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A DIAGNOSTICS PROBLEM OF A-POSTERIOR PROBABILITY DETERMINATION VIA BAYES' FORMULA OBTAINED IN THE MULTI-OPTIONAL HYBRID FUNCTIONS ENTROPY CONDITIONAL OPTIMIZATION WAY

The paper theoretically considers the possibility of the multi-optional hybrid functions entropy conditional optimization principle applicability with the purpose of discovering one more substantiated reason for the Mr. Thomas Bayes' Theorem Formula existence, as well as the reasons for the formula optimality. The Bayes' formula for a hypothesis' a-posterior probability is obtained on the basis of multi-optional optimality doctrine with taking into account the degree of uncertainty, in the view of the entropy member, for a certain type hybrid-optional effectiveness functions. With the help of the variational principle it is shown the optimality of the Bayes' formula. The approach is applicable to the variety of issues since it has a significance of a plausible explanation for phenomena stipulated by multi-optionality.

Keywords: *Bayes' formula, a-posterior probability, hypothesis, optimization, entropy extremization principle, multi-optionality, hybrid optional function, optimal distribution, variational problem.*

Introduction. Engineering diagnostics predictions at maintenance influence a lot the decisions made in order to avoid damages got due to the problems of friction and wear encountered in operation of, let us say, aeronautical engineering [1] when one should take into consideration the issues of reliability, risks and so on [2].

State of the problem. Safety endeavors, undoubtedly related with risks and reliabilities [2], therefore probabilities [3], in aeronautical engineering operation generally [1] and in its particular sorts of aircraft noise assessment, prediction and control [4], progressive techniques development [5; 6], accompanying processes evolutions for radio flight support operational systems [7], issues of aircraft navigation and piloting [8], have instigated through decades and will definitely continue to instigate the scientifically grounded research in the fields of knowledge discovering.

In this respect, the identified research gap is still the lack of the newly emerged theories (like of the subjective preferences' one, the foundation stone is the Subjective Entropy Maximum Principle (SEMP) developed by the impressively eminent theoretician, Professor Kasianov V. A. (National Aviation University, Kyiv, Ukraine) [9, 10], and for present days being undergone the evolution into the doctrine of multi-optional conditional optimality of special hybrid-optional effectiveness functions uncertainty [11–25]) connections to the well-known concepts having already been developed through the centuries (regarding the presented paper objectives it is the diagnostics based upon the Mr. Thomas Bayes' Theorem Formula [3]).

Problem setting. According to the state of the problem, it is required to find the Mr. Thomas Bayes' Theorem Formula [3] following a certain variational principle of multi-optional conditional optimality of special hybrid-optional effectiveness functions uncertainty [11–25], similar to SEMP [9;, 10].

Purpose of the paper. The presented paper is aimed at discovering the substantiated reasons for an a-posterior probability of a hypothesis existence and to demonstrate, on such an example, the multi-optional hybrid functions entropy conditional optimization principle applicability.

Multi-optional concept. One can present the process of random events happenings as a multi-optional problem. Generally speaking it is supposed that occurrence of an event, let us designate it as A, has a possibility to be realized with just each one of the set's hypothesis of H_i and only. That is the classical problem setting according to the reference of [3], as well as which can be found in many study books in multiple of interpretations and with a brilliant collections of numerous examples.

However, for us now, it is important to treat the conditions of the complete group of disjunctive events (hypothesis) of H_i, the probability of which is indicated as a rule as $P(H_i)$, and conditional probability of $P(A|H_i)$ of the event of A occurrence on condition that the hypothesis of H_i has been realized, denoted in the traditional manner as well as that, like multi-optional chances with an optimal, in a certain respect, and objectively existing on some needed to be revealed reason, hybrid multi-optional function of $f[\cdot]$, pertaining with a-posterior probability of the hypothesis of H_i realization on condition that the diagnostic event of A have already taken place, $P(H_i|A)$ stands for the corresponding probability of such case.

The things to be taken into consideration here are: 1) "optionality's" or option's (optional) effectiveness of the probability of the events of H_i and A happening in conjunction, i.e. $P(H_i \,\text{and}\, A)$, in a logarithmic style: $\ln[P(H_i)P(A|H_i)]$, allowing a representation of that in a linear combination: $\ln[P(H_i)]+\ln[P(A|H_i)]$; with 2) taking into account the corresponding multi-optional hybrid effectiveness function of $f[\cdot]$, of the a-posterior probability: $f\left[P_{H_i}^{(posterior)}\right]$; and 3) uncertainty of the hybrid-optional effectiveness function of $f[\cdot]$.

The most important here is to understand that there must be some optimality in the framework of the nature things "optionality". The approach similar to seeking after preferences in subjective analysis, SEMP [9, 10], and applied to hybrid optional optimal distribution densities findings [11-25], allows implementing the objective functional of the following kind:

$$G_f = -\sum_{i=1}^{n} f\left[P_{H_i}^{(posterior)}\right]\ln\left\{f\left[P_{H_i}^{(posterior)}\right]\right\}+\beta\sum_{i=1}^{n} f\left[P_{H_i}^{(posterior)}\right]\left\{\ln[P(H_i)]+\ln[P(A|H_i)]\right\}+ \\ +\gamma\left[\sum_{i=1}^{n} f\left[P_{H_i}^{(posterior)}\right]-1\right], \quad (1)$$

where β and γ are the internal structural parameters of the hybrid optional functions $f\left[P_{H_i}^{(posterior)}\right]$ distribution (conditional optimal distribution of the a-posterior probabilities functions with respect to the functions' degree of uncertainty and regarding to the logarithmic values of conjunctive events of H_i and A chances corresponding probabilities) as an uncertain Lagrange multipliers for the options' (optional) effectiveness:

$\ln[P(H_i)]+\ln[P(A|H_i)]$ and normalizing condition envisaged with last member of the functional of (1), together β and γ are analogous to the parameters characterizing a system's intrinsic hybrid optimal optional behavior [11-25], likewise for the active element's psych [9, 10], SEMP, (endogenous parameter for the functions of the optional effectiveness $\ln[P(H_i)]+\ln[P(A|H_i)]$ and uncertain Lagrange multiplier for the normalizing condition $\sum_{i=1}^{n} f\left[P_{H_i}^{(posterior)}\right]-1$ respectively).

Thus, we propose to use an optimization method which resembles SEMP of subjective analysis, but the proposed method differs absolutely from SEMP [9, 10], since, being applied for the optional effectiveness $\ln[P(H_i)]+\ln[P(A|H_i)]$, the method does not imply or consider any of active elements of the system at all [11-25]. Only objectively existing characteristics of the H_i and A chances probabilities, however, presupposed with the background of the parameter of β and of the a-posterior probabilities distribution uncertainty are utilized.

The first member of the objective functional (1) is the exact distribution uncertainty parameter in the view of the optional hybrid effectiveness functions' entropy like also discussed at [11-25].

The necessary conditions of functional (1) extremum existence yield

$$\frac{\partial G_f}{\partial f\left[P_{H_i}^{(posterior)}\right]} = -\ln f\left[P_{H_i}^{(posterior)}\right]-1+\beta\ln[P(H_i)P(A|H_i)]+\gamma=0, \tag{2}$$

$$f\left[P_{H_i}^{(posterior)}\right]=e^{\gamma-1+\beta\ln[P(H_i)P(A|H_i)]}=e^{\gamma-1}e^{\ln[P(H_i)P(A|H_i)]^{\beta}}=e^{\gamma-1}[P(H_i)P(A|H_i)]^{\beta}, \tag{3}$$

$$\sum_{j=1}^{n} f\left[P_{H_j}^{(posterior)}\right]=1=\sum_{j=1}^{n} e^{\gamma-1}[P(H_j)P(A|H_j)]^{\beta}, \tag{4}$$

$$f\left[P_{H_i}^{(posterior)}\right]=\frac{[P(H_i)P(A|H_i)]^{\beta}}{\sum_{j=1}^{n}[P(H_j)P(A|H_j)]^{\beta}}. \tag{5}$$

Which (equation (5)) is, at the parameter of

$$\beta=1 \tag{6}$$

value, absolutely exactly nothing more than the a-posterior probability of a diagnosis (by Mr. Thomas Bayes' Theorem Formula) [3]. That is

$$f\left[P_{H_i}^{(posterior)}\right]\Big|_{(\beta=1)} \equiv P(H_i|A). \tag{7}$$

$$G_f\Big|_{(\beta=1)} \equiv G_{[P(H_i|A)]} = -\sum_{i=1}^{n} P(H_i|A)\ln[P(H_i|A)]+\sum_{i=1}^{n} P(H_i|A)\ln[P(H_i)P(A|H_i)]+ +\gamma\left[\sum_{i=1}^{n} P(H_i|A)-1\right]. \tag{8}$$

Discussion on the proposed approach. The proposed approach (1-8) is different from the entirely probabilistic way of Bayes' formulae derivation [3]. It has been taken into account the multi-optional conditional optimality of the special hybrid-optional effectiveness functions uncertainty [11-25] doctrine. Internal parameter of the optimization β influences the optimal solution significantly. It might be considered as the value bearing the meanings of subjectivisms (likewise in SEMP [9, 10]).

Thus, in the way of (1-8), we have found the functional value (8), to which the Bayes' formulae of a-posterior probabilities of hypotheses deliver the optimal value.

Conclusions. It is discovered an explanation for the Bayes' formula in terms of the multi-optional conditional optimality doctrine for the special hybrid-optional effectiveness functions uncertainty. Parameters of the objective functional need further investigation.

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ДІАГНОСТИЧНА ПРОБЛЕМА ВИЗНАЧЕННЯ АПОСТЕРІОРНОЇ ЙМОВІРНОСТІ ЧЕРЕЗ ФОРМУЛУ БАЙЕСА, ОТРИМАНУ ШЛЯХОМ УМОВНОЇ ОПТИМІЗАЦІЇ ЕНТРОПІЇ БАГАТООПЦІЙНИХ ГІБРИДНИХ ФУНКЦІЙ

Стаття розглядає теоретично можливість застосування принципу умовної оптимізації ентропії багато-опційних гібридних функцій з метою відкриття іще однієї обґрунтованої причини існування формули теореми м-р Байеса, а також причин оптимальності тієї формули. Формула Байеса для апостеріорної ймовірності гіпотези отримується на основі доктрини багато-опційної оптимальності з урахуванням ступеня невизначеності, у вигляді ентропійного члена, певного типу гібридно-опційних функцій ефективності. За допомогою даного варіаційного принципу показано оптимальність формули Байеса. Даний підхід є таким, що має можливість бути застосовуваним до найширшого кола питань, оскільки він має значущість правдоподібного пояснення для явищ, обумовлених багато-опційністю.

Ключові слова: формула Байеса, апостеріорна ймовірність, гіпотеза, оптимізація, принцип екстремізації ентропії, багато-опційність, гібридна опційна функція, оптимальний розподіл, варіаційна задача.

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