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[1]M. M. Komnatska,
[2]A. M. Klipa,
[3]Y. A. Berezanskyi,
[4]I. V. Kiliian

ON ONE APPROACH OF OBSERVER-BASED FLIGHT CONTROL SYSTEM DESIGN

Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine
E-mails: [1]martakomnatska@gmail.com, [2]aklipa@mail.ru, [3,4]yariksoffice@gmail.com

Abstract**—***This paper presents a novel approach of observer design for aircraft motion control. The novelty of design procedure is based on the application of linear matrix inequality technique. The design procedure treats the design of both observer and controller by solving the set of linear inequalities simultaneously. The proposed approach is free of observer poles placement location. Simulation results demonstrate the validity and effectiveness of the proposed design approach.*

Index Terms—Flight control system; linear matrix inequalities; observer design; state feedback; separation principle; state estimation.

I. INTRODUCTION

The motion of aircraft is considered in a non-uniform atmosphere. Therefore, the use of control strategy is necessary for completing aircraft mission successful. The application of modern control theory requires all the state variables to be available. Thus, the control systems developed due to modern control theory application increase the complexity of the system. To overcome the requirement of complete state space vector measurements, observer-based control systems design has been considered. The development of observer-based control systems reduces the requirement of full phase vector measurements. Observers avoid complexity to the system and require only computational resources [1]. The observer design was originally proposed in works [2] – [4]. Lately, the numbers of observer-based control system design approaches were proposed [5] – [7].

The design of observer-based flight control systems are successfully applied in the area of small Unmanned Aerial Vehicles (UAV), satisfying manifold requirements imposed on it [5], [8], [9]. In [5] the design strategy involves observer design without reducing the robustness and performance of the system. The required level of performance and robustness is kept due to mixed $\mathbf{H}_2/\mathbf{H}_\infty$ optimization technique.

The survey on observer design is given in [10]. It is shown three main observer design results connected with reduced order, under separation principle and observers for input fault detection and identification.

The autopilot design is also may be performed basing on the available information about the output variables. This circumstance leads to the problem of static output feedback (SOF) controller design. The main advantage of SOF design is that it requires only available signals from the plant to be controlled. Unfortunately, the output feedback problem is much more difficult to solve in comparison to state feedback control problem [11].

The motivation for this research arises from a desire to reduce the number of sensors necessary for multivariable flight control system (FCS) design for civil aircraft. The research concerns on finding appropriate solution under linear matrix inequalities (LMIs) approach [12] for aircraft control during flight envelope.

It is known that the design procedure of observer deals with selecting desired region poles location. Moreover, the observer eigenvalues should be faster up to ten times in comparison to plant eigenvalues. It results in the observer sensitivity to noisy measurement, which is not desirable. To overcome this difficulty the procedure of observer design basing on Lyapunov approach is proposed.

The main result of this paper is the FCS design via LMI technique, where the observer gains and controller structure are defined by solving the set of LMIs, simultaneously.

To demonstrate the validity and efficiency of the proposed approach, the longitudinal motion of the aircraft is considered as a case study.

II. PROBLEM STATEMENT

Let us consider a problem of FCS design with incomplete state vector measurement. The aircraft dynamics is represented by the following set of equations

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}; \\ \mathbf{y} = \mathbf{C}\mathbf{x}, \end{cases} \qquad \mathbf{x}(0) = \mathbf{x}_0, \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^{\mathbf{n}}$ is the state space vector; $\mathbf{u} \in \mathbf{R}^{\mathbf{m}}$ is the control vector; $\mathbf{y} \in \mathbf{R}^{\mathbf{p}}$ is the observation vector.

Besides that, the state space matrices of the controlled plant have the following dimensions $\mathbf{A} \in \mathbf{R}^{\mathbf{n} \times \mathbf{n}}, \mathbf{B} \in \mathbf{R}^{\mathbf{m} \times \mathbf{n}}, \mathbf{C} \in \mathbf{R}^{\mathbf{p} \times \mathbf{n}}$. It could be seen that number of measuring variables $\mathbf{p}$ is less than number of all phase coordinates, $\mathbf{n}$. Thus, to design the FCS the full state space vector is necessary to be restored.

In this paper we develop the procedure of full-order state observer design with further state feedback construction such that the performance of closed-loop system satisfies selected performance criterion. Thus, the FCS design is performed under the well-known separation principle [5].

