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THE SPECIAL CASE OF OPERATIVE PROGRAMMING METHOD OF FLIGHT ROUTE

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Abstract*—The paper presents description algorithm of the trajectory motion correction of an unmanned aerial vehicle. The part operations of algorithm are shown in detail. Example of calculation of returning the unmanned aerial vehicle to a predetermined route line is given.*

Index Terms—Navigation; intermediate route points; correction; turning radius; great circle coordinates; given routing line.

I. INTRODUCTION

One way to ensure trouble-free use of unmanned aerial vehicles (UAVs) