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GRAPH THEORY APPLAYING FOR QUANTITATIVE ESTIMTION OF UAV'S GROUP FLIGHT INTO AERIAL PHOTOGRAPHY

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Abstract*—*The paper deals with the problem of performance of Unmanned Aerial Vehicles group flights for decision of different tasks. In paper criteria of efficiency for the performance of tasks of Unmanned Aerial Vehicles group flight are analyzed. It was used graph theory for quantity estimation of effectiveness of Unmanned Aerial Vehicles group flight. Also it was intended flight group performance criteria for all types of Unmanned Aerial Vehicles connections (fully connected, a star, ring, tree, with a common tire, mixed, cellular).

Index Terms—Unmanned Aerial Vehicle; efficiency; reliability and topologies survivability; graph theory.

I. INTRODUCTION

Unmanned aerial vehicle (UAV) has several advantages, namely low operating cost, good concealment, resistance and flexibility, simplicity and availability of technology compared to aircraft with pilot on board and UAV can be used in cases where the usage of piloted aircraft is impractical, expensive or risky [1], [2]. The main advantages of using UAVs are tasks that involve risk to humans and efficiency in solving economic problems.

Practice shows that often aerial photographs and cartographical materials of small plots with an area of 30 square kilometers, with the scale of 1:5000 and larger are claimed for the purposes of such as landscape design / development of architectural and environmental solutions; monitoring of forest fires, traffic congestion agricultural fields; the design of local line network (power lines, roads local roads, gas and oil pipelines); assessment of profitability and more [2], [3].

Optical systems in unmanned aerial technology used to review the area and monitoring the situation on the area, obtaining detailed images of terrain and objects in them, identifying objects in the area and visually invisible [1], [2]. Firstly let us define the criteria for aerial photography:

– ground resolution (how many centimeters on the ground will match 1 pixel image);

– scale of produced photo plan (pre-select of the focal length and height limits of photographing the lower and upper bar – as the possible difference in this altitude, due to the relative instability of the aircraft);

– the working area of the image (will depend on parameters such as focal length and height of the photograph).

Decomposition of the technological processes by creating aerial photo for using group of UAV's is shown on Fig. 1 [2], [3].

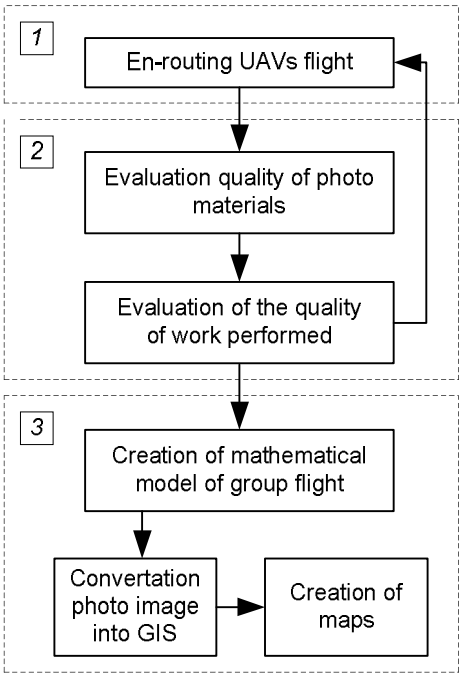

Fig. 1. Decomposition of process of performance the UAV's group flight aerial photo

Stages of technological processes by creating aerial photo for using group of UAV':

1. Carrying out UAV's flight route and getting the original aerial photographs [2], [3].

2. Analysis of the resulting photographic material and evaluation of the quality of performed work. For estimation of the quality of work let us use multi-criteria relationship, taking into account the quality and the quality of aerial photographs, covering the study area. When the result is negative – it will be decided on additional flights [2], [3].

3. Construction of mathematical models of the flight and the determination of the models parameters of external orientation. Exterior orientation data derived from the mathematical modeling of the UAV's

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flight transmitted into the external system for further processing and creation of photo plan [2], [3].

There are three main types of aerial photographs quality criteria [3]:

1. The presence of distortions associated with poor visual perception: a violation of the white balance of the color image [3].

2. The presence of the distortion associated with the excess of the permissible limits of external orientation of aerial photographs: exceeding the tolerance of the optical axis of the aerial photos of the average of the route; exceeding the permissible extent of the difference of adjacent aerial photographs [3].

3. The presence of conditions that do not meet the requirements for the degree of overlap of technology of allied aerial photographs [3].

Quality rating of scanning flights is done by the following criteria:

– the quality of the photographic image;

– the value of the longitudinal and transverse overlap;

– avoidance of the vertical axis of the camera;

– a straight path;

– avoidance of specified flight altitude of the aircraft.

Aerial identified deficiencies are eliminated. Block layout is photographed on a small scale – getting a reproduction block layout. It is used for the preliminary studying of the area.

Obviously, the effectiveness of group flying UAVs in monitoring forest fires, search and rescue operations in the processing of agricultural crops, relay communications and the movement of goods is much higher than in single UAV flights. In this sense, the use of UAVs is more appropriate: to relay communications in those places – where the antenna coverage cannot be set because of difficult terrain, agriculture (group of spraying fields), with aerial photography (group survey large areas, monitoring of forest fires, patrol areas, etc.), moving cargo. Also, UAVs was used for military purposes since 1961 [1].

The disadvantages of UAV include the limited capacity due to the small size UAV that can be compensated for the use of group flight [4]. In addition, the group of small and light UAV effectively interact appear more useful qualities: speed task, increasing the density of surface area when spraying crops, and so on [5]. For example, usage group of UAV important to improve the performance targets: photo/video monitoring, agricultural work, retransmission of communications, cargo movement, as well as in the other cases where a group of UAVs quickly and efficiently perform the task.

II. Problem Statement

Let we have a group from n UAVs. They are performed different tasks purposes. To manage a group of UAVs is expedient to apply graph theory as mathematical tools for modeling. It is proposal to represent UAV's group's configuration as a graph with n tops (UAVs). They connected by m arcs between each other – information data channels for UAV's control. It is necessary to calculate the efficiency of UAV structure using methods of graph theory.

Consider the problem of usage of UAV's group flight by aerial photography like an example. It is proposed to build a scheme with group of UAVs, the objectives of which are to maximize the overlap area for map information.

So, the main problem is lack of optimization UAV's group operations depending of the task purpose.

III. An Overview of Existing Solutions

Advantages of UAV's are to perform the tasks associated with the risk for man and effectiveness in solving economic problems. In this sense, the use of UAVs is more appropriate: to relay communications in those places - where the antenna coverage cannot be set because of difficult terrain, agriculture (group of spraying fields), with photo/video monitoring (group survey of large areas, monitoring of forest fires, patrol areas, etc.), moving cargo [4]. Obviously is the use of UAVs for military purposes.

Comparing usage of UAV's group with one UAV we have more useful properties: faster coverage of area fragment and consequently more effective at photo/video monitoring, relay communications, agricultural operations [5] – [7].

But despite of a number of advantages there are some drawbacks, namely the main problem associated with the usage of airspace allocation of the frequency range for UAVs management and transmission of information from the board to the ground; lack of optimization of the structure of the UAV. Therefore, the foregoing has been optimized by graph theory.

In [4], [6] – [9] investigated ununiformed group consisting of UAV helicopter and aircraft types, is complex information exchange through the use of flight control channels, carrier channel, as well as interaction autonomous system elements together. It is noted the complexity of problems in the case management group UAV, which is the inappropriate use of classical control theory.

To manage a group of UAVs is expedient to apply graph theory as mathematical tools for modeling

and optimization of complex structure: in mathematics to solve complex equations; in physics for constructing electronic circuits; in construction for the most rational distribution facilities and the construction of roads; in biology to solve problems of genetics; in the economy for finding solutions minimal cost; in information technology to determining the effectiveness topology of local and global computer networks and other applications [4] – [5].

IV. Topology of an Aircraft Group

For a UAV's group flight advisable to apply graph theory [8]. The group structure may have a different configuration, location and connections between network nodes, the most common of which are: fully connected, a star, ring, tree, with a common tire, mixed, cellular [7], [8]. From UAV's topology, which is performed by the flight of an aircraft group depends on the effectiveness of the task purpose.

