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AERONAUTICAL ENGINEERING MAINTENANCE PERIODICITY OPTIMIZATION WITH THE HELP OF SUBJECTIVE PREFERENCES DISTRIBUTIONS

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Abstract

Purpose: *The goal of this research is to investigate the possibility of the objectively existing aeronautical engineering maintenance optimal periodicity determination in the different from the probabilistic methods way. In this paper there is a scientifically proven explanation for the mentioned above periodicity optimization with the help of subjective preferences distributions.* ***Methods:*** *The described approach proposes to consider functioning of an aeronautical engineering system with possible degrading failure on the multi-alternativeness basis. The objective effectiveness functions in such a case are intensities of damage and failure which corresponds with the set of considered operational alternatives. It allows obtaining the related subjective preferences functions implying the preferences proportionality to the alternative intensities of damage and failure.* ***Results:*** *The found expression for the optimized maintenance periodicity is the same as yielded by probabilistic methods and subjective preferences functions deliver the extremal (maximal) value to the operational-maintenance purpose functional. While this objective functional undergoes its maximum for optimal both canonical and proposed view preference distributions, the subjective entropy of the preference does not have any exremum in the vicinity at all.* ***Discussion:*** *The proposed approach model parameters interpretation, with the help of the carried out calculation experiments and plotting necessary diagrams, means that the damage and failure intensities are the self-measured subjective preferences obtained with the use of the subjective entropy maximum principle. They are expressed so in the terms of the subjective analysis theory.*

Keywords: airworthiness; aeronautical engineering; alternative; damage intensity; degrading failure; extremal; failure intensity; maintenance periodicity; optimization; preferences distribution; subjective entropy maximum principle; variational problem.

1. Introduction

Aeronautical engineering maintenance periodicity is an important technical operation parameter that is designated in order to keep aircraft airworthiness at the required level (Smirnov et al. 1990; Kroes, Wild 1994).

Optimal values of the periodicity are determined on the basis of different criteria including probabilistic, economical, etc. (Dmitriyev et al. 2005; Zaporozhets et al. 2011; Solomentsev et al. 2016; Smirnov et al. 1990; Tamarin 2002; Pallos 2001).

There is a chance to try an attempt for solving the problem of optimal aeronautical engineering maintenance periodicity determination with the use of subjective preferences theory (Kasianov 2013; Kasyanov, Szafran 2015; Goncharenko 2016a).

1. **Problem formulation** is that it is a challenging problem to discover an optional way of finding the aeronautical engineering maintenance optimal periodicity with the application of the new approach proposed in the framework of subjective analysis.

2. **Analysis of the latest researches and publications**

There is a postulated principle of subjective entropy maximum in subjective analysis (Kasianov 2013). The principle is called: "Subjective Entropy Maximum Principle" (SEMP). It is very productive; it has been used in the variety of solutions of problems (Kasianov 2013; Kasyanov, Szafran 2015;

Goncharenko 2014; 2016a; 2016b); nevertheless, it still needs more applications and substantiations.

The unsolved parts of the general problems of aeronautical engineering operation and maintenance (Dmitriyev et al. 2005; Zaporozhets et al. 2011; Solomentsev et al. 2016; Smirnov et al. 1990; Tamarin 2002; Pallos 2001; Kroes, Wild 1994) include multi-alternativeness of the related situations.

3. **Task setting**

The task setting for the presented paper is to obtain the objectively existing optimal aeronautical engineering maintenance periodicity in the different from the probabilistic way.

2. Main material

Following the methods described at the book of (Smirnov et al. 1990) one can find that according to (Smirnov et al. 1990, pp. 170-172) at the work of a product at a random moment of time τ there happens a damage, the further development of which leads to the failure at the moment of time t. Maintenance has to be performed in the interval (τ, t) at the moment t_p.

Thus, there is a system with a degrading failure.

1. **Problem setting**

The optimal time interval t_p^*, accepting, for instance, the law of the damages appearance times distribution, as well as of their development times to the failure happening as the exponential ones with the corresponding intensities of λ_1 and λ_2, for the scheduled maintenance works performance will be found at the maximum of the probability of the conjoint events of the damage occurrence and failure not happening realization by the formula of (Smirnov et al. 1990, P. 171):

$$t_p^* = \frac{\ln\lambda_1 - \ln\lambda_2}{\lambda_1 - \lambda_2}. \quad (1)$$

It appears from the extremization of the conditional probability:

$$P_{D\overline{F}}(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \quad (2)$$

of the two conjoint events happening, namely, damage D and not failure $\overline{F}$.

This can be derived from the normalizing condition for the probabilities:

$$P_{DF}(t) + P_{D\overline{F}}(t) + P_{\overline{D}F}(t) + P_{\overline{D}\overline{F}}(t) = 1, \quad (3)$$

where the probability of the two conjoint events happening: damage D and failure F

$$P_{DF}(t) = 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right], \quad (4)$$

the probability of the two conjoint events happening: not damage $\overline{D}$ and failure F

$$P_{\overline{D}F}(t) = 0, \quad (5)$$

and the probability of the two conjoint events happening: not damage $\overline{D}$ and not failure $\overline{F}$

$$P_{\overline{D}\overline{F}}(t) = e^{-\lambda_1 t}. \quad (6)$$

The other way of the problem solving implies not the probabilistic but subjective preferences, SEMP, approach. The corresponding intensities of λ_1 and λ_2, for the considered problem setting can be represented as certain parameters of the multi-alternativeness.

Therefore, we may use the apparatus of preferences functions.

2. **Problem solution**

The optimal value of (1) can be obtained with the use of SEMP (Kasianov 2013; Goncharenko 2016b).

Supposedly, the subjective preferences entropy functional has the view of (Goncharenko 2016b):

$$\Phi_\pi = -\sum_{i=1}^{2}\pi_i(\lambda_i)\ln\pi_i(\lambda_i) + t_p^*\sum_{i=1}^{2}\pi_i(\lambda_i)\lambda_i + \gamma\left[\sum_{i=1}^{2}\pi_i(\lambda_i) - 1\right], \quad (7)$$

where $\pi_i(\cdot)$ – preferences functions; γ – normalizing coefficient.

The first member of equation (7) is the subjective entropy of the preferences.

The necessary conditions for the functional (7) extremum existence

$$\frac{\partial\Phi_\pi}{\partial\pi_i(\cdot)} = 0 \quad (8)$$

in accordance with (Kasianov 2013; Goncharenko 2016b) yield

$$\frac{\partial\Phi_\pi}{\partial\pi_i(\cdot)} = -\ln\pi_i(\cdot) - 1 + t_p^*\lambda_i + \gamma = 0, \quad \forall i = \overline{1,2}. \quad (9)$$

This inevitably means in turn (Goncharenko 2014; 2016a; 2016b)

$$\ln \pi_1(\cdot) - t_p^* \lambda_1 = \gamma - 1 = \ln \pi_2(\cdot) - t_p^* \lambda_2 . \quad (10)$$

From where

$$\ln \pi_1(\cdot) - \ln \pi_2(\cdot) = t_p^* (\lambda_1 - \lambda_2) . \quad (11)$$

And

$$t_p^* = \frac{\ln \pi_1(\cdot) - \ln \pi_2(\cdot)}{\lambda_1 - \lambda_2} . \quad (12)$$

Thus, we have got the law of subjective conservatism (Goncharenko 2014; 2016b) if the values of parameters t_p^*, λ_1, and λ_2 are given.

In case

$$\pi_i(\lambda_i) = x\lambda_i , \quad (13)$$

where x – unknown, uncertain multiplier in type of the Lagrange one, we obtain with the help of the procedure considered through (7)–(13) the needed optimal periodicity (1).

Indeed, substituting for their values equations (13) into expression (12) it yields

$$t_p^* = \frac{\ln \frac{x\lambda_1}{x\lambda_2}}{\lambda_1 - \lambda_2} , \quad (14)$$

finally, formula (1).

The sense of the uncertain multiplier x becomes obvious with the use of the normalizing condition of the initial functional (7). That is

$$x\lambda_1 + x\lambda_2 = 1 . \quad (15)$$

Hence,

$$x = \frac{1}{\lambda_1 + \lambda_2} . \quad (16)$$

Remarkable here is that the cognitive function has the view of

$$t_p^* \frac{\sum_{i=1}^{2} \lambda_i^2}{\sum_{i=1}^{2} \lambda_i} . \quad (17)$$

Moreover, in case expressed with (7)–(17) the optimal value of the sought maintenance interval t_p^*, traditionally obtained through the procedure of (1)–(6), has been got for the given values of preferences $\pi_i(\cdot)$ and at this the optimum of t_p^* makes the preferences of $\pi_i(\cdot)$ also be optimal for the objective functional (7).

3. **Application of the subjective entropy extremization principle**

From conditions of (9) it is possible to get the canonical distributions of the preferences functions for the considered problem setting.

Then

$$\ln \pi_i(\cdot) = \gamma - 1 + t_p^* \lambda_i , \quad \pi_i(\cdot) = e^{\gamma - 1 + t_p^* \lambda_i} . \quad (18)$$

Applying the normalizing condition

$$\pi_1(\cdot) + \pi_2(\cdot) = 1 = e^{\gamma - 1}\left(e^{t_p^* \lambda_1} + e^{t_p^* \lambda_2} \right),$$

$$e^{\gamma - 1} = \frac{1}{e^{t_p^* \lambda_1} + e^{t_p^* \lambda_2}} , \quad \pi_i(\cdot) = \frac{e^{t_p^* \lambda_i}}{\sum_{j=1}^{2} e^{t_p^* \lambda_j}} . \quad (19)$$

The preferences functions described with equations (19) deliver the conditional extremum (maximum) to the subjective entropy or the maximum to the purpose functional (7).

Indeed

$$\frac{\partial^2 \Phi_\pi}{\partial \pi_i(\cdot)^2} = -\frac{1}{\pi_i(\cdot)} < 0 , \quad (20)$$

at any “point”, since preferences functions always have positive values $]0 \ldots 1[$ in the postulated view functional for multi-alternative consideration, like (7), including the “point” suspected for the extremum on condition of (8).

Moreover, although the preferences expressed with (19) are different from the preferences functions described with equations (13), but in case of (14)-(16) the preferences have the same value.

4. **Numerical experiment**

Computer simulation for the accepted data: $\lambda_1 = 5 \cdot 10^{-3}$ h^{-1}; $\lambda_2 = 1 \cdot 10^{-3}$ h^{-1}; $t = 0 \ldots 1.5 \cdot 10^3$ h is illustrated in Figures 1-4.

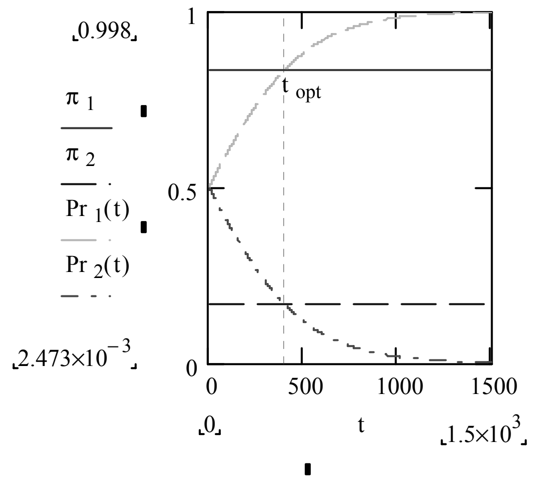

Fig. 1. Optimal distributions of the preferences functions

In Fig. 1 the designations of π_1 and π_2 stand for the preferences functions obtained with the equations of (13) with respect to condition (16). The functions of $\Pr_1(t)$ and $\Pr_2(t)$ are obtained by the formulas of (19). The value $t_{opt} \approx 402$ h (see Figure 1) has been found by equation (1).