III. OBSERVER-BASED FLIGHT CONTROL SYSTEM DESIGN

It is known that the observer estimates the state variables based on measurement of the output $\mathbf{y}$ and control $\mathbf{u}$ variables [2] – [4]. Let us consider the procedure of observer-based FCS design under LMI approach.

Consider linear time-invariant system given by (1). Assume that the states $\mathbf{x}$ are approximated by the states $\tilde{\mathbf{x}}$. The observer model takes into account feedback information about observation error and can be represented as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\left(\tilde{\mathbf{y}}(t) - \mathbf{y}(t)\right) \\ &= \mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\mathbf{C}\left(\tilde{\mathbf{x}}(t) - \mathbf{x}(t)\right), \end{aligned} \tag{2}$$

where $\left(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)\right) = \mathbf{e}(t)$ is the difference between the real and estimated states (observation error); $\mathbf{L}$ is the observer gain matrix that has to be chosen such that the observation error approaches zero as time increases. From (1) and (2) the observation error dynamics equation takes the following form

$$\dot{\mathbf{e}}(t) = \left(\dot{\mathbf{x}}(t) - \dot{\tilde{\mathbf{x}}}(t)\right);$$

$$\begin{aligned} \dot{\mathbf{x}}(t) - \dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ &- \left(\mathbf{A}\tilde{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\mathbf{C}\left(\tilde{\mathbf{x}}(t) - \mathbf{x}(t)\right)\right) \\ &= \left(\mathbf{A} + \mathbf{L}\mathbf{C}\right)\mathbf{e}(t). \end{aligned} \tag{3}$$

The error decays to zero if it is possible to find observer gain matrix $\mathbf{L}$ such that $\left(\mathbf{A} + \mathbf{LC}\right)$ is asymptotically stable. Moreover, the eigenvalues of $\left(\mathbf{A} + \mathbf{LC}\right)$ are the same as those of $\left(\mathbf{A} + \mathbf{LC}\right)^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}} + \mathbf{C}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}$.

The final goal is to control the motion of the plant basing on the estimated states. Thus, for the state feedback control based on observed state variables $\tilde{\mathbf{x}}$, namely

$$\mathbf{u} = \mathbf{K}\tilde{\mathbf{x}}, \tag{4}$$

where $\mathbf{K}$ is the constant state feedback gain matrix that assures that the system is asymptotically stable, the state equation becomes

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}\tilde{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \\ &+ \mathbf{B}\mathbf{K}\left(\mathbf{x}(t) - \mathbf{e}(t)\right) = \left(\mathbf{A} + \mathbf{BK}\right)\mathbf{x}(t) - \mathbf{BK}\mathbf{e}(t). \end{aligned} \tag{5}$$

Combining together (3) and (5), we obtain

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \mathbf{BK} & -\mathbf{BK} \\ 0 & \mathbf{A} + \mathbf{LC} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix}. \tag{6}$$

Equation (6) describes the dynamics of the observed state feedback control system. The characteristic equation for the system is

$$\left|s\mathbf{I} - \mathbf{A} - \mathbf{BK}\right|\left|s\mathbf{I} - \mathbf{A} - \mathbf{LC}\right| = 0.$$

It is possible to rewrite the system dynamics in terms of plant and observer states, respectively.

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{BK} \\ -\mathbf{LC} & \mathbf{A} + \mathbf{BK} + \mathbf{LC} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \tilde{\mathbf{x}}(t) \end{bmatrix}.$$

It is supposed also that the obtained solution given by (4) minimizes the performance index as

$$\begin{aligned} J &= \int_0^{\infty} \left(\tilde{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{x}}(t) + \mathbf{u}(t)^{\mathrm{T}}\mathbf{R}\mathbf{u}(t)\right) dt \\ &= \int_0^{\infty} \tilde{\mathbf{x}}(t)^{\mathrm{T}}\left(\mathbf{Q} + \mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\right)\tilde{\mathbf{x}}(t)\, dt, \end{aligned} \tag{7}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are diagonal matrices, weighting each state and control variables, respectively.

This cost depends on the trajectory $\tilde{\mathbf{x}}(t)$ taken, such that the worst trajectory will correspond to the worst cost [13].

