The effect of stalled flow hysteresis in the process of streamlining a cylindrical solid, isolated backswept wing of a model aircraft and a decelerating compressor cascade is analyzed in the given article. The structure and the performances of stalled flows depending on the speed of the flow, angles of attack and on the direction of their change are considered. Generalized results of research of hysteresis effect occurrence in aerodynamic performances of compressor cascades and its influence on compressor blade rings performances that operate in real conditions of periodic circular inlet distortion induced by aerodynamic trail behind the blade cascade of the compressor inlet guide vane are demonstrated.

**Keywords:** compressor cascades, hysteresis, separated flows, circular inlet distortion, aerodynamic trail, aerodynamic performances.

**Introduction**

At supercritical angles of attack of the gas-turbine engine compressor airfoil cascades there appear intensive areas of flow distortion on the blade surface that have a negative effect on the parameters and performances of compressor cascades. In turn, the structure of the stalled flows depends on their kinematic performances — velocity, angles of attack and on such flow parameters as pressure, density and temperature, and is subject to the direction of their change within certain values interval.

Research of the stalled flows in the cascades is brought about by the necessity of problem solving connected with prediction of the mentioned above flow separation modes occurrence in compressor cascades, definition of ways to extend the range of compressors blade rings unstalled streamlining [1; 2] and increase of their gas-dynamic stability margins [3].

Under different directions of flow change parameters, identical collection of parameters there exist various aerodynamic performances. One possible reason for this occurrence is aerodynamic hysteresis that is conditioned by reforming of the streamlining structure under the change of at least one of such parameters as speed of the unstalled flow, angle of attack, Reynolds number, and in the conditions of unsteady flow — frequency and oscillatory amplitude.

**Problem statement**

The given paper is intended for analysis of unstalled flows hysteresis in streamlining of a cylindrical solid, isolated backswept wing of a model aircraft and a decelerating compressor cascade and the degree of gas-turbine engine axial compressor with the purpose of further account of the effect of hysteresis on the performances of compressor blade rings and their efficiency.

**Hysteresis at streamlining a cylindrical solid and isolated aircraft wing**

At subsonic and supersonic speed, the streamlining of a cylindrical solid, the longitudinal axis of which is located along the direction of flow, is accompanied by creation of unstalled flow areas behind the face part. With alteration of Mach number \( M \), the structures of streamlining are transformed [4]. At low subsonic speeds, the flow stalls around the front edge of the cylinder creating a non-isolated flow on the surface of the solid. With the increase in \( M \), number in the range of subsonic speed the area of the separation flow is being pressed down to the cylindrical surface, the cross sectional dimensions and pressure in the area decrease. The further increase in the speed induces changes on the outer streamlined structure. In front of the solid and on the outer edge of the stalled area separated shocks take...
place, however the discontinuity of the stalled flow is retained.

Almost instant reforming of the streamlining structure occurs under somewhat critical number \( \dot{\alpha} \approx 1.14 \), when instead of a developed stalled flow that closes behind, there appears a local area of stall in the front part of cylinder. This is accompanied by occurrence of a system of shocks around the streamlined surface that are caused by attachment of the local stalled flow and deceleration of the outer flow. The further increase in the speed of undisturbed flow retains the same structure of the streamlined solid.

With the decrease in \( \dot{\alpha} \) to the value of \( \dot{\alpha} = 1 \) the type of streamlined structure is maintained as the disturbance factor which leads to reforming of the flow structure is absent. However, if \( \dot{\alpha} < 1 \), the disturbance factor appears. In fact, deceleration of the supersonic flow occurs due to the shockwave. If the velocity of streaming decreases below the sound velocity, the local supersonic flow transfers to the subsonic through a direct shock wave. The interaction of the shock wave with the boundary layer on the surface of the solid induces its separation. The point of separation with the decrease in velocity shifts to the front part. When the oscillating area overlaps with the area of separation that is induced by the shock wave, the flow structure reforms, the pressure increases drastically and the separation initiated at the front edge extends over the surface of the entire cylinder. Reforming in such flow structures is performed under critical Mach number \( \dot{\alpha} = \dot{\alpha}_{c1} \approx 0.9 \) lower in value than \( \dot{\alpha}_{c1} \).

Therefore, under different directions of change in Mach number, under an increase of \( \dot{\alpha} \) in the range from 0.9 to 1.14 and under a decrease ranging from 1.14 to 0.9, there occur dissimilar streamlining structures because reforming of flow structures at direct and reverse change of \( \dot{\alpha} \) numbers derives from a different initial state of the system, and additional energy inlet is obtained from the approach flow to provoke variations.

Ambiguity of flow structures is revealed with the change of the angle of attack. Let us consider this hypothesis on the example of streaming of the backswept high-aspect model aircraft wing [5].

With an increase in the angle of attack the areas of separated streaming on the wing alter their geometrical parameters, interact with the flows on other parts of the model aircraft surface. Such interaction is overdue regarding the change in kinematic performances and behaves differently at various angles of attack: under an increase of the angle (impact stroke) induces a breakup in the initial structures and under a decrease of the angle (back stroke) causes recovery of initial structures. This leads to a dependence of aerodynamic performances on the direction of the change of the angle of attack, or, in other words, to hysteretic dependences of aerodynamic forces and moments on the angle of attack. The transition range of the angles of attack, where a breakup of the attached flow occurs and a developed separated flow is created, can be characterized by a number of steady separated flows. Under a change in the flow, as an example, an extreme decrease in the lift ratio \( \bar{n} \) can be observed. At a reverse deviation of the model aircraft wing, the angles of attack that induce a hopping increase in aerodynamic forces can be observed. These angles differ from the angles of attack that induce drastic decrease in the value of \( \bar{n} \) at direct deviation of the wing. Therefore, hysteresis in aerodynamic forces of the model aircraft wing can be multiple and can contain several subareas that are separated from each other by internal boundaries.

Figure 1 shows the dependence of the lift ratio \( \bar{n} \) on the angle of attack \( \alpha \) for a model aircraft with a rectangular wing with a high aspect \( \lambda = 10 \), a thickness-chord ratio \( \bar{t} = 17 \% \) and with Reynolds number \( \text{Re} = 0.33 \cdot 10^6 \) that was obtained under the change of the angles of attack in the range of \(-3^\circ \leq \alpha \leq 20^\circ \). It is obvious that beyond the section of the angles of attack \( 8,5^\circ \leq \alpha \leq 17^\circ \) the dependence is single-valued, and within the given section it is double-valued. The upper bound of the double-valued area under the increase in the angle of attack ranges from \( \alpha = -3^\circ \) to \( \alpha = 20^\circ \) and the lower bound ranges from \( \alpha = 20^\circ \) to \( \alpha = -3^\circ \).

![Fig. 1. Change of \( \bar{n} \) under increase and decrease in the angle of attack](image)

Let us consider the effect of hysteresis on the performances of the decelerating compressor cascades and the axial compressor stages. In the cascades of aerofoils that are streamlined by unsteady flows periodical shifts of cascades from subcritical to supercritical angles of attack and back are possible under specified even conditions. These shifts are accompanied by the effect of hysteresis that is characterized
by a return of compressor cascade from the stalled mode to the non-stalled mode.

**The effect of hysteresis on aerodynamic performances of compressor cascades under their streamlining by a periodical unsteady flow**

As aerodynamic performances of compressor cascades are significantly influenced by the processes that occur in the boundary layers on the blades, it is practically important to regard the effect of periodical flow non-uniformity at inlet to cascades on the type of compressor cascades streamlining and the flow in the boundary layer on the blades [6; 7].

In streamlining of the blade disks of multistage compressors, that operate under the conditions of periodical circular flow non-uniformity caused by aerodynamic trails behind the blade cascade of the compressor inlet guide vane, unsteady streamlining of compressor cascades takes place (in calculation of the unsteady flow the values of velocity and pressure contain a steady and an oscillatory component).

The effect from outer flow oscillation on the velocity profile and temperatures of the steady average movement is studied in [8]. In this work the behavior of the boundary layer is considered under the condition that the velocity on the boundary is given as follows:

\[ W = W_0 \left[ 1 - \lambda \cos \omega \left( \frac{x}{W_0} - \frac{x}{W_0} \right) \right], \]

where \( W_0 \) is the speed of undisturbed flow; \( x \) is a specific dimension of a streamlined body; \( \omega \) is an angular frequency; \( \lambda \) is a speed parameter.

The most significant effect on the flow in the boundary layer, in particular in the boundary layer areas near the walls is caused by low-frequency oscillations which impose upon the main stream and high-frequency disturbances and do not affect the boundary layer flow. Common for all theoretical researches is application of a set of empirical ratios for solving the equations which describe the processes in unsteady boundary layers.

In [8] the results of researches of circular cylinders drag during their accelerated motion are discussed. Acceleration or deceleration of the flow that moves around the cylinder causes significant change to the structure of boundary layer and, correspondingly, change to the position of the point of the boundary layer separation: during acceleration of the flow about the cylinder \( (\partial W/\partial x) > 0 \), the point of separation shifts backwards, that causes the decrease of an aerodynamic trail width and as a result profile drag caused by viscosity forces in the flow (the coefficient of profile drag \( c_i \) decreases); during deceleration of the flow \( (\partial W/\partial x) < 0 \) the point of the boundary layer separation shifts forward that causes widening of the aerodynamic trail behind the cylinder and profile drag increases. These conclusions characterize changes of profile drag instantaneous coefficient of the cylinder which oscillates in the flow.

In [9] the influence of relative acceleration of the flow on airfoil aerodynamic performances is investigated and it is shown that acceleration enhances non-separated flow about the surfaces and deceleration initiates the flow separation.

It is quite evident that there is an analogy between considered phenomenon and streamlining of the compressor cascades by periodically non-uniform flow. When the angle of attack \( \alpha \) is increased (deceleration of the flow) the separation point moves to the leading edge of a blade and during decrease of angle of attack (flow acceleration) it moves to the trailing edge and, correspondingly, periodically the coefficient of cascade profile drag \( c_{i\alpha} \) changes.

In aerofoil cascades which are streamlined by the periodic non-uniform flow under some certain conditions there is a possibility of transition of cascades to supercritical modes of operation with further recovery or non-recovery of the initial mode. The cascades’ transition to the supercritical modes and their return to subcritical modes are usually accompanied by hysteresis effect. The transition from separation mode to the non-separated mode of streamlining the compressor cascades returns not along normal characteristic (upper curve, fig.2) but along separation one (lower curve).

![Fig. 2. Hysteresis of decelerating cascades performances](image)

The typical modes that shall be used to analyze the effect of hysteresis are demonstrated on fig.2: \( i_{cr} \) stands for the critical angle of attack which characterizes the maximum rating of the cascade when it is streamlined by a steady flow (point \( A \) in fig.2); \( \alpha_0 \) is the angle of attack that represents a transfer from the stalled performance to the non-stalled mode (point \( D \) in fig.2); \( \alpha \) – angle of attack that demonstrates the cascade transfer to the stalled streamlining after a
rapid increase in the angle of attack at the cross of aerodynamic trails by the blades (point B in fig.2). To characterize the peculiarities of blade rings streamlining by a periodically non-uniform circular flow we shall regard two parameters [10].