In order to discover, or better to say make sure, that the preferences distributions of (13) with respect to condition (16) and (19) deliver extremum to the objective functional (7) we make the obtained preferences functions an arbitrary infinitesimal variation of $\delta = [-0.02 \ldots 0.02]$ (see Figure 2).

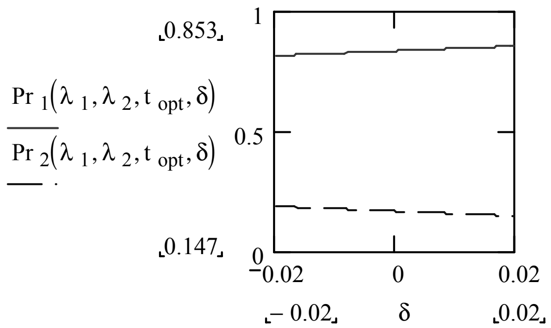

Fig. 2. Variated optimal preferences functions distributions

Here, in Figure 2, the functions of $\Pr_1(\cdot)$ and $\Pr_2(\cdot)$ are obtained by the formulas of (19) as the functions of the four independent variables but at the fixed values; and at the value of $t_{opt} \approx 402$ h found by equation (1) they coincide with the preferences functions of (13) (see Fig. 1).

5. Analysis of the obtained results

As it was expected the variated objective maintenance periodicity functional (7) has the maximal value at the optimal subjective preferences distributions, that is, at the optimal subjective preferences with the variation of $\delta = 0$ (see Fig. 3).

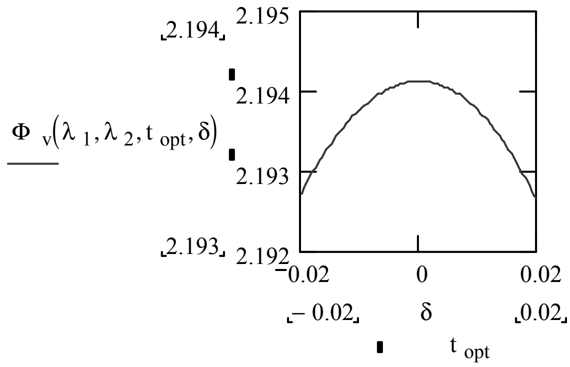

Fig. 3. Maximal value of the objective maintenance periodicity functional

However the subjective entropy of the preferences is not extremized at the optimal preferences distributions (see Fig. 4).

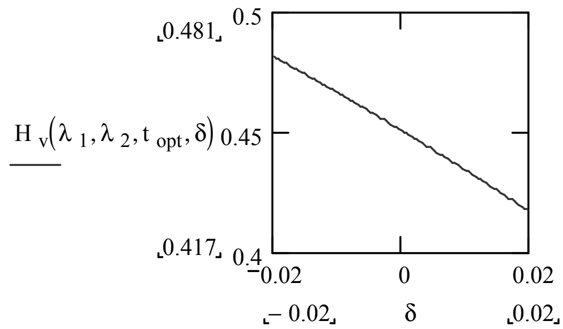

Fig. 4. Subjective entropy of the preferences

6. Discussion

The disputable issue here is that the preferences functions might be expressed in some other than with the equations of (13) way. Although it is quite enough to state that SEMP has got one more substantiation, the approach expressed with the equations of (7)–(20) and illustrated with Figures 1-4, for application described with the traditional approach of (1)–(6) to solving multi-alternative problems in conditions of uncertainty.

The supposition that the preferences (13) of the alternatives are proportional with the corresponding to the alternatives values of λ_1 and λ_2 has appeared to be fruitful.

Optimal maintenance periodicity is found with the help of the law of subjective conservatism (Goncharenko 2014; 2016b).

In accordance with the objective maintenance periodicity functional (7) the damage and failure intensities λ_1 and λ_2 has happened to be the self-measured preferences (13).

3. Conclusions

On the basis of the subjective entropy maximum principle it was obtained the optimal value of aeronautical engineering maintenance periodicity in case of degrading failure, which had been known before from the probability extremization.

In further researches it should be considered more than the two alternative case.

