In this article, model of passenger flow distribution in regard to their waiting time at city railway transport stations is constructed. For modelling, developed method is used, in which it is fill that the distribution of passengers in regard to waiting time is consistent with function of Erlang distribution of k order. Taken as a basis the results of experiment that was carried out on the existing network of city railway transport in Kiev.

Keywords: model; distribution; passenger flow; waiting time; station; city railway transport.

Problem

At the present stage of development and growth of the cities-millionaires and their agglomerations, actual there is a question to research of temporal parameters of public conveyance in urban space. Passenger waiting time on landing implementation in vehicle of the city railway transport is one of the main components of time expenditure at movement in city space. Need of waiting the vehicle at public transport stops arises from divergence of time between the moment of occurrence of necessity a movement and the moment of possibility of its satisfaction.

Passenger flow distribution in regard to their waiting time at city railway stations quantitatively describes one of the most important quality index of the public conveyance. Namely, quality of implementation the transportation function of this transport system [1]. Character of passengers quantity distribution allows setting frequency of their accumulation during definite traffic interval between vehicles. Knowing its expected average value it is possible to estimate the value of certain percent by means of the theoretical distribution. Conversely, knowing the value of certain percent (for example, in the form of preassigned value or efficiency function) it is possible to define the average value of distribution.

When scheduling work of transport in cities, sets the objective of providing a certain availability level in time main points of destination of trip, which is determined by “superior limit” or “standard” value of duration of the trip. In other words, in normative documents like SBN (State Building Norms) timing of movement is set for 80 or 90 % of city habitants, i.e. for the majority of city transport users. Therefore, at our sight, is the expedient a definition of value of the waiting time for bigger number of passengers of the city railway transport, that will allow really to estimate non-productive costs of the personal time of passengers, and level of service by this transport. In addition, knowledge of passenger flow distribution in regard to the waiting time is very important for development or updating of the vehicles movement schedule of the city railway transport.

However, as well known, for realization of the analysis and the estimate of research results of definite phenomenon or object it is necessary to construct it model. Obviously, that the model is chosen more successful, the better it will reflect the most characteristic features of research object. Therefore, the more successful will be the research.

Analysis of last researches and publications

In the majority of scientific works of these themes dominates creation of models the waiting time distribution in regard to such parameters, as the movement interval between the vehicles of public transport [2; 3] and intensity of approach of the passengers at public transport stops [4; 5]. In addition, the main attention is paid to determination of average value the passengers waiting time of public transport vehicles.

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Exception is the work [3], in which attention is paid to research and ascertainment of behaviors and characteristics of distribution elements of this parameter at stops of six bus routes. By the author of work is confirmed with through experiment, that the distribution of the waiting time elements is consistent with the law of normal distribution. On basis of received practical results in this work, the actual average value of waiting time by passengers at stops of these bus routes was calculated.

Analyzing modern scientific works it is established, that the most widespread method of modeling processes of transportation the passengers in city transport systems is imitation modeling. For example, using this method parameters of distribution density the passenger's waiting time at stops of public transport routes were established [6].

As for studying of passenger flows, the majority of scientific experts researched passenger flow distribution on temporal periods during day on intercity routes of public transport [7; 8; 9].

However, nowadays, a small amount of researches is devoted to studying of various models of passenger flows distribution in regard to waiting time. Though, successful attempts of creation the model of passenger flows distribution in regard to waiting time at station platforms and establishment of the actual waiting time) will allow us to achieve our research purpose.

For research of process accumulation passengers on platforms and establishment of the actual waiting time, experiment was made at stations of existing network of city railway transport in Kyiv during four weeks in weekdays [8]. The results of experiment is given for one of city railway stations. Processing of primary data was realized in the program «Excel Microsoft».

First stage of model construction of passenger flow distribution in regard to waiting time at station consists in processing statistical data of the experiment. It’s known, that a large number of statistical data demands their systematization, which we carry out in tabular form. For simplification of work with the observation results representing empirical distribution, we compile them in a statistical row.

By-turn, first stage of processing statistical data consists in their association in categories. Digit group length is determined by G. A. Stregerss’ formula [9]:

$$I_c = \frac{t_{w_{\text{max}}} - t_{w_{\text{min}}}}{1 + 3.2 \lg n},$$

where $t_{w_{\text{max}}}, t_{w_{\text{min}}}$ — maximum and minimum value of waiting time respectively, min; $(1 + 3.2 \lg n)$ — digit count; $n$ — general number of observation.

$$I = \frac{21.0 - 0}{1 + 3.2 \cdot lg 1869} = 1.831 \text{ (min)}.$$

We accept close to received interval value of 2 minutes that is more convenient for calculations. Thus, digit count makes $R = 21/2 \approx 11$. After determination of digit length and digit count, we carry out grouping of interval (Table 1). Waiting time of passengers at station is continuous variate. Its numerical characteristics are the mathematical expectation, the dispersion, the standard deviation and the variation coefficient.

The mathematical expectation is determined by formula [10]:

$$M(t_w) = \sum_{i=1}^{n} t_i p_i,$$

where $p_i$ — probability value of random waiting time variable.

$$M(t_w) = 7,956 \text{ (min)}.$$
### Table 1

<table>
<thead>
<tr>
<th>№</th>
<th>Digits, min $t_w$</th>
<th>Mean value of interval, min $\bar{t}_w$</th>
<th>Cell frequency, pass. $m_i$</th>
<th>Relative frequency, $p_i$</th>
<th>Accumulative relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0–2</td>
<td>1,0</td>
<td>152</td>
<td>0,081</td>
<td>8,1</td>
</tr>
<tr>
<td>2</td>
<td>2–4</td>
<td>3,0</td>
<td>278</td>
<td>0,149</td>
<td>14,9</td>
</tr>
<tr>
<td>3</td>
<td>4–6</td>
<td>5,0</td>
<td>299</td>
<td>0,16</td>
<td>16,0</td>
</tr>
<tr>
<td>4</td>
<td>6–8</td>
<td>7,0</td>
<td>269</td>
<td>0,144</td>
<td>14,4</td>
</tr>
<tr>
<td>5</td>
<td>8–10</td>
<td>9,0</td>
<td>268</td>
<td>0,143</td>
<td>14,3</td>
</tr>
<tr>
<td>6</td>
<td>10–12</td>
<td>11,0</td>
<td>222</td>
<td>0,119</td>
<td>11,9</td>
</tr>
<tr>
<td>7</td>
<td>12–14</td>
<td>13,0</td>
<td>170</td>
<td>0,091</td>
<td>9,1</td>
</tr>
<tr>
<td>8</td>
<td>14–16</td>
<td>15,0</td>
<td>112</td>
<td>0,06</td>
<td>6,0</td>
</tr>
<tr>
<td>9</td>
<td>16–18</td>
<td>17,0</td>
<td>67</td>
<td>0,036</td>
<td>3,6</td>
</tr>
<tr>
<td>10</td>
<td>18–20</td>
<td>19,0</td>
<td>27</td>
<td>0,014</td>
<td>1,4</td>
</tr>
<tr>
<td>11</td>
<td>20–22</td>
<td>21,0</td>
<td>5</td>
<td>0,003</td>
<td>0,3</td>
</tr>
<tr>
<td>∑</td>
<td></td>
<td></td>
<td>1869</td>
<td></td>
<td>1,000</td>
</tr>
</tbody>
</table>

The weighted average value of waiting time will accept value [10]: $\bar{t}_w = M(t_w) = 8,0$ min. The dispersion of waiting time is determined by formula [10]:

$$D(t_w) = M\left(\bar{t}_w^2\right) - \left[M(t_w)\right]^2,$$

where $M\left(\bar{t}_w^2\right)$ — the second initial time, min$^2$;

$$M\left(\bar{t}_w^2\right) = \sum_{i=1}^{k} \bar{t}_w^2 p_i;$$

$$M(t_w) = 84,12 \text{ (min)};$$

$$D(t_w) = 84,12 - 7,956^2 = 20,822 \text{ (min$^2$)}.$$

The standard deviation [10]:

$$\delta = \sqrt{D(t_w)}.$$

$\delta = \sqrt{20,822} = 4,563 \text{ (min)}$.

This means that waiting time deviates from the weighted average value by an average on 4,6 minutes.

Last numerical characteristic is the variation coefficient.

It characterizes degree of flow irregularity [9]:

$$\nu = \frac{\delta}{M(t_w)},$$

$$\nu = \frac{4,563}{7,956} = 0,57.$$

This means that waiting time deviates from the average value on 57 %. So, we defined all necessary components for model construction of passenger flow distribution in regard to waiting time.

### Modeling passenger flow distribution in regard to waiting time

For modeling passenger flow distribution in regard to waiting time at city railway transport stations, we will use already developed methodology by Yan Tsibulka’s [1].

The author of this methodology for modeling passengers’ distribution according to traveling time accepts the function, which was deduced from Erlang $k$-distribution.

We will use this methodology in view of positive results of author's model.

It’s known, that the histogram of a statistical row is a statistical analog to the graph of theoretical density distribution [10].

I.e., the graphic presentation of passengers’ distribution in regard to waiting time (Fig. 1) will help us to determine by means of which theoretical distribution we will be able to approximate the statistical row received during data processing.

The analysis of Fig. 1 shows, that the distribution according to data of table 1 is asymmetric histogram with displaced in left top.

So, the analysis of this histogram enables to assume hypothetically that this statistical row can be described by means Erlang distribution model of order $k$ for a continuous random variable.

