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GEOMETRIC ATTITUDE DETERMINATION USING VECTOR MEASUREMENTS

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Abstract—*The geometric algorithms of attitude determination using vector measurements are suggested. Matrix or quaternion algebras are not used. The comparison with algorithm QUEST is fulfilled.*

Index Terms—Attitude determination; algorithm.

I. INTRODUCTION

In the article on the basis of information about two vectors the geometric algorithm of body attitude determination is examined. In the known matrix algorithms [1], [2] angles of orientation are determined from the matrix of orientation. At the using of quaternion the axis of turn and angle of turn of the body are determined from a quaternion of turn [3], [4]. On the proposed algorithms matrix or quaternion algebras are not used.

II. PROBLEM STATEMENT

Suppose we know the projections of two normalized vectors in reference ($\vec{e}_{on}$, $\vec{s}_{on}$) and body ($\vec{e}_n$, $\vec{s}_n$) coordinate systems. A problem is determine the axis unite vector and rotation angle of the body coordinate system relative to the reference coordinate system. I.e. vectors are immobile but their projections in two noted coordinate systems are variable.

Change the problem definition. Suppose that the vectors $\vec{e}_n$ and $\vec{s}_n$ rotate in the direction opposite to rotation of body coordinate system relative reference coordinate system. The trajectories of the ends of the vectors are circles with centers on the axis of rotation (Fig. 1).

III. SOLUTION OF THE PROBLEM

Specify vectors $\vec{a}_1$ and $\vec{a}_2$ that characterize displacements of the ends of the vectors (Fig. 1). These vectors are situated in the planes of the noted circles and are perpendicular to the axis of rotation.

Consider possible cases.

1. Vector $\vec{a}_1$ or vector $\vec{a}_2$ equals zero. Suppose, that $\vec{a}_1 = 0$. In this case vector $\vec{e}$ does not change, i.e. the axis of rotation coincides with the vector $\vec{e}_{on}$.

Specify the unit vector $\vec{b} = \vec{e}_{on}$ (Fig. 2), directed along the axis of rotation. The axis unite vector of rotation which characterizes direction of rotation define as follows

$$\vec{b}_{1n} = zn \cdot \vec{b}, \tag{1}$$

where

$$zn = -\text{sign}\left(\vec{v}_1 \cdot \vec{v}_2\right);$$
$$\vec{v}_1 = \vec{b} \times (\vec{s}_n + \vec{s}_{on});$$
$$\vec{v}_2 = \vec{s}_n - \vec{s}_{on}.$$

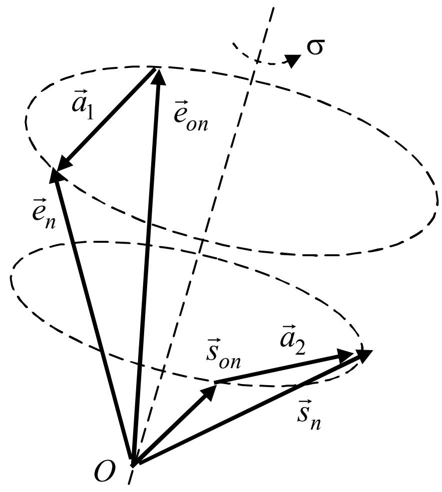

Fig. 1. Common case

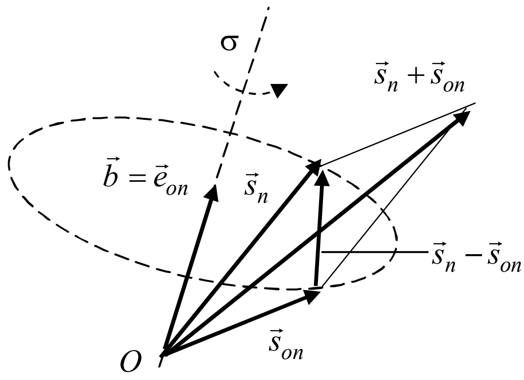

Fig. 2. Case 1

Here the sign of "–" is accepted because of we examine the rotation of vectors, but not system of coordinate.

The rotation angle is defined as a dihedral angle between the plane, formed by the vectors $\vec{b}_{1n}$ and $\vec{s}_{on}$, and the plane, formed by the vectors $\vec{b}_{1n}$ and $\vec{s}_n$. For this purpose specify vectors

$$\vec{p}_1 = \vec{b}_{1n} \times \vec{e}_{on}; \quad \vec{p}_2 = \vec{b}_{1n} \times \vec{e}_n, \tag{2}$$

which determine perpendiculars to the corresponding planes. Then a rotation angle is an angle between these vectors

$$\sigma = \arccos(\vec{p}_1 \cdot \vec{p}_2) / \text{norm}(\vec{p}_1) / \text{norm}(\vec{p}_2). \tag{3}$$

A similar analysis takes place if $\vec{a}_2 = 0$. In this case

$$\vec{p}_1 = \vec{b}_{1n} \times \vec{s}_{on}; \quad \vec{p}_2 = \vec{b}_{1n} \times \vec{s}_n. \tag{4}$$

2. The vectors $\vec{a}_1$ and $\vec{a}_2$ are nonparallel, i.e. $\vec{a}_1 \times \vec{a}_2 \neq 0$. The vector of axis rotation will define thus

$$\vec{b} = \vec{a}_1 \times \vec{a}_2. \tag{5}$$

The axis unite vector $\vec{b}_{1n}$ is searched after a equation (1), and rotation angle – after a equation (3).

3. Vectors $\vec{a}_1$ and $\vec{a}_2$ are parallel (Fig. 3). Here $\overrightarrow{OA_1} = \vec{e}_{on}$, $\overrightarrow{OB_1} = \vec{e}_n$, $\overrightarrow{OA_2} = \vec{s}_{on}$, $\overrightarrow{OB_2} = \vec{s}_n$ or $\overrightarrow{OA_3} = \vec{s}_{on}$, $\overrightarrow{OB_3} = \vec{s}_n$. At first consider the case, when vectors $\vec{a}_1$ and $\vec{a}_2$ coincide after direction. Triangles OA_1B_1 and OA_2B_2 are similar, therefore

$$\mu = \frac{\text{norm}(\vec{a}_2)}{\text{norm}(\vec{a}_1)} = \frac{A_2B_2}{A_1B_1} = \frac{B_1O_1 - B_1B_2}{B_1O_1},$$

or $B_1O_1 = \dfrac{B_1B_2}{1-\mu}$. In a vector form it can be written down as $\overrightarrow{B_1O_1} = \dfrac{\overrightarrow{B_1B_2}}{1-\mu} = \dfrac{1}{1-\mu}(\vec{s}_n - \vec{e}_n)$.

Then

$$\vec{b} = \overrightarrow{OO_1} = \overrightarrow{OB_1} + \overrightarrow{B_1O_1} = \vec{e}_n + \frac{1}{1-\mu}(\vec{s}_n - \vec{e}_n). \tag{6}$$

Consider the case, when vectors $\vec{a}_1$ and $\vec{a}_2$ are opposite after direction. Then

$$\mu = \frac{B_1B_3 - B_1O_1}{B_1O_1}, \text{ or } B_1O_1 = \frac{B_1B_2}{1+\mu}.$$

In a vector form it can be written down as

$$\overrightarrow{B_1O_1} = \frac{\overrightarrow{B_1B_2}}{1+\mu} = \frac{1}{1+\mu}(\vec{s}_n - \vec{e}_n).$$

Then

$$\vec{b} = \overrightarrow{OO_1} = \overrightarrow{OB_1} + \overrightarrow{B_1O_1} = \vec{e}_n + \frac{1}{1+\mu}(\vec{s}_n - \vec{e}_n). \tag{7}$$

It is possible connect expressions (5) and (6) in one equation

$$\begin{aligned} \vec{b} &= \vec{e}_n + \frac{1}{1-\text{sign}(\vec{a}_1 \cdot \vec{a}_2)\mu}(\vec{s}_n - \vec{e}_n) \\ &= \vec{e}_{on} + \frac{1}{1-\text{sign}(\vec{a}_1 \cdot \vec{a}_2)\mu}(\vec{s}_{on} - \vec{e}_{on}). \end{aligned} \tag{8}$$

The axis unit vector of rotation $\vec{b}_{1n}$ is searched after a equation (1) and rotation angle – after a equation (3).

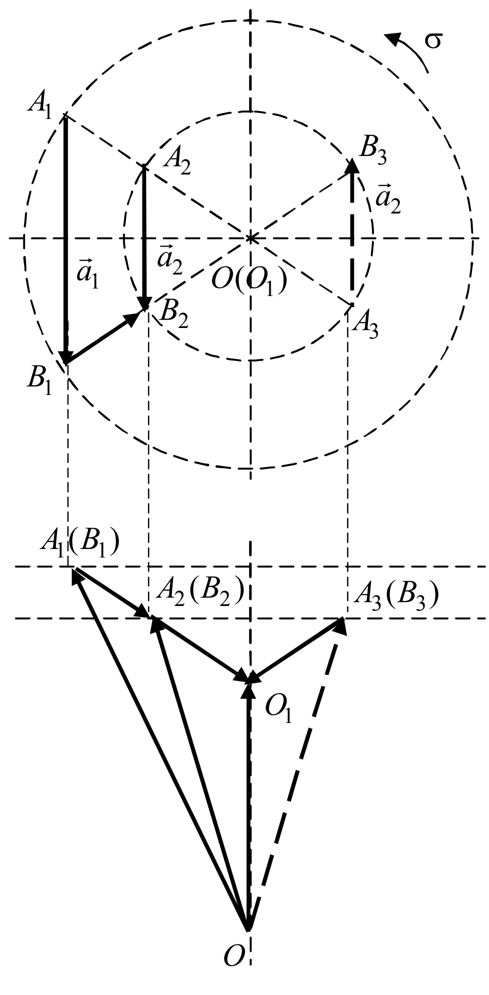

Fig. 3. Case 3

The disadvantage of this algorithm is that cross product $\vec{b} = \vec{a}_1 \times \vec{a}_2$ which determines the axis of rotation, under certain circumstances equals zero.

Specify the third vector $\vec{c}_{on} = \vec{e}_{on} \times \vec{s}_{on}$ unparallel to the vectors $\vec{e}_{on}$ and $\vec{s}_{on}$, and determine vector $\vec{b}$ which always doesn't equals zero

$$\vec{b} = \vec{a}_1 \times \vec{a}_2 + \vec{a}_2 \times \vec{a}_3 + \vec{a}_3 \times \vec{a}_1, \qquad (9)$$

where $\vec{a}_3 = \vec{c}_n - \vec{c}_{on}$.

The angle of rotation is determined by the equation (3). To choose the vectors $\vec{p}_1, \vec{p}_2$ we do next. From three modules $|\vec{a}_1|$, $|\vec{a}_2|$, $|\vec{a}_3|$ select the biggest and after this assume such vectors: $\vec{e}_{on}, \vec{e}_n$ if $|\vec{a}_1|$ is the biggest, $\vec{s}_{on}, \vec{s}_n$ is the $|\vec{a}_2|$ is the biggest: $\vec{c}_{on}, \vec{c}_n$ is the $|\vec{a}_3|$ is the biggest.

For the estimation of accuracy of algorithm with three vectors will estimate influence of error of vector $\vec{s}$. As vectors are normalized, then only an error which is perpendicular to the vector influences on the accuracy. Therefore vector of error, by a size 0,001, will form perpendicular to the vector $\vec{s}$ with variable direction with a step 10° in plane, perpendicular to the vector $\vec{s}$. The new value of vector $\vec{s}$ is normalized.

We have the good results for all cases.The results of simulation for the cases 2 ($\psi = 10°$; $\theta = 20°$; $\varphi = 30°$; $\vec{e}_o = [10\ 3\ 50]'$; $\vec{s}_o = [2\ 0\ 1]'$) are represented in Fig. 4.

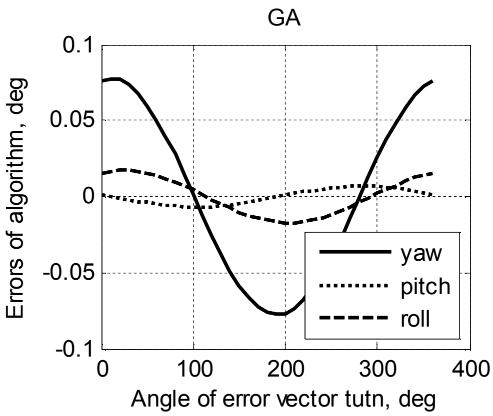

Fig. 4. Errors of geometric algorithm

For comparison the error of determination of orientation for this case is calculated by means of the QUEST algorithm (Fig. 5). We can see that the accuracy of the proposed algorithm is equivalent to the accuracy of the algorithm QUEST.

Note that three vectors may be chosen as in TRIAD algorithm

$$\vec{e}_n;\ \vec{m}_n = \frac{\vec{e}_n \times \vec{s}_n}{|\vec{e}_n \times \vec{s}_n|};\ \vec{c}_n = \vec{m}_n \times \vec{e}_n.$$

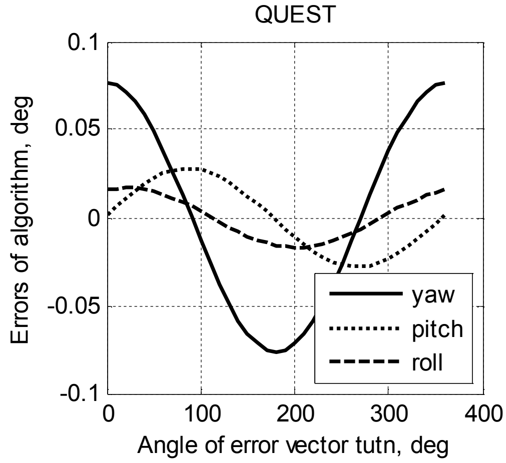

Fig. 5. Errors of algorithm QUEST

VI. CONCLUSION

The geometric algorithm on the base of three vectors is the effective method of attitude determination. His advantage is simplicity (matrix or quaternion algebras not used). After accuracy this algorithm is equivalent to the QUEST algorithm.

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Л. М. Рижков. Геометричне визначення орієнтації на основі вимірювання векторів

Запропоновано геометричне визначення орієнтації на основі вимірювання векторів. Матрична або кватерніонна алгебри не використовуються. Виконано порівняння з алгоритмом QUEST.

Ключові слова: визначення орієнтації; алгоритм.

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Л. М. Рыжков. Геометрическое определение ориентации на основе измерения векторов

Предложено геометрическое определение ориентации на основе измерения векторов. Матричная или кватернионная алгебры не используются. Выполнено сравнение с алгоритмом QUEST.

Ключевые слова: определение ориентации; алгоритм.

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