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Оптимізація періодичності технічного обслуговування авіаційної техніки за допомогою розподілів суб'єктивних переваг

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Мета: Метою даної роботи є дослідити можливість визначення об'єктивно існуючої оптимальної періодичності технічного обслуговування авіаційної техніки іншим від ймовірнісних методів шляхом. У даній статті міститься науково доведене пояснення для оптимізації вищезгаданої періодичності за допомогою розподілів суб'єктивних переваг. **Методи дослідження:** Описаний підхід пропонує розглядати функціонування системи авіаційної техніки із можливою поступовою відмовою на основі багатоальтернативності. Об'єктивні функції ефективності у такому разі – це параметри потоків ушкоджень та відмов, що відповідає множині експлуатаційних альтернатив, яка розглядається. Це дозволяє отримати відповідні функції суб'єктивних переваг за умов пропорційності тих переваг до альтернативних параметрів потоків ушкоджень та відмов. **Результати:** Знайдений вираз для оптимізованої періодичності технічного обслуговування є таким самим як і отриманий через ймовірнісні методи, а функції суб'єктивних переваг доставляють екстремальне (максимальне) значення експлуатаційному цільовому функціоналу технічного обслуговування. У той час, коли такий цільовий функціонал перебігає свого максимуму, у випадку як канонічного так і пропонованого виду розподілу переваг, суб'єктивна ентропія тих переваг не має жодного екстремуму поблизу взагалі. **Обговорення:** Інтерпретація параметрів моделі, що стосується запропонованого підходу, за допомогою виконаних розрахункових експериментів та побудови необхідних діаграм, означає, що параметри потоків ушкоджень та відмов то є само-вимірні суб'єктивні переваги отримані із використанням принципу максимуму суб'єктивної ентропії. Вони таким чином виражаються в термінах теорії суб'єктивного аналізу.

Ключові слова: авіаційна техніка; альтернатива; варіаційна задача; екстремаль; льотна придатність; оптимізація; параметр потоку відмов; параметр потоку ушкоджень; періодичність технічного обслуговування; поступова відмова; принцип максимуму суб'єктивної ентропії; розподіл переваг.
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Оптимизация периодичности технического обслуживания авиационной техники с помощью распределений субъективных предпочтений

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Цель: Целью данной работы является исследовать возможность определения объективно существующей оптимальной периодичности технического обслуживания авиационной техники иным, нежели вероятностными методами, путем. В данной статье содержится научно доказанное пояснение для оптимизации вышеупомянутой периодичности с помощью распределений субъективных предпочтений. **Методы исследования:** Описанный подход предлагает рассматривать функционирование системы авиационной техники с возможным постепенным отказом на основе многоальтернативности. Объективные функции эффективности в таком случае – это параметры потоков повреждений и отказов, что соответствует множеству рассматриваемых эксплуатационных альтернатив. Это позволяет получить соответствующие функции субъективных предпочтений при условии пропорциональности этих предпочтений альтернативным параметрам потоков повреждений и отказов. **Результаты:** Найденное выражение для оптимизированной периодичности технического обслуживания является таким же самым, как и полученное вероятностными методами, а функции субъективных предпочтений доставляют экстремальное (максимальное) значение эксплуатационному целевому функционалу технического обслуживания. В то время, когда такой целевой функционал претерпевает свой максимум, в случае как канонического, так и предложенного вида распределения предпочтений, субъективная энтропия этих предпочтений не имеет ни одного экстремума в окрестности вообще. **Обсуждение:** Интерпретация параметров модели, что касается предложенного подхода, с помощью выполненных расчетных экспериментов и построения необходимых диаграмм, означает, что параметры потоков повреждений и отказов являются само-измеренными субъективными предпочтениями, полученными с использованием принципа максимума субъективной энтропии. Они таким образом выражаются в терминах теории субъективного анализа.

Ключевые слова: авиационная техника; альтернатива; вариационная задача; летная годность; оптимизация; параметр потока отказов; параметр потока повреждений; периодичность технического обслуживания; постепенный отказ; принцип максимума субъективной энтропии; распределение предпочтений; экстремаль.

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