Generally, Erlang distribution with defined value of order $k$ is possible to approximate the flows with different irregularity degree with the variation coefficient of intervals from zero to one.
Erlang distribution of order $k$ is one of general distribution of continuous variables and it is characterized by two parameters: the flux level $\lambda$ and Erlang parameter $k$. In turn [10]:

$$\lambda = \frac{1}{t_w};$$  \hspace{1cm} (7)

$$k = \frac{[M(t_{wf})]^2}{D(t_w)}.$$  \hspace{1cm} (8)

The function of Erlang distribution is [10]:

$$F(t_w) = \frac{\int_0^{t_{wf}} (\lambda k)^k t_w^{k-1} e^{-\lambda t_w} dt_w}{(k-1)!},$$  \hspace{1cm} (9)

where $t_{wf}$ — final value of intervals, min.

By computed result is established:

$$\lambda = \frac{1}{8} = 0.125 \text{ (pass./min)};$$

$$k = \frac{(7.956)^2}{20,822} = 3.$$  

Therefore, the function of Erlang distribution (9) the third order will have the following appearance:

$$F(t_w) = 1 - \frac{1}{2} \left( 9\lambda^2 t_{wf}^2 + 6\lambda t_{wf} + 2 \right) e^{-3\lambda t_{wf}}.$$  \hspace{1cm} (10)

The computed results the parameters and function of Erlang distribution the third order behind statistical data are shown in table 2.

On Fig. 2 are shown results of statistical cumulative and function of Erlang distribution, that is modeling the accumulate relative frequency of passenger flow distribution in regard to waiting time.

**Table 2**

<table>
<thead>
<tr>
<th>Digits, min $t_w$</th>
<th>Final value of interval, min $t_{wf}$</th>
<th>Parameter of theor. Distribution, $\lambda t_{wf}$</th>
<th>Function (10) $F(t_w)$</th>
<th>Difference of theor. and emp. values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>2.0</td>
<td>0.25</td>
<td>0.041</td>
<td>4.1</td>
</tr>
<tr>
<td>2–4</td>
<td>4.0</td>
<td>0.50</td>
<td>0.191</td>
<td>19.1</td>
</tr>
<tr>
<td>4–6</td>
<td>6.0</td>
<td>0.75</td>
<td>0.391</td>
<td>39.1</td>
</tr>
<tr>
<td>6–8</td>
<td>8.0</td>
<td>1.00</td>
<td>0.577</td>
<td>57.7</td>
</tr>
<tr>
<td>8–10</td>
<td>10.0</td>
<td>1.25</td>
<td>0.723</td>
<td>72.3</td>
</tr>
<tr>
<td>10–12</td>
<td>12.0</td>
<td>1.50</td>
<td>0.826</td>
<td>82.6</td>
</tr>
<tr>
<td>12–14</td>
<td>14.0</td>
<td>1.75</td>
<td>0.895</td>
<td>89.5</td>
</tr>
<tr>
<td>14–16</td>
<td>16.0</td>
<td>2.00</td>
<td>0.938</td>
<td>93.8</td>
</tr>
<tr>
<td>16–18</td>
<td>18.0</td>
<td>2.25</td>
<td>0.968</td>
<td>96.8</td>
</tr>
<tr>
<td>18–20</td>
<td>20.0</td>
<td>2.50</td>
<td>0.980</td>
<td>98.0</td>
</tr>
<tr>
<td>20–22</td>
<td>22.0</td>
<td>2.75</td>
<td>0.99</td>
<td>99.0</td>
</tr>
</tbody>
</table>

Fig. 2. The accumulative statistical passengers distribution in regard to waiting time and its model
Comparison of numerical values the empirical and the theoretical accumulative distributions is shown an insignificant deviation, that isn’t exceed 5%. As for their graphic representation, we are observe accurate imposition of accumulative curves the model and the statistical distribution.

With the help of accumulative graphic the passenger flow distribution in regard to waiting time (Fig. 2) we will establish required value for 80 and 90 % of passengers. On average, the waiting time according to statistical data and its model compiles:

For 80 % of passengers $t_{w}^{90} = 11$ min.

For 90 % of passengers $t_{w}^{90} = 13$ min.

Conducted calculation is analyzed, it’s obvious that waiting time is not appreciably, but is unacceptable for bigger number of users by city railway transport services in Kyiv.

**Conclusions**

Thus, authors constructed the model of dependence passengers uplift from value of waiting time, by means of theoretical function of Erlang distribution of the third order. Comparison data of statistical accumulative relative frequency of passengers and its theoretical model was shows, that their difference is less than 5%. So, the hypothesis is valid that passenger flow distribution in regard to waiting time at city railway transport stations is described by received model. This statement also was confirmed when comparing the data of empirical and theoretical results in graphic representation.

By means of received model it was established the waiting time indicator at city railway transport stations for 80 and 90 % of passengers. I.e. the main purpose of conducted research was achieved. So, according to results, waiting time indicator for users of city railway transport services is not considerable, but is not unacceptable. Authors recommend holding events for reduction of this temporary indicator of passengers city railway transport. These measures will allow raising appeal of this type of public transport and a level of service quality. Therefore, further research will be focus on search of solutions specified problem. As for model of passengers’ distribution in regard to their waiting time, it can be applied only for types of public transport which work according to a fixed schedule with considerable intervals.

**REFERENCES**


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