It is known that the observed-state feedback control system design consists of two stages: (1) to design a state feedback control law assuming that all states are available; (2) to design a state estimator to estimate states of the system. Replace the states in state feedback control law from stage (1) by the state estimates. Further, they can be combined to form the observed-state feedback control system. This principle of independent state feedback and observer design is referred to as separation principle [5]. Moreover, the observer design deals with choice of poles location. They are usually chosen such that the observer response is much faster that the system response, but very fast observers possess noise. The proposed approach solves the problem of observed-state feedback design under LMI technique. The main advantage of the proposed design procedure is that there is no need to define the observer poles location. The solution of this problem via LMIs gives the constant state feedback gain matrix $\mathbf{K}$ and observer gain matrix $\mathbf{L}$ by solving a set of LMIs simultaneously. The proposed design procedure is very simple and utilizes Lyapunov approach.

The simultaneous observer and controller design can be formulated with following theorem.

Theorem. The observer-based system (6) is said to be statically stable by means of state feedback (4) if there exist matrices $\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} > 0$, $\mathbf{M}$ and $\mathbf{X}_2 = \mathbf{X}_2^{\mathrm{T}} > 0$, $\mathbf{Z}$ and satisfy the following conditions:

$$\begin{bmatrix} \mathbf{X}_1\mathbf{A}^{\mathrm{T}} + \mathbf{A}\mathbf{X}_1 + \mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\mathbf{M} & \mathbf{X}_1\mathbf{Q}^{1/2} & \mathbf{M}^{\mathrm{T}}\mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2}\mathbf{X}_1 & -\mathbf{I} & 0 \\ \mathbf{R}^{1/2}\mathbf{M} & 0 & -\mathbf{I} \end{bmatrix} < 0,$$

$$\mathbf{X}_1 = \mathbf{X}_1^{\mathrm{T}} > 0, \tag{8}$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{X}_2 + \mathbf{X}_2\mathbf{A} + \mathbf{C}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}} + \mathbf{Z}\mathbf{C} < 0,$$

$$\mathbf{X}_2 = \mathbf{X}_2^{\mathrm{T}} > 0. \tag{9}$$

Proof. Let $\mathbf{V}_1(\mathbf{x},t) = \mathbf{x}(t)\mathbf{P}_1\mathbf{x}^{\mathrm{T}}(t)$ with $\mathbf{P}_1 = \mathbf{P}_1^{\mathrm{T}} > 0$ be a candidate Lyapunov function. The closed loop system (6) preserves stability and minimizes the performance index (7) if:

$$\dot{\mathbf{V}}_1(\mathbf{x},t) + \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)R\mathbf{u}(t) < 0. \tag{10}$$

The condition (10) leads to the following inequality:

$$\mathbf{x}^{\mathrm{T}}(t)\left\{\mathbf{A}^{\mathrm{T}}\mathbf{P}_1 + \mathbf{P}_1\mathbf{A} + \mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{P}_1 + \mathbf{P}_1\mathbf{B}\mathbf{K} + \mathbf{Q} + \mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\right\} \times \mathbf{x}(t) < 0.$$

Pre-multiplying and post-multiplying right and left sides above written inequality by $\mathbf{P}^{-1}$:

$$\mathbf{P}_1^{-1}\mathbf{A}^{\mathrm{T}} + \mathbf{A}\mathbf{P}_1^{-1} + \mathbf{P}_1^{-1}\mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\mathbf{K}\mathbf{P}_1^{-1} + \mathbf{P}_1^{-1}\mathbf{Q}\mathbf{P}_1^{-1} + \mathbf{P}_1^{-1}\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\mathbf{P}_1^{-1} < 0. \tag{11}$$

Let us define the following change of variables $\mathbf{X}_1 = \mathbf{P}_1^{-1}$, $\mathbf{M} = \mathbf{K}\mathbf{P}_1^{-1}$, $\mathbf{K} = \mathbf{M}\mathbf{P}_1$ and rewrite inequality (11) as

$$\mathbf{X}_1\mathbf{A}^{\mathrm{T}} + \mathbf{A}\mathbf{X}_1 + \mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\mathbf{M} + \mathbf{X}_1\mathbf{Q}\mathbf{X}_1 + \mathbf{X}_1\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\mathbf{X}_1 < 0. \tag{12}$$

By applying Shur's Lemma to inequality (12) it is possible to rewrite it as matrix inequality:

$$\begin{bmatrix} \mathbf{X}_1\mathbf{A}^{\mathrm{T}} + \mathbf{A}\mathbf{X}_1 + \mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{B}\mathbf{M} & \mathbf{X}_1\mathbf{Q}^{1/2} & \mathbf{M}^{\mathrm{T}}\mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2}\mathbf{X}_1 & -\mathbf{I} & 0 \\ \mathbf{R}^{1/2}\mathbf{M} & 0 & -\mathbf{I} \end{bmatrix} < 0.$$

This part of the proof considers the design stage (1) according to the separation principle. The second part of the proof considers stage (2) of the design procedure connected with observer development.

Let $\mathbf{V}_2(\mathbf{e}(t),t) = \mathbf{e}(t)\mathbf{P}_2\mathbf{e}^{\mathrm{T}}(t)$ with $\mathbf{P}_2 = \mathbf{P}_2^{\mathrm{T}} > 0$ be a candidate Lyapunov function. The observer gains can be found if the following inequality is hold:

$$\mathbf{e}^{\mathrm{T}}(t)\left\{(\mathbf{A} + \mathbf{L}\mathbf{C})^{\mathrm{T}}\mathbf{P}_2 + \mathbf{P}_2(\mathbf{A} + \mathbf{L}\mathbf{C})\right\}\mathbf{e}(t) < 0,$$

or

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}_2 + \mathbf{P}_2\mathbf{A} + \mathbf{C}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{P}_2 + \mathbf{P}_2\mathbf{L}\mathbf{C} < 0.$$

The use of the following change of variables $\mathbf{X}_2 = \mathbf{P}_2$, $\mathbf{P}_2\mathbf{L} = \mathbf{Z}$ reduces to the next LMIs:

$$\mathbf{A}^{\mathrm{T}}\mathbf{X}_2 + \mathbf{X}_2\mathbf{A} + \mathbf{C}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}} + \mathbf{Z}\mathbf{C} < 0,$$

$$\mathbf{X}_2 = \mathbf{X}_2^{\mathrm{T}} > 0.$$

Thus, the observer gains can be evaluated as

$$\mathbf{L} = \mathbf{X}_2^{-1}\mathbf{Z}.$$

IV. CASE STUDY

The state space linearized longitudinal model of large four-fanjet Boeing 747 aircraft flying about equilibrium point (Mach=0.198) is used as a case study.

The main geometrical characteristics of the given aircraft are:

– wing reference area, $S = 5500\,ft^2$;
– wing span, $b = 195.68\,ft$;
– mean geometric chord, $\overline{c} = 27.31\,ft$;

The moments of inertia:

$$I_x = 18.2\times10^6\,slug - ft^2;$$

$$I_y = 33.1\times10^6\,slug - tf^2;$$

$$I_z = 0.97\times10^6\,slug - ft^2;$$

The state space vector of Boeing 747 longitudinal channel comprises the following variables: $\mathbf{x} = [V_t, \alpha, q, \theta, h]^{\mathrm{T}}$, where V_t is the true airspeed of aircraft, α is the angle of attack, q is the pitch, θ is the pitch angle rate and h is the altitude. The control input vector $\mathbf{u} = [\delta_e, \delta_{th}]^{\mathrm{T}}$ is represented by elevator deflection and throttle lever displacement correspondingly.

It is considered operating mode with true airspeed at V_t = 67.4 m/s. The linear model in state space is represented by the matrices $[\mathbf{A}, \mathbf{B}]$:

$$\mathbf{A}=\begin{bmatrix} -0.0209 & 0.0018 & 0 & -9.81 & 0 \\ 0.0030 & -0.5120 & 1.0000 & 0 & 0 \\ 0.0002 & -0.1108 & -0.3736 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \\ 0 & -67.3922 & 0 & 67.3922 & 0 \end{bmatrix};$$

$$\mathbf{B}=\begin{bmatrix} 0.9590 & 0.0001 \\ -0.0953 & 0 \\ -0.3764 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

The output vector of measured variables is given as follows $\mathbf{y}=[q,\ \theta,\ h]^{\mathrm{T}}$. Thus, the observation matrix has the following structure:

$$\mathbf{C}=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Disturbance, $\boldsymbol{\upsilon}$ affecting the longitudinal motion of the aircraft involves the following components: the true airspeed, V_t, angle of attack, α and pitch rate, q, so that $\boldsymbol{\upsilon}=\left[V_{t_g},\ \alpha_g,\ q_g\right]^{\mathrm{T}}$. In order to simulate the atmospheric turbulence the Dryden filter is used [13]. It is considered that aircraft flies at moderate turbulence. To generate a signal with the correct characteristics, a unit variance, band-limited white noise signal is passed through appropriate forming filter.