For example: for making aerial photography of a fragment area with the help of 5 UAV's let us imagine group flight scheme as an undirected graph G (n; m), which has n nodes (UAV's) and m arcs (connections), that is shown in Figure 1. So, fully connected topology will analyze the effectiveness of the task group UAV (Fig. 2). It corresponds fully connected topology network, where each node is directly connected to all other nodes [7], [8]. For UAV's groups flight applicable criteria of reliability, represented as a structure using graph theory, connectivity, structural redundancy, uneven distribution relationships, structural compactness, degree of centralization in the system survivability (Table I) [8].

Table I

Criteria of Reliability UAV's Group Structure

N	Criteria of reliability	Symbol
1	Graph Connectivity	L
2	Structural redundancy	R
3	The uneven distribution of connections	ε^2
4	Structural compactness	D
5	Level of system centralazing	δ
6	Durability	K

Undirected graph G (n; m) is considered to be connected, if from any node (UAV's) there is the way to other nodes (the path may consist of any number of ribs). In our case the graph is connected, because all UAV's are connected between each other.

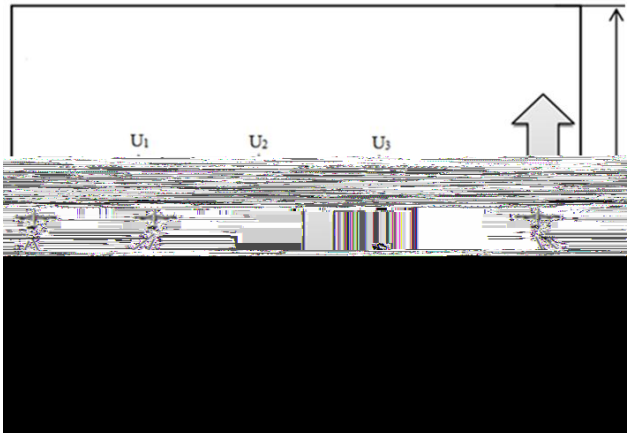

Fig. 2. Presentation a group the UAV's in a fully connected network proceeding aerial photography, $n = 5$; $m = 10$

We represent a group of UAVs using adjacency matrix (Table II), let us calculate criteria of the topology of the structures of the UAV's group.

Table II

Matrix of adjacency of fully connected UAV's group $A = \|Uij\|$

Tops of graph G (n; m)	Tops of graph G (n; m)					
	U_1	U_2	U_3	U_4	U_5	$\sum_j U_{ij}$
U_1	1	1	1	1	1	5
U_2	1	1	1	1	1	5
U_3	1	1	1	1	1	5
U_4	1	1	1	1	1	5
U_5	1	1	1	1	1	5

Connectivity meets the following conditions:

$$L \geq L_{\min},$$

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n} U_{ii} \geq n - 1 = 10,$$

$$L_{\min} = n - 1 = 4,$$

where n is number of UAV's in group; $L_{\min}$ is the minimum number of required connections of UAV's group; U_{ij} are the tops of graph G (n; m), $i = \overline{1,n}$.

So if inequality observed $L \geq L_{min}$, then the scheme is connected.

Determination of *structural redundancy R,* it is exceeding the total number of connections over the minimum necessary.

$$R = \frac{1}{2}\left[\sum_{i=1}^{n}\sum_{i=1}^{n} U_{ii}\right]\frac{1}{n-1} - 1 = 1.5,$$

where U_{ii} are the tops of graph G (n; m), $i=\overline{1,n}$; $j=\overline{1,n}$; n is the number of UAV's in group;

If $R > 0$ it is maximum redundancy, $R = 0$ – minimal redundancy, $R < 0$ – incoherent system [7]. For group of the UAV's as fully connected structure we have $R = 1.5 > 0$? so it is maximum redundancy.

Uneven distribution characterizes connections ε^2 non opportunities structure that has m connections and n tops to achieve maximum connectivity of our structure (Table III):

$$\varepsilon^2 = \sum_{i=1}^{n}\left(\rho_i - \overline{\rho}\right)^2 = \sum_{i=1}^{n}\rho_i^2 - 4\frac{m^2}{n} = 0,$$

where $\rho = \left\|\rho_{ij}\right\|$ is the element of matrix of an incidence; m is the number of the ribs of the UAV's fully connected group structure; n is number of the tops of the UAV's fully connected group structure

If $\varepsilon^2 = 0$, so the fully connected structure of UAV's group has even distribution.

TABLE III

MATRIX OF AN INCIDENCE OF FULLY CONNECTED UAV'S GROUP $\rho = \left\|\rho_{ij}\right\|$

Tops $i=\overline{1,n}$	**Ribs of the graph *G*(*n*; *m*),** $j=\overline{1,m}$											
	1	2	3	4	5	6	7	8	9	10	$\sum_{j=1}^{n}\rho$	$\sum_{j=1}^{n}\rho^2$
U_1	1	1	0	0	0	1	1	0	0	0	4	16
U_2	0	1	1	0	0	0	0	1	1	0	4	16
U_3	0	0	1	1	0	0	1	0	0	1	4	16
U_4	0	0	0	1	1	1	0	0	1	0	4	16
U_5	0	0	0	0	1	1	1	0	0	0	4	16

Structural compactness parameter indicates the proximity to each other via a minimum chain length d_{ij}.

$$D = \sum_{i=1}^{n}\sum_{j=1}^{m} d_{ij} = 30.$$

where d_{ij} is the minimum distance between tops in graph G (n; m) of the UAV's group structure.

The structural compactness index characterizes the diameter of the structure: d is maximum value of d_{ij}.

Values d and D_{rel} integrally characterize the inertia in the system, at equal values of R and their increase reflects the growing number of connections (Table IV). Putting distance matrix whose elements d_{ij} is defined as the minimum distance between nodes i and j. Total intimacy of the elements in the UAV's group equal 30: it means if the meaning of D is more lees so the system is more compactness. The relative rate from:

$$D_{\text{rel}} = \frac{D}{D_{\min}} - 1 = \frac{30}{20} - 1 = 0.5,$$

$$D_{\min} = n(n-1) = 20.$$

TABLE IV

DISTANCE MATRIX $D = \left\|d_{ii}\right\|$

Tops of graph *G* (*n*; *m*),	**Tops of graph *G*(*n*; *m*),**					
	U_1	U_2	U_3	U_4	U_5	$\sum_{j=1}^{n} d_{ij}$
U_1	0	1	2	2	1	6
U_2	1	0	1	2	2	6
U_3	2	1	0	1	2	6
U_4	2	2	1	0	1	6
U_5	1	2	2	1	0	6
$\sum_{i=1}^{n}\sum_{i=1}^{n} d_{ij}$						30

Centralizing level of system is determined by the index of centrality, calculated for a group of the equation:

$$\delta = (n-1)(2Z_{\max} - n)\frac{1}{Z_{\max}(n-2)} = 0,$$

$$Z_i = \frac{D}{2}\left(\sum_{i=1}^{n} d_{ii}\right)^{-1} = 5, \quad i=\overline{1,n}, \quad i \neq j,$$

where $Z_{\max}$ is the maximum rate of Z_i; n is number of UAV's in group; Z_i is the index of centrality of the system; D is the structural compactness of the UAV's fully connected group structure.

To assess the degree of irregularity (C_i) load elements of UAVs group structures and the degree of centralization system uses the notion of the centrality of its individual elements C_i, calculated using the equation:

$$C_i = \frac{\sum_{i=1}^{n}\sum_{i=1}^{n} d_{ii}}{2\sum_{i=1}^{n} d_{ii}} = 5,$$

where d_{ii} is the minimum distance between tops of structure U.

Maximum centrality of any of the knots (UAV's)

Relative peripherality of a knot:

$$\Pi_i = C_{\max} - C_i = 0.$$

Durability of the group is defined by the number of states, in which the network keeps working.

Durability can be regarded as the most objective and adequate indicator which enables the best to evaluate all aspects of structural and functional reliability of networks, which is in the environment

that is constantly changing and subjected to permanent modernization in order to improve the quality of its operation. [5].

$$K = \frac{\sum_{i=1}^{n} U_i - 2(n-1)}{2(n-1)} = 2.125,$$

$K > 0$, the loss of at least one communication of structure remains operational, where n is the number of UAV's in group; U_{ii} are the tops of graph $G(n; m)$, $i = \overline{1, n}$.

Using graph theory we can determine the effectiveness of group structures in different types of UAV's formation. The type of formation (structure of UAV's group), which performed the flight of an aircraft's group depends on the effectiveness of their task purpose.

It is intended flight group performance criteria for all types of connections UAV (Table V, Fig. 3).

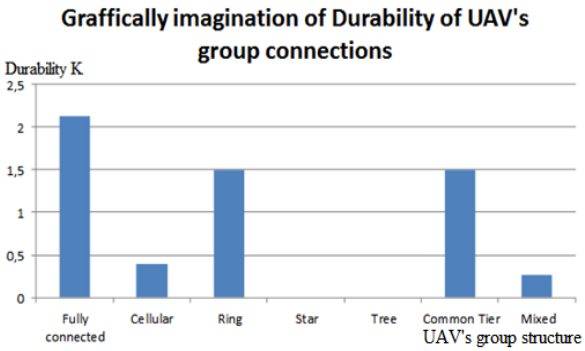

Fig. 3. Graphically representation of group durability

TABLE V

PERFORMANCE CRITERIA OF GROUP FLIGHTS FOR DIFFERENT TOPOLOGIES OF UAVS GROUPS

Scheme	Connectivity	Marks							
		R	ε^2	***D***	D_{rel}	***d***	δ	**Π**	***K***
Fully connected	+	1.5	0	30	0.5	2	0	0	2.125
Cellular	+	0.4	47.3	68	1.26	3	0.23	4.63	0.4
Ring	+	2.5	105	50	1.5	4	0	0	1.5
Star	-	0	13.3	50	0.6	2	1	22.5	0
Tree	-	0	5.42	96	1.28	4	0.0325	4.38	0
Common tire	+	2.5	-23.8	80	3	6	0.33	3.3	1.5
Mixed	+	1.26	-9.14	755	3.14	9	0.67	101.98	0.26

V. CONCLUSIONS

UAV's groups configuration was represented as a graph. It was calculated the efficiency of UAV structure using graph theory. Also, was performed optimization of the UAV's group operations depending of the task purpose.

Thus, the creation of topology groups UAV advisable to focus on fully connected topology, as the most effective. Further research should be directed to the solution of practical problems of implementation of group management in driving the UAV, which leads to more efficient use of UAVs, namely the possibility of an adjustment plan and optimize the flight route, based on data already obtained from other UAVs; enhance the probability of success of the task; a significant gain in time; simultaneous examination of the territory and the simultaneous increase in the area of monitoring; The possibility of setting different tasks for members of multi-UAV considering efficiency topology groups.

REFERENCES

[1] Y. Ivanov, A. Bruises, and I. Filatov. *Integration of information-measuring devices of aircraft*: Proc. manual for schools, Engineering, Leningrad, 1984.

[2] M. Yarlikov. *Statistical theory of navigation*. Moscow: Radio and Communications, 1985.

[3] V. Sineglazov, and F. Zakharin. "Peculiarities of future air navigation system based on inertial navigation systems/global positioning system", *Electronics and Control Systems*, vol. no. 4(34), Kyiv: National Aviation University, 2012, pp. 84–90.

[4] M. Filyashkin, V. Rogozhin, and A. Skripets. *Inertial-satellite navigation systems*. Kyiv: National Aviation University, 2009, pp. 199–268.

[5] N. Deputat, Y. Smislova, and M. Filyashkin. "Correction inertial navigation systems speed and positional correctors", *Proceedings VI International Scientific Conference "AVIA 2004"*, vol. 2, Kyiv: National Aviation University, 2004, pp. 67–70.

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Т. Ф. Шмельова, Д. І. Бондарєв. Застосування теорії графів для групових польотів БПЛА під час проведення аерофотозйомки

Проаналізовано критерії ефективності для виконання завдань польоту групи БПЛА і застосування теорії графів для кількісної оцінки ефективності групового польоту БПЛА. Розраховано критерії ефективності групового польоту для всіх типів з'єднань БПЛА (повнозв'язна, зірка, кільце, дерево, загальна шина, чарункова, змішана).
Ключові слова: безпілотний літальний апарат; ефективність; надійність; живучість топологій.

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Т. Ф. Шмельова, Д. І. Бондарев. Применение теории графов для групповых полетов БПЛА при проведении аэрофотосъемки

Проанализированы критерии эффективности для выполнения задач полета группы БПЛА и применения теории графов для количественной оценки эффективности группового полета БПЛА. Рассчитаны критерии эффективности группового полета для всех типов соединений БПЛА (полносвязная, звезда, кольцо, дерево, общая шина, ячеистая, смешанная).
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