1. Parameter of inability to calculate flow by the angle of attack \( \tilde{t} = i' / i_{cr} \). The given parameter accounts for the instant change of bladed disks streamlining during the passage over the aerodynamic trail. During the passage of the airfoil over the aerodynamic trail at the subcritical mode of compressor cascade operation the angle of attack has the value of \( i' > i_{cr} \) which induces earlier (as for air intake volume) flow separation in the blade ring. The higher the intensity of the aerodynamic trail, the higher is the value of inability to calculate parameter at the specified angle of attack and the more significant is the difference between the angles \( i' \) and \( i_{cr} \). It should be accounted in defining the boundary of the stall in the blade ring of the operating wheel of the axial compressor stage under its streamlining by the periodical non-uniform oscillating flow caused by the aerodynamic trail behind the blades of compressor inlet guide vane.

2. Parameter of inability to calculate by hysteresis \( \tilde{i}_0 = i_0 / i_{cr} \). The given parameter accounts for the difference in the passage from the stalled performance to the non-stalled one between the critical angle of attack \( i_{cr} \) and the angle of attack \( i_0 \), under which the process of non-stalled blade ring streamlining by the periodical non-uniform oscillating flow.

\[ \tilde{i}_0 = \frac{i_0}{i_{cr}} \]

Introduction of the parameters \( \tilde{t} \) and \( \tilde{i}_0 \) is stipulated by the fact that after the blade ring passes over the aerodynamic trail the angle of attack decreases up to the subcritical value of \( \tilde{t} \), the cascade should transfer into the non-stalled mode of streamlining. However, according to the research results, recovery of the non-separated mode of blade ring streamlining takes place under the angles of attack \( \tilde{i}_0 \) that are significantly lower than the critical angles of attack \( i_{cr} \) which initiated the flow separation.

The higher the intensity of the aerodynamic trail (the higher is the value of inability to calculate according to the angle of attack \( \tilde{t} \)), the more significant is the difference between the angles \( i' \) and \( i_{cr} \). This is explained by the inert processes of reforming the flow under the change of the angles of attack and appears as a hysteresis loop in quasi stationary performances of the compressor cascades. Hysteresis is revealed differently under a different value of inability to calculate by the angle of attack and depends on the intensity of aerodynamic trail, kinematic performances and geometrical parameters of the blade rings.

Figure 3 shows the dependence of the inability to calculate by hysteresis \( \tilde{i}_0 \) on the parameter of inability to calculate by the cascade angle of attack \( \tilde{t} \): for single row cascades with viscosity of \( (b/t) \leq 0.5 \) the effect of hysteresis is revealed with superseding of the critical values of \( i_{cr} \) by the angle of attack \( \tilde{t} \) by 10–20%. In this case during the shift of the cascade from the supercritical mode of streamlining to the subcritical (non-stalled) angle of attack that recovers the uniform initial streamlining by 3-18% less than \( i_{cr} \); for single row cascades with viscosity of \( (b/t) \leq 2.0 \) the effect of hysteresis is revealed under the value of inability to calculate \( \tilde{t} = 1.12 – 1.25 \) and stipulates a decrease of \( i_0 \) by 4 – 20% (in the range of angle of attack shift \( i_0 < i < i_{cr} \)); for single row cascades with viscosity of \( (b/t) \leq 1.0 \) the effect of hysteresis is revealed under the value of inability to calculate \( \tilde{t} = 1.25 – 1.35 \) and implies a decrease of \( i_0 \) by 2 – 13% (in the range of angle of attack shift \( i_0 < i < i_{cr} \)).

![Fig.3. Dependence of hysteresis on inability to calculate cascade streamlining](image-url)

The research has shown no effect of hysteresis under lower values of the inability to calculate.

Demonstration of hysteresis effect in typical decelerating cascades under the inability to calculate flow \( \tilde{t} > 1.25 \) has a number of peculiarities: for single row cascades of aerofoils with the angles of midline curve \( \theta = 30^\circ \) the change of viscosity from \( (b/t) = 0.5 \) to \( (b/t) = 2.0 \) leads to a neglect of hysteresis effect. If under \( (b/t) = 0.5 \) \( \tilde{t} = 1.25 \) hysteresis appears in the angle of attack \( i_0 \) is by 40-48% less than \( i_{cr} \) and the initial streamlining is recovering then at \( (b/t) = 2.0 \) this discrepancy equals 8-12%.
At low values of viscosity \((b/t) \leq 1.0\) an increase in the angle of slip from \(\varphi = 0^\circ\) to \(\varphi = 30^\circ\) leads to a decrease of \(\tau_b\) corresponding the values from 0.6 to 0.52. At \((b/t) > 1.5\) a decrease of the angle of slip results in less obvious effect of hysteresis and \(\tau_b\) changes from 0.85 to 0.98. The results of research of cascades composed of aerofoils with different angles of midline curve \(\theta_0 = 10^\circ; \theta_2 = 20^\circ; \theta_3 = 30^\circ\) showed that an increase in the angle of aerofoil curve for the single row cascades causes a more significant effect of hysteresis. Moreover, the effect is the most significant in the cascades with low viscosity.

**Hysteresis of performances of axial compressor stage**

To assess the effect from hysteresis on the performances of the blade rings of axial compressor stages we shall introduce a parameter of hysteresis [11]:

\[
\tilde{q} \lambda = \frac{q \lambda_n - q \lambda_s}{q \lambda_s},
\]

where \(q \lambda_s\) — value of expense parameter that corresponds to the shift from the non-separated performance to a separated one at \(Sr = 0\) (fig. 4); \(q \lambda_n\) — point of shift from the separated performance to a non-separated one.

\[\pi_s^{*}\]

\[\pi_{te} = \text{const}\]

![Fig. 4. Hysteresis of performances of axial compressor stage](image)

Parameter of hysteresis assesses the effect from the periodical oscillating flow non-uniformity during the shift from the non-separated mode of compressor stage to a separated one and vice versa.

It is convenient to present the performances of axial compressor stages in the process of analysis of stalled and non-stalled modes of operation as a correlation \(\pi_n = f(\tau_n, q(\lambda))\), where \(\tau_n\) is the reduced frequency of rotor revolving; \(q \lambda\) is the value of air flow. Actually, a decrease in \(q \lambda\) is accompanied by a decrease in the angles of attack in blade rings.

Non-separated (regular) performance (upper curve in fig. 4) corresponds with streamlining of blade rings at subcritical modes, subcritical angles of attack and, accordingly, under absence of a developed stall of flow. The separating performance (lower curve, fig.4) corresponds with the constant separating streamlining of the blade ring. Periodical shift of axial compressor stage from the separating to non-separating mode \((BC)\) and back \((AD)\) under a periodical oscillating flow non-uniformity in front of the operating wheel occurs at different values of the expenses function. In fact, the stage shifts from separated to non-separated performance conditioned by air flow value is larger than at the beginning of the separation. The angles of attack of blade rings in this case are lower than critical angles of attack that corresponded with the initial values of the developed separation. Aerodynamic performances largely depend on the processes that take place in the boundary layer on the blades. Lag of gas-dynamic processes in the boundary layer under periodical change of the angle of attack resulting from periodical oscillating flow non-uniformity in front of cascades are the main reason for effect of hysteresis on the performances of compressor cascades.

The research of unsteady streamlining of compressor cascades, when kinematic parameters of flow and aerodynamic parameters of aerofoils periodicaly change, should be carried out after a detailed study of hysteresis effect on performances of compressor cascades under their slow transfer to the supercritical modes and back.

To assess the effects of flow non-uniformity on occurrence of separated streamlining of blade rings we introduce a parameter of inability to calculate [11]:

\[
\tilde{q} \lambda = \frac{q \lambda_s - q \lambda_n}{q \lambda_n},
\]

where \(q \lambda_s\) — beginning of separation at \(Sr \neq 0\;\); where \(q \lambda_n\) — value of the expense parameter that corresponds to the shift from the non-separated to separated mode at \(Sr = 0\).

The given parameter describes the effect on the flow stability in interblade channels of axial compressor of periodical flow non-uniformity.

Fig. 5 illustrates the generalized correlations \(\tilde{q} \lambda = f(\pi_s^{*})\) and \(\tilde{q} \lambda = f(Sr)\) for the researched stages. Analysis of these correlations shows that the stages of axial compressors the oscillating periodical non-uniformity leads to a neglect of the hysteresis.
effect at values of Strouhal numbers \( Sr > 0.3 \pm 0.4 \). This can be explained by the fact that for the blade rings that are streamlined by a periodically non-uniform oscillating flow stable separated streamlining happens at lower angles of attack than for the blade rings that are streamlined by a uniform stable flow. In such cases, point \( A \) that corresponds to the stall occupies a position of \( A' \) and the hysteresis loop in the stage characteristics practically vanishes (fig. 5).

![Fig. 5. Dependence of the hysteresis parameter \( \tilde{q}(\lambda) \) and of the inability to calculate \( \tilde{q}(\lambda) \) on intensity of flow unsteadiness at inlet to the axial compressor stage: - \( \Delta - \bar{\eta}_{w}=1.0 \); - \( \circ - \bar{\eta}_{w}=0.835 \); - \( \ast - \bar{\eta}_{w}=0.665 \)](image)

Periodical non-uniformity of the flow leads to earlier flow separation of the axial compressor: at values \( Sr > 0.25 \pm 0.35 \) the flow separation (shift of stages performances from non-stalled to the stalled ones) occurs at 10–15% higher values of air flow than in other stages that are streamlined by steady flows. The effect from the change of Strouhal number on performances of stage is similar to the effect from periodical flow non-uniformity on the performances of two-dimensional cascades. In fact, the cascades experience significant effect under the range \( Sr = 0.3 \pm 0.5 \), whereas the range for significant effect in axial stages is \( Sr = 0.3 \pm 0.4 \).

**Conclusions**

1. Unsteady streamlining of a cylindrical solid, isolated backswept wing of a model aircraft, compressor cascades, blade rings of axial compressor stage under the change of kinematic flow parameters is accompanied by the effect of hysteresis. The basis of hysteresis is the influence of viscous effects on the flow in the boundary layer on the streamlined surfaces. As hysteresis significantly affects the aerodynamic performances of aerofoils, streams in the decelerated channels of compressor cascades and the performances of axial compressor stages, the problem of defining the generalized parameters for connecting the effect of hysteresis in the aforementioned elements presents immediate interest.

2. Overall analysis of the research results lets us conclude that under the values of the inability to calculate \( \tilde{q}(\lambda) > 1.1 \pm 1.2 \) the performances of typical compressor cascades the effect of hysteresis occurs in quasi-stationary streamlining which defines the degradation of compressor cascades efficiency under their flow operation which shows particular non-uniformity at subcritical and critical values of the angles of attack. For typical compressor cascades at values of Reynolds numbers \( Re = 10^{5} \pm 10^{6} \) the range of significant effect of periodical oscillating flow non-uniformity on the cascades’ performances corresponds to the values of Strouhal numbers \( Sr = 0.2 \pm 0.4 \).

3. At streamlining of the axial compressor stages by the periodically non-uniform flow the effect of hysteresis in the pressure lines streaming does not show under \( Sr > 0.2 \pm 0.3 \).

In the stages of axial compressors with inlet guide vanes the flow separation from the blade surface happens 3-5% earlier (according to the parameter of loss \( q(\lambda) \)) than in the stages without inlet guide vanes. This is explained by the initiation of stall from the blade surface by the periodically non-uniform flow at subcritical angles of attack of blade rings.

4. With the purpose of decreasing the hysteresis effect on the performances of the blade rings, promotion of their stalling performances it is necessary to gain velocity and pressure recovery at compressor inlet by way of affecting the aerodynamic trails behind the inlet guide vane.
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