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A. V. GONCHARENKO

National Aviation University, Kyiv, Ukraine

GENERALIZATION FOR THE DEGRADING STATE MAXIMAL PROBABILITY IN THE FRAMEWORK OF THE HYBRID-OPTIONAL ENTROPY CONDITIONAL OPTIMALITY DOCTRINE

The paper theoretically considers the possibility of the multi-optional hybrid functions entropy conditional optimization doctrine applicability with the purpose of discovering the substantiated reasons for the degrading state maximal probability, however in a not probabilistic way, as well as the reasons for the generalized formula optimality. The maximum of the degrading state probability is obtained with taking into account the degree of uncertainty for a certain type hybrid-optional effectiveness functions. With the help of the variational principle it is shown the optimality of the formula. The approach has a significance of a plausible explanation for phenomena stipulated by the multi-optionality.

Keywords: *probability maximum, optimization, entropy doctrine, multi-optionality, hybrid optional function, optimal distribution, variational problem.*

Introduction. Because of the friction processes and resulting wear an engineering unit goes into damaged and after that into failure state. Reliability is a very significant factor [1]. Even if it not just friction and wear results, but also, for instance, functional coatings application [2] or power losses of gear systems [3]. In order to prevent those negative consequences periodical maintenance is carried out [1, 4, 5].

State of the problem. The optimal periodicity is designated, prescribed, and scheduled in accordance with various subjectively preferred requirements [6-8]. One of such strategies envisages the optimal periodicity predetermined and established by the probabilities of the engineering devices acceptable states. The states dynamical characteristics with the maximum probability of a non-failure state can sometimes be considered as a proper criterion for the optimal periodicity of the engineering units' maintenance [9-12].

Problem setting. The proposed approach (doctrine) likewise in [10-12] is based upon the Jaynes' principle [13] and subjective entropy maximum principle [6-8, 14-16]. It resembles [17], however in actual fact follows [11].

It is considered a general case with the following three states: "0" designates the up state of the system; "1" – damage due to friction and wear; "2" – failure. Parameters of λ_{ij} and μ_{ji} are intensities of the flows of the corresponding events causing the system to transfer from state to state.

Purpose of the paper. Considering a simplified system of the possible discrete states randomly changed in time deemed to be a continuum [17], it is proposed to find the maximum of the probability through the multi-optional hybrid functions entropy conditional optimization doctrine.

Traditional concept. The solution is obtained with the help of the corresponding system of the linear ordinary differential equations of the first order by Erlang [17], where P_i are the probabilities of the corresponding states and t is time.

For example, at the initial conditions of

$$P_0\big|_{t=t_0} = 1, \qquad P_1\big|_{t=t_0} = P_2\big|_{t=t_0} = 0, \qquad t_0 = 0, \tag{1}$$

for the Laplace transformants (images) F_i of the probabilities of P_i, one has

$$F_0 = \frac{p^2 + pa_1 + b_1}{p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right)}, \tag{2}$$

where p is complex parameter (variable) of the Laplace transformation;

$$a_1 = \mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10}, \qquad b_1 = \lambda_{12}\mu_{20} + \mu_{10}\mu_{20} + \mu_{10}\mu_{21}, \tag{3}$$

$$e_1 = \mu_{20} + \mu_{21} + \lambda_{12} + \mu_{10} + \lambda_{01} + \lambda_{02}, \qquad c_1 = \lambda_{01}\mu_{20} + \lambda_{01}\mu_{21} + \lambda_{02}\mu_{21}, \tag{4}$$

$$d_1 = \lambda_{01}\lambda_{12} + \lambda_{02}\lambda_{12} + \lambda_{02}\mu_{10}. \tag{5}$$

$$F_1 = \frac{p\lambda_{01} + c_1}{p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right)}, \qquad F_2 = \frac{p\lambda_{02} + d_1}{p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right)}. \tag{6}$$

And for the initial functions of P_i (or originals) the method of (1-6) yields

$$P_0(t) = \frac{b_1}{k_1 k_2} + \left(1 - \frac{k_2 + a_1 + \frac{b_1}{k_2}}{k_2 - k_1} - \frac{b_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{k_2 + a_1 + \frac{b_1}{k_2}}{k_2 - k_1}\right) e^{k_2 t}, \tag{7}$$

where

$$k_1 = \frac{-e_1 + \sqrt{e_1^2 - 4 f_1 g_1}}{2 f_1}, \quad k_2 = \frac{-e_1 - \sqrt{e_1^2 - 4 f_1 g_1}}{2 f_1}, \quad f_1 = 1, \quad g_1 = b_1 + c_1 + d_1. \tag{8}$$

The two other probabilities, except (7), but with (8), are

$$P_1(t) = \lambda_{01} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} + \frac{c_1}{k_1 k_2} + \left(-\frac{c_1}{k_2(k_2 - k_1)} - \frac{c_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{c_1}{k_2(k_2 - k_1)}\right) e^{k_2 t}. \tag{9}$$

$$P_2(t) = \lambda_{02} \frac{e^{k_1 t} - e^{k_2 t}}{k_1 - k_2} + \frac{d_1}{k_1 k_2} + \left(-\frac{d_1}{k_2(k_2 - k_1)} - \frac{d_1}{k_1 k_2}\right) e^{k_1 t} + \left(\frac{d_1}{k_2(k_2 - k_1)}\right) e^{k_2 t}. \tag{10}$$

The first derivatives of either Eq. (9) or (10) lead to the wanted optimal solutions.

Multi-optional concept. Consider the options essential to the system.

Objective functional

$$\Phi_h = -\sum_{i=1}^{3} \left[x F_1^{(i)}\right] \ln\left[x F_1^{(i)}\right] - \frac{t_p^*}{\lambda_{01}} \sum_{i=1}^{3} \left[x F_1^{(i)}\right]\left(M_{12}^{(i)}\right) + \gamma \left[\sum_{i=1}^{3} \left[x F_1^{(i)}\right] - 1\right], \tag{11}$$

where x is an unknown parameter; $h_i = xF_1^{(i)}$ is the multi-optional hybrid functions depending upon the options effectiveness functions of $F_1^{(i)}$; t_p^*/λ_{01} is the intrinsic parameter of the system and the process, which is the ratio of the optimal (delivering the sought maximal value to the probability) time t_p^* of the maintenance periodicity to the flow intensity λ_{01}; $M_{12}^{(i)}$ is the algebraic addition of the initial elementary intensities matrix $\mathbf{M}$, formed in the style likewise from the Erlang's system, element of m_{12}; γ is the parameter, coefficient, function (uncertain Lagrange multiplier, weight coefficient) for the normalizing condition. In the objective functional of (11)

$$F_1^{(i)} = \frac{M_{12}^{(i)}}{\Delta(\mathbf{M})} = \frac{k_i \lambda_{01} + c_1}{p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right)}, \tag{12}$$

where $\Delta(\mathbf{M})$ is the determinant of the initial elementary intensities matrix $\mathbf{M}$; k_i are the roots of (8); and $k_3 = 0$. In equation (12)

$$M_{12}^{(i)} = k_i \lambda_{01} + c_1, \qquad \Delta(\mathbf{M}) = p\left(p^2 + pe_1 + b_1 + c_1 + d_1\right) . \tag{13}$$

Consider an extremum existence necessary conditions for the objective functional of (11):

$$\frac{\partial \Phi_h}{\partial h_i} = \frac{\partial \Phi_h}{\partial \left[xF_1^{(i)}\right]} = 0, \qquad \forall i \in \overline{1,3}. \tag{14}$$

$$\ln\left[xF_1^{(1)}\right] + \frac{t_p^*}{\lambda_{01}}\left(\lambda_{01}k_1 + c_1\right) = \gamma - 1 = \ln\left[xF_1^{(2)}\right] + \frac{t_p^*}{\lambda_{01}}\left(\lambda_{01}k_2 + c_1\right). \tag{15}$$

$$t_p^* = \frac{\ln\left(k_1\lambda_{01} + c_1\right) - \ln\left(k_2\lambda_{01} + c_1\right)}{k_2(\cdot) - k_1(\cdot)}. \tag{16}$$

Now one ought to say that for the situation when the probability of $P_2(t)$ undergoes the extremum instead of the probability of $P_1(t)$, the problem, **due to the symmetry**, has a symmetrical solution:

$$t_p^* = \frac{\ln\left(\lambda_{02}k_1 + d_1\right) - \ln\left(\lambda_{02}k_2 + d_1\right)}{k_2 - k_1}. \tag{17}$$

Thus, the same approach as (11-16) is applicable to $F_2^{(i)}$ with yielding the parallel to the Eq. (16) results (17).

The optimal for the objective functional (11) distribution of the multi-optional hybrid functions is as follows

$$h_i[\cdot] = \frac{e^{-\frac{t_p^*}{\lambda_{01}}\left[M_{12}^{(i)}(\cdot)\right]}}{\sum_{j=1}^{3} e^{-\frac{t_p^*}{\lambda_{01}}\left[M_{12}^{(j)}(\cdot)\right]}}. \tag{18}$$

Conclusions. It is discovered that the system, according to the developing stationary Poison flow process, has the possible states optimal options related with either the system of parameters $\{k_i, \lambda_{02}, d_1\}$ or $\{k_i, \lambda_{01}, c_1\}$ values (16-18) for the initial moment probability of the state "0" being equaled to "1".

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А. В. ГОНЧАРЕНКО

УЗАГАЛЬНЕННЯ ДЛЯ МАКСИМАЛЬНОЇ ЙМОВІРНОСТІ ДЕГРАДАЦІЙНОГО СТАНУ В РАМКАХ ДОКТРИНИ УМОВНОЇ ОПТИМАЛЬНОСТІ ГІБРИДНО-ОПЦІЙНОЇ ЕНТРОПІЇ

Стаття розглядає теоретично можливість застосування доктрини умовної оптимізації ентропії багатоопційних гібридних функцій з метою відкриття обґрунтованих причини існування максимальної ймовірності деградаційного стану, однак не ймовірнісним шляхом, а також причин оптимальності тієї узагальненої формули. Максимум ймовірності деградаційного стану отримується з урахуванням ступеня невизначеності певного типу гібридно-опційних функцій ефективності. За допомогою даного варіаційного принципу показано оптимальність формули. Даний підхід має значущість правдоподібного пояснення для явищ, обумовлених багатоопційністю.

Ключові слова: максимум ймовірності, оптимізація, ентропійна доктрина, багатоопційність, гібридна опційна функція, оптимальний розподіл, варіаційна задача.

Гончаренко Андрій Вікторович – д-р техн. наук, доцент, професор кафедри збереження льотної придатності авіаційної техніки, Навчально-науковий аерокосмічний інститут, Національний авіаційний університет, проспект Космонавта Комарова, 1, м. Київ-58, Україна, 03680, тел.: +380669550225, E-mail: andygoncharenco@yahoo.com.