The transfer functions of forming filter according to standard MIL–F–8785C [13], [14] used in simulation to account external disturbances have the following structure:

$$H_u(\mathrm{s})=\sigma_u\sqrt{\frac{2L_u}{\pi V}}\frac{1}{1+\frac{L_u}{V}s};\qquad H_w(\mathrm{s})=\sigma_w\sqrt{\frac{L_w}{\pi V}}\frac{1+\frac{\sqrt{3}L_w}{V}s}{\left(1+\frac{\mathrm{L}_w}{V}\mathrm{s}\right)^2};\qquad H_q(\mathrm{s})=\frac{\pm\frac{\mathrm{s}}{V}}{\left[1+\left(\frac{4b}{\pi V}\right)\mathrm{s}\right]}H_w(\mathrm{s}).$$

The transfer function of forming filter along the variable w is possible to rewrite in terms of the variable the angle of attack, α according to the phase vector. Thus, for small angles

$$\alpha=\frac{w}{U_0},\ \text{where } U_0=V_t.$$

Parameters appearing in the transfer functions of the forming filter are given as follows [13], [14]:

$$b = 59.64\ \text{m};\ L_V= L_w= 533.3\ \text{m};$$

$$\sigma_V= \sigma_w=1.542\ \text{m/s},$$

where b is the aircraft wingspan; L_V, L_w are the appropriate turbulence scale lengths; σ_V, σ_w are the appropriate turbulence intensities. The computation of these values depends on the altitude at which the aircraft is flying, wingspan and type of turbulence according to standard MIL–F–8785C [14].

The weighting matrices $\mathbf{Q}$, $\mathbf{R}$ in (7) have the structure:

$$\mathbf{Q}=\mathrm{diag}\left(0.024\cdot[0.8001\ 0.1288\ 0.055\ 250.11\ 0.01]\right);$$

$$\mathbf{R}=\mathrm{diag}\left(\begin{bmatrix}2 & 0.4\end{bmatrix}\right).$$

By applying the proposed approach of observer-based controller design under LMI technique, the state feedback gain matrix $\mathbf{K}$ and observer gain matrix $\mathbf{L}$ are found. Their numerical values are given below:

– state feedback gain matrix:

$$\mathbf{K}=\begin{bmatrix} 0.2005 & 5.2133 & -22.4525 & -23.4080 & -0.0377 \\ 0.000027 & -0.00019 & 0.000017 & -0.00020 & 0.0000012 \end{bmatrix};$$

– observer gain matrix:

$$\mathbf{L}=\begin{bmatrix} 0.0182 & -9.8723 & 17.3514 \\ 0.9210 & 0.0688 & -77.8686 \\ 0.1263 & 0.4942 & -0.0855 \\ 0.4815 & 0.5000 & 30.0388 \\ -0.0002 & 33.6840 & 0.6971 \end{bmatrix}.$$

Table 1 reflects standard deviations of the aircraft outputs.

TABLE 1

STANDARD DEVIATIONS OF BOEING 747 OUTPUTS IN A STOCHASTIC CASE

Plant	Standard deviation				
	σ_V, m/sec	σ_α, °	σ_ϑ, °	σ_q, °/sec	σ_h, m
V = 67.4 m/s	0.1634	0.0936	0.0458	0.0134	0.7403

