_d = \sum_{k=0}^{\infty} \left[ x_k^T \cdot Q \cdot x_k + u^T \cdot R \cdot u \right]
\]

where $x_k$ – the state space vector, $u$– the input vector, $R, Q$ – weighting matrices.

2. $H_2$ -norms of a model in stochastic case:

\[
J_d = \mathbb{E}_M \sum_{k=0}^{\infty} \left[ x_k^T \cdot Q \cdot x_k + u^T \cdot R \cdot u \right]
\]

where $\mathbb{E}_M$ mathematical expectation operator.

3. $H_\infty$ -norm:

\[
\| \mathbf{T} \|_\infty = \sup_{\omega \in \Omega} \sigma(\mathbf{T}(j\omega)) \quad \text{where} \quad \sigma - \text{maximum singular value on the current frequency.}
\]

Flight various uncertainties could be external and/or internal, structural and/or unstructured, which produce certain deviation from the nominal behavior to perturbed one. The design algorithm is to estimate the performance and robustness of the closed loop system using $H_2$-norm of the sensitivity function and $H_\infty$-norm of the complementary sensitivity function, and then try to find the compromise between those two properties.

For this reason, we use a multi-objectives optimization problem, based on insert several objectives in one cost function and try to satisfy them at the same time [4,9].Therefore the penalty function (PF), restricting location’s area of the closed loop system poles in the predefined region $M$ in the complex plan. The penalty function is demonstrated as vertical section through Real axes on the figure 2b.

Fig. 1. The scheme of a pitch hold feedback loop

Fig. 2. Penalty function in the complex plan:

\[ a – \text{closed loop system poles;} \quad b – \text{PF} \left( d_m \right) – \text{vertical section through Real axes} \quad \text{«PF – Re»} \]

The penalty function is function of this minimum distance $d_m$:
where $P$ is a big number (for instance, $P=10^4-10^6$).

The expression of cost function is given as,

$$J_{\Sigma}=\lambda_d J_d^2 + \lambda_s J_s^2 + \lambda_p J_p^2 + \lambda_\omega J_\omega^2 + \lambda_{\omega_p} J_{\omega_p}^2 + \lambda \left[ \left\| T_{12} \right\|_{\infty}^2 + \left\| T_{22} \right\|_{\infty}^2 \right] + \text{Pf}(1)$$

where $\lambda_d$, $\lambda_p$, $\lambda_s$, $\lambda_\omega$, $\lambda_{\omega_p}$ – the LaGrange factors, $J_d$ and $J_p$ define the $H_2$-norms of the models in deterministic cases for particular DP range. $J_s$ and $J_{\omega_p}$ – define the performances of the stochastic models. $\left\| T_{12} \right\|_{\infty}$ and $\left\| T_{22} \right\|_{\infty}$ are the $H_\infty$-norms and gives the estimation of the robustness of the particular two parametrically disturbed plants. Pf is the penalty function. The cost functions in the expression (1) is the function of controller gain vector $\hat{C}_a$, that’s why the result of optimization procedure will be following $\hat{C}_a^* = \arg \min J_{\Sigma}(\hat{C}_a)$, $\hat{C}_a \in M_c$, where $M_c$ – stability range of controller gains.

The block diagram in fig.3, describes the algorithm of parametric optimization procedure.

### 7. Case study

The state space vector of the longitude channel is $x = [v_T, \alpha, q, \theta]^T$, where $v_T$ is the true speed, m/s, $\alpha$ is the angle of attack, deg, $q$ is the pitch rate, deg/s, $\theta$ is the pitch angle, deg. The control input vector is $u = [\delta_e]^T$ represented by elevator deflections. The nonlinear model of the Aerosonde is linearized for range of operating conditions respected to range of DP from 200 to 650 m/s, with a granularity of 50 kg/m$^2$.

Stochastic disturbance is modeled by means of Dryden filter [10]. The state space matrices $A$ and $B$ in general form filled with longitude coefficients and stability derivatives are given below:

$$A = \begin{bmatrix}
X_v + X_{v_T \cos \alpha} X_{v_T - g_x \cos \theta} & 0 \\
Z_v + Z_{v_T \sin \alpha} Z_{v_T - g_x \sin \theta} & V_T + Z_q \\
0 & 0 & 0 & 1 \\
M_v + M_{v_T} & M_{v_T} & 0 & M_q
\end{bmatrix}$$

$$B = \begin{bmatrix}
X_{\alpha} \\
Z_{\alpha} \\
0 \\
M_{\alpha}
\end{bmatrix}$$

---

**Fig. 3.** The block scheme of parametric optimization algorithm
where \( X_v = -\frac{qS}{mV_T} (2C_L - C_{Dv}) \)

\[
X_{\alpha} = \frac{qS}{m} (C_L - C_{D\alpha}); \quad X_{\delta e} = \frac{qS}{m} C_{D\delta e};
\]

\[
Z_v = -\frac{qS}{mV_T} (2C_L - C_{Lv});
\]

\[
Z_{\alpha} = -\frac{qS}{m} (C_D - C_{L\alpha});
\]

\[
Z_{\delta e} = \frac{qS \bar{\sigma}}{m} C_{L\delta e}; \quad Z_q = -\frac{qS \bar{\sigma}}{2mV_T} C_{Lq};
\]

\[
M_v = \frac{qS \bar{\sigma}}{J_y} (2C_m + C_{mv}); \quad M_{\alpha} = \frac{qS \bar{\sigma}}{J_y} C_{m\alpha};
\]

\[
M_{\delta e} = -\frac{qS \bar{\sigma}}{J_y} C_{m\delta e};
\]

\[
M_q = -\frac{qS \bar{\sigma}}{J_y} \frac{\bar{\sigma}}{2V_T} C_{mq};
\]

\( \gamma = \gamma_e, \alpha = \alpha_e \) are equilibrium (steady-state) conditions.

As seen from the description of space matrices coefficients, the aircraft flight dynamic depends on DP changes.

The controller gains after providing optimization procedure are shown in table 1.

**Table 1. Controller gains**

<table>
<thead>
<tr>
<th>DP, kg m(^{-2}) s(^{-1})</th>
<th>( P_{pid} )</th>
<th>( I_{pid} )</th>
<th>( D_{pid} )</th>
<th>( K_q )</th>
<th>( K_\theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.5665</td>
<td>0.2186</td>
<td>-0.0721</td>
<td>0.0365</td>
<td>-4.7635</td>
</tr>
<tr>
<td>250</td>
<td>0.3691</td>
<td>0.4343</td>
<td>-0.0455</td>
<td>0.0473</td>
<td>-6.3766</td>
</tr>
<tr>
<td>300</td>
<td>0.6456</td>
<td>0.2798</td>
<td>-0.1348</td>
<td>0.2108</td>
<td>-2.2564</td>
</tr>
<tr>
<td>350</td>
<td>0.6154</td>
<td>0.2849</td>
<td>-0.1676</td>
<td>0.1219</td>
<td>-2.7315</td>
</tr>
<tr>
<td>400</td>
<td>0.5277</td>
<td>0.1793</td>
<td>-0.0781</td>
<td>0.3353</td>
<td>-3.0210</td>
</tr>
<tr>
<td>450</td>
<td>0.5536</td>
<td>0.186</td>
<td>-0.0805</td>
<td>0.3740</td>
<td>-2.1004</td>
</tr>
<tr>
<td>500</td>
<td>0.2244</td>
<td>0.1330</td>
<td>-0.0577</td>
<td>0.0494</td>
<td>-6.4949</td>
</tr>
<tr>
<td>550</td>
<td>0.3242</td>
<td>0.1559</td>
<td>-0.0616</td>
<td>0.199</td>
<td>-3.7701</td>
</tr>
<tr>
<td>600</td>
<td>0.4180</td>
<td>0.1747</td>
<td>-0.0246</td>
<td>0.2343</td>
<td>-3.6411</td>
</tr>
<tr>
<td>650</td>
<td>0.4233</td>
<td>0.2171</td>
<td>-0.0195</td>
<td>0.1934</td>
<td>-5.1648</td>
</tr>
</tbody>
</table>

Performance indices for DP range and standard deviations of the UAV outputs are shown in tables 2 and 3.

**Table 2. Standard deviations of the UAV outputs**

| DP, kg m\(^{-2}\) s\(^{-1}\) | Standard deviations of the UAV outputs in a stochastic case |
|---|---|---|---|---|---|
| \( \sigma_v, m/s \) | \( \sigma_{\alpha}, \mathrm{deg} \) | \( \sigma_{q}, \mathrm{deg/s} \) | \( \sigma_{\theta}, \mathrm{deg} \) | \( \sigma_{cl}, \mathrm{deg} \) |
| 200 | 0.9909 | 0.2151 | 0.0221 | 0.0348 | 0.0342 |
| 250 | 1.0719 | 0.2193 | 0.0298 | 0.0358 | 0.0321 |
| 300 | 1.0746 | 0.2436 | 0.0524 | 0.0362 | 0.0609 |
| 350 | 1.2066 | 0.2414 | 0.0275 | 0.0353 | 0.0572 |
| 400 | 1.0630 | 0.1735 | 0.0221 | 0.0354 | 0.0294 |
| 450 | 1.2538 | 0.2061 | 0.0159 | 0.0346 | 0.0243 |
| 500 | 1.2423 | 0.2441 | 0.03 | 0.0373 | 0.0550 |
| 550 | 1.0376 | 0.1763 | 0.0238 | 0.0357 | 0.0296 |
| 600 | 1.1926 | 0.1879 | 0.0175 | 0.0353 | 0.0226 |
| 650 | 1.1057 | 0.1609 | 0.0152 | 0.0355 | 0.0225 |

**Table 3. Performance indices for DP range**

<table>
<thead>
<tr>
<th>H(_2)-norm</th>
<th>Deterministic case</th>
<th>Stochastic case</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{\infty} )-norm</td>
<td>1.0829</td>
<td>1.2287</td>
</tr>
<tr>
<td>1.5169</td>
<td>1.6150</td>
<td>3.5608</td>
</tr>
<tr>
<td>1.3955</td>
<td>1.1423</td>
<td>2.5850</td>
</tr>
<tr>
<td>1.4645</td>
<td>1.6239</td>
<td>3.0458</td>
</tr>
<tr>
<td>1.3712</td>
<td>1.1691</td>
<td>2.8391</td>
</tr>
<tr>
<td>1.0713</td>
<td>1.1447</td>
<td>1.8986</td>
</tr>
<tr>
<td>1.6482</td>
<td>1.7871</td>
<td>2.8533</td>
</tr>
<tr>
<td>1.3592</td>
<td>1.2211</td>
<td>2.7598</td>
</tr>
<tr>
<td>0.8107</td>
<td>1.1208</td>
<td>2.0588</td>
</tr>
<tr>
<td>0.7919</td>
<td>1.2114</td>
<td>2.2432</td>
</tr>
</tbody>
</table>

The values of \( H_{\infty} \)-norm are appropriate, the difference between \( H_{2}\)-norm in stochastic and deterministic case is very small.

The pitch angle step plots for the DP 300, 600 kg m\(^{-2}\) s\(^{-1}\) are shown on the figure 4.
Given step responses show that settling time values and the maximum overshoot values reflect the stability of close-loop system.

8. Conclusion

The design results prove the effectiveness of the proposed control method. The required flight performances are respected as well as the robustness of the control law. It can be seen that the handling quality of the models for each DP range are satisfied. The flight usage range is satisfied within UAV flight envelope.

In addition, the author highly appreciates professor A.A. Tunik in helping to provide the research.

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Received 02 March 2015.

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Received 02 March 2015.

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