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A. V. GONCHARENKO

National Aviation University, Kyiv, Ukraine

EXPONENTIAL DISTRIBUTION DENSITY DERIVED WITH THE HELP OF THE MULTI-OPTIONAL HYBRID FUNCTIONS ENTROPY CONDITIONAL OPTIMIZATION

The paper theoretically considers the possibility of the multi-optional hybrid functions entropy conditional optimization principle applicability with the purpose of discovering the substantiated reasons for the exponential distribution density existence. Exponential distribution density for a random value is obtained on the basis of multi-optional optimality concept with taking into account the degree of uncertainty, in the view of the entropy member, for a special sort hybrid-optional effectiveness functions rather than on the basis of probabilistic speculations and derivations. With the help of the variational principle it is shown the optimality of the distribution density. The approach is applicable to the variety of issues since it has a significance of a plausible explanation for phenomena stipulated by multi-optionality.

Keywords: *exponential distribution, distribution density, parameter of distribution, optimization, entropy extremization principle, multi-optionality, hybrid optional function, optimal distribution, variational problem.*

Introduction. The issues of friction and wear are tightly connected; and they finally result in problems of reliability. The regular model to predict the technical state of engineering equipment is based upon the Poisson law of the events flow and exponential distribution law of the time interval between failures.

State of the problem. Touching the problem of friction and wear related with maintenance and reliability [1], one can notice that, either it is the general problem of operation, like in aircraft and its powerplant maintenance and repair field of industry [2–4] or accompanying the aircraft operation noise troubles [5], the random values accordingly to Poisson law of the events flow and exponential law of the time interval between failures distribution really transfer the related systems from state into state.

The identified research gap here deals not only with the protective coatings for turbine blades [3; 4] or aircraft noise concerns [5], but also with the scientifically plausible explanation of uncovering the reasons for the observed phenomena even if they have already had some solutions obtained from other ideas.

Problem setting. It is required to find exponential distribution density following a certain variational principle, similar to the Subjective Entropy Maximum Principle (SEMP) developed by the outstanding theoretician, Professor Kasianov V. A. (National Aviation University, Kyiv, Ukraine) [6] and nowadays being transformed into the concept of multi-optional optimality of special hybrid-optional effectiveness functions uncertainty [7–23].

Purpose of the paper. The goal of the presented paper is to discover the substantiated reasons for exponential distribution density existence and demonstrate, on such an example, the multi-optional hybrid functions entropy conditional optimization principle applicability.

Traditional theory approach. As it is well-known the probability density of an exponentially distributed continuous random value is given with the formula of

$$f(x) = \lambda e^{-\lambda x}, \tag{1}$$

where $f(x)$ is the density of the exponential distribution; x is the continuous random value; λ is the parameter of the distribution of x by exponential law.

One way of (1) derivation, it can be found in many study books, is as follows. We present the derivation's concise version with emphasizing the principal moments.

Consider at the axis Ot of time t a simplest flow Π of events as an unlimited sequence of random points. Distinguish an arbitrary fragment of time with the duration of τ. In probabilities theory it was proven that, at the conditions of: stationary, absence of after-action, and ordinariness, the number of points getting into the fragment of τ is distributed accordingly to the Poisson law with the expectation of

$$a = \lambda\tau, \tag{2}$$

where λ is the flow density. The probability that in time interval τ there happens exactly m events is equaled to

$$P_m(\tau) = \frac{(\lambda\tau)^m}{m!} e^{-\lambda\tau}, \qquad \text{then} \qquad P_0(\tau) = e^{-\lambda\tau}. \tag{3}$$

This is the probability that the fragment will be empty (there will be no one event). The important characteristic of the flow is the law of distribution of the interval length between two arbitrary neighboring events: T. Consider a random value of T and find its function of distribution:

$$F(t) = P(T < t) \qquad \text{and} \qquad 1 - F(t) = P(T \ge t). \tag{4}$$

This is the probability that at the fragment of the length of time t, starting at the moment t_k of appearance of one of the events of the events flow, there will not appear any one of the following events. Since the simplest flow has no after-action, then the presence of some event at the beginning of the fragment (at the point of t_k) in no way exerts an influence upon the probability of those or other events appearances in the future. Therefore the probability of $P(T \ge t)$ can be calculated by the formula of (4):

$$P_0(t) = e^{-\lambda t}, \qquad \text{from where} \qquad F(t) = 1 - e^{-\lambda t},\ (t > 0). \tag{5}$$

Differentiate the second equation of (5) it gives the density of the distribution (1):

$$f(t) = \lambda e^{-\lambda t}, \qquad (t > 0). \tag{6}$$

Multi-optional concept. On the other hand one can present the process of random points' distribution along the Ox axis as a multi-optional problem. The things to be taken into consideration in this case are: 1) "optionality" of the values of x; with 2) taking into account the value of the flow density λ; and 3) uncertainty of supposed random value x probability distribution density $f(x)$.

The most important here is to understand that there must be some optimality in the framework of the nature things "optionality". The approach similar to seeking after preferences in subjective analysis [6], and applied to hybrid optional optimal distribution densities findings [7–23], allows implementing the objective functional of the following kind:

$$G_f = \int_0^\infty \left[-f(x)\ln f(x) - \lambda f(x)x\right]dx + \gamma\left[\int_0^\infty f(x)dx - 1\right] - \ln\Delta x, \tag{7}$$

where γ is the internal structural parameter of the hybrid optional distribution function $f(x)$ (random value x probability distribution density) as an uncertain Lagrange multiplier for the normalizing condition envisaged with the second equation of (5), together λ and γ are analogous to the parameters characterizing a system's intrinsic hybrid optimal optional behavior [7-23], likewise for the active element's psych [6] (endogenous parameter for the function of the optional effectiveness x and uncertain Lagrange multiplier for the normalizing condition $\int\limits_0^\infty f(x)dx - 1$ respectively); Δx is the degree of accuracy at the hybrid optional function distribution density entropy (analogous to the subjective entropy of the preferences) determination [7-23].

Thus, we propose to use an optimization method which resembles SEMP of subjective analysis [6], but the proposed method differs absolutely from SEMP [6], since, being applied for a continuous optional value x, the method does not imply or consider any of active elements of the system at all [7-23]. Only objectively existing characteristics of a continuous random value probability distribution density, however, presupposed with the background of the density of the probability distribution uncertainty are utilized. The first integral member of the objective functional (7) is the exact distribution uncertainty parameter in the view of the distribution's optional function's entropy like also discussed at [7-23]. The sign "minus" in front of the hybrid optional effectiveness function: $\lambda f(x)x$ (the second integral member of the objective functional (7)) means the existence of relatively higher density distribution $f(x)$ values in areas pertaining with lower optional effectiveness function: x.

The necessary conditions of functional (7) extremum existence are given in the view of the well-known Euler-Lagrange equation. Then

$$\frac{\partial F^*}{\partial f'(x)} \equiv 0 \quad \text{and} \quad \frac{\partial F^*}{\partial f(x)} = 0, \qquad \frac{\partial F^*}{\partial f(x)} = -\ln f(x) - 1 - \lambda x + \gamma = 0, \tag{8}$$

where F^* is the underintegral function of the integral of (7); $f'(x)$ is the first derivative of the sought after probability density distribution function of $f(x)$ with respect to x. The third equation of (8) yields

$$\ln f(x) = \gamma - 1 - \lambda x, \ f(x) = e^{\gamma-1-\lambda x} = e^{\gamma-1}e^{-\lambda x}, \qquad \int\limits_0^\infty f(x)dx = 1 = \int\limits_0^\infty e^{\gamma-1}e^{-\lambda x}dx,$$

$$e^{\gamma-1} = \frac{1}{\int\limits_0^\infty e^{-\lambda x}dx}, \qquad f(x) = \frac{1}{\int\limits_0^\infty e^{-\lambda x}dx} e^{-\lambda x}. \tag{9}$$

The integral in the denominator of (9) yields

$$\int\limits_0^\infty e^{-\lambda x}dx = -\frac{1}{\lambda}e^{-\lambda x}\bigg|_0^\infty = -\frac{1}{\lambda}e^{-\lambda\cdot\infty} + \frac{1}{\lambda}e^{-\lambda\cdot 0}, \ -\frac{1}{\lambda}e^{-\lambda\cdot\infty} \to 0 \text{ and } \frac{1}{\lambda}e^{-\lambda\cdot 0} = \frac{1}{\lambda}\cdot 1. \tag{10}$$

Therefore finally

$$\int\limits_0^\infty e^{-\lambda x}dx = 0 + \frac{1}{\lambda} = \frac{1}{\lambda} \qquad \text{and} \qquad f(x) = \frac{1}{\frac{1}{\lambda}}e^{-\lambda x}. \tag{11}$$

At last (1) again, but obtained in a different, (7-11), from probabilistic way of derivation (2-6).

Discussion on the proposed approach. Proposed approach engaging an uncertainty measure in type of entropy, applied for distribution density hybrid optional functions optimization, allows finding exponential distribution density, without probabilities determination, in a new multi-optional way. The accepted suppositions are the spreading of a random value having the existence of the distribution density uncertainty suspected in delivering an extremal value to some objective functional.

As a result, it is revealed that exponential distribution density is optimal for an objective functional including the distribution's density entropy, as well as taking into account, with the flow density, the higher probability density for smaller magnitudes of the random value. This is so obvious in widely spread exponential distribution.

Conclusions. Such approach and interpretations broaden the horizons of scientific explanations for occurring exponential distribution optimality; and it encourages further research in the field of hybrid optional functions optimal distributions.

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А. В. ГОНЧАРЕНКО

ЩІЛЬНІСТЬ ЕКСПОНЕНЦІАЛЬНОГО РОЗПОДІЛУ ВИВЕДЕНА ЗА ДОПОМОГОЮ УМОВНОЇ ОПТИМІЗАЦІЇ ЕНТРОПІЇ БАГАТО-ОПЦІЙНИХ ГІБРИДНИХ ФУНКЦІЙ

Стаття розглядає теоретично можливість застосування принципу умовної оптимізації ентропії багато-опційних гібридних функцій з метою відкриття обґрунтованих причин існування щільності експоненціального розподілу. Щільність експоненціальний розподілу випадкової величини отримується на основі концепції багато-опційної оптимальності з урахуванням ступеня невизначеності, у вигляді ентропійного члена, спеціального сорту гібридно-опційних функцій ефективності, а не на основі ймовірнісних міркувань та виведень. За допомогою даного варіаційного принципу показано оптимальність даної щільності розподілу. Даний підхід є таким, що має можливість бути застосовуваним до найширшого кола питань, оскільки він має значущість правдоподібного пояснення для явищ, обумовлених багато-опційністю.

Ключові слова: експоненціальний розподіл, щільність розподілу, параметр розподілу, оптимізація, принцип екстремізації ентропії, багато-опційність, гібридна опційна функція, оптимальний розподіл, варіаційна задача.

Гончаренко Андрій Вікторович – д-р техн. наук, доцент, професор кафедри збереження льотної придатності авіаційної техніки, Навчально-науковий аерокосмічний інститут, Національний авіаційний університет, проспект Космонавта Комарова, 1, м. Київ-58, Україна, 03680, тел.: +380669550225, +380684417071, +380933382376, Skype name: andygoncharenco, E-mail: andygoncharenco@yahoo.com.