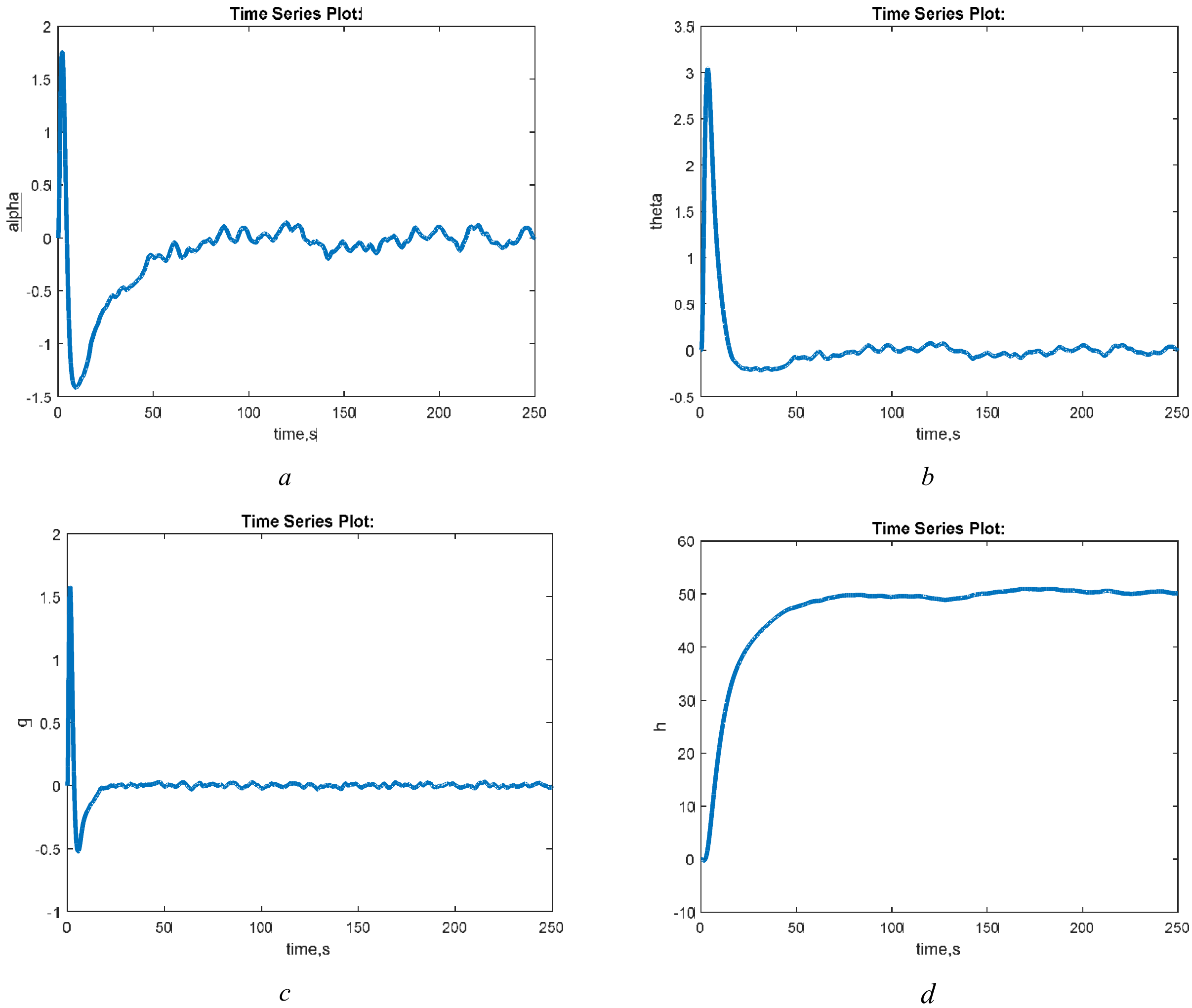
Performance indices of closed loop system with observed state feedback in a loop are given in Table 2.

The simulation results of the closed loop system taking into account the influence of the random wind, simulated according to the standard Dryden model of turbulence confirm the efficiency of proposed approach. Results of the simulation are shown in Figure.

TABLE 2

PERFORMANCE INDICES OF CLOSED-LOOP SYSTEM

Performance Index	Plant
	$V = 67.4$ m/s
$\mathbf{H}_2$-norm	0.6194
$\mathbf{H}_\infty$-norm	1.2791



Simulation results of longitudinal motion control in the presence of external disturbances:
a is the angle of attack, deg; *b* is the pitch angle, deg; *c* is the pitch rate, deg/s; *d* is the altitude, m

CONCLUSIONS

As far as the incomplete state space vector is available for measuring, the flight control system for aircraft can be easily designed by applying observer. Thus, the unavailable states can be suitable approximated by restored states. The proposed solution is very simple and uses Lyapunov approach. The proposed design procedure can be solved efficiently by applying LMI optimization technique. The main advantage of the proposed approach is that there is no need to define the region of observer poles placement. The proposed approach permits to define the observer gains and state feedback gain matrix directly from a set of LMIs, simultaneously.

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Komnatska Marta. Ph. D. Associate Professor.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2007).
Research interests: control systems and data processing.
Publications: 38.
E-mail: martakomnatska@gmail.com

Klipa Antonina. Ph. D. Associate Professor.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2002).
Research interests: systems identification and control.
Publications: 52.
E-mail: aklipa@mail.ru

Berezanskyi Yaroslav. 4th year of study student.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2015).
Research interests: control systems and data processing.
E-mail: yarikfreerun@gmail.com

Kiliian Ivan. 4th year of study student.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2015).
Research interests: control systems and data processing.
E-mail: ivkilnau@gmail.com

М. М. Комнацька, А. М. Кліпа, Я. А. Березанський, І. В. Кіліян. Про один підхід до синтезу системи управління польотом зі спостерігачем

Представлено новий підхід до синтезу системи управління польотом зі спостерігачем. Новизна запропонованого підходу ґрунтується на застосуванні апарату лінійних матричних нерівностей. Процедура синтезу передбачає одночасне визначення спостерігача та регулятора шляхом розв'язання системи лінійних матричних нерівностей. Запропонований підхід не потребує вибору полюсів спостерігача. Результати моделювання демонструють обґрунтованість та ефективність запропонованого підходу.

Ключові слова: система управління польотом; лінійні матричні нерівності; синтез спостерігача; зворотний зв'язок за станом; теорема розділення; оцінювання станів.

Комнацька Марта Миколаївна. Кандидат технічних наук. Доцент.
Кафедра систем управління літальних апаратів, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна (2007).
Напрямок наукової діяльності: системи управління та обробка інформації.
Кількість публікацій: 38.
E-mail: martakomnatska@gmail.com

Кліпа Антоніна Миколаївна. Кандидат технічних наук. Доцент.
Кафедра систем управління літальних апаратів, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна (2002).
Напрямок наукової діяльності: ідентифікація систем та управління.
Кількість публікацій: 52.
E-mail: aklipa@mail.ru

Березанський Ярослав Андрійович. Студент 4-го курсу.
Кафедра систем управління літальних апаратів, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна (2015).
Напрямок наукової діяльності: системи управління та обробка інформації.
E-mail: yarikfreerun@gmail.com

Кіліян Іван Васильович. Студент 4-го курсу.
Кафедра систем управління літальних апаратів, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна (2015).
Напрямок наукової діяльності: системи управління та обробка інформації.
E-mail: ivkilnau@gmail.com

М. Н. Комнацкая, А. Н. Клипа, Я. А. Березанский, И. В. Килиян. Об одном подходе к синтезу системы управления полетом с наблюдателем

Представлен новый поход к синтезу системы управления полетом с использованием наблюдателя. Новизна предложенного подхода основывается на применении аппарата линейных матричных неравенств. Процедура синтеза предусматривает одновременное определение наблюдателя и регулятора путем решения системы линейных матричных неравенств. Предложенный подход не требует выбора полюсов наблюдателя. Результаты моделирования показывают обоснованность и эффективность предложенного подхода.

Ключевые слова: система управления полетом; линейные матричные неравенства; синтез наблюдателя; обратная связь по состоянию; теорема разделения; оценивание состояний.

Комнацкая Марта Николаевна. Кандидат технических наук, доцент.
Кафедра систем управления летательных аппаратов, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина (2007).
Направление научной деятельности: системы управления и обработка информации.
Количество публикаций: 38.
E-mail: martakomnatska@gmail.com

Клипа Антонина Николаевна. Кандидат технических наук, доцент.
Кафедра систем управления летательных аппаратов, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина (2002).
Направление научной деятельности: идентификация систем и управление.
Количество публикаций: 52.
E-mail: aklipa@mail.ru

Березанский Ярослав Андреевич. Студент 4-го курса.
Кафедра систем управления летательных аппаратов, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина (2015).
Направление научной деятельности: системы управления и обработка информации.
E-mail: yarikfreerun@gmail.com

Килиян Иван Васильевич. Студент 4-го курса.
Кафедра систем управления летательных аппаратов, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина (2015).
Направление научной деятельности: системы управления и обработка информации.
E-mail: ivkilnau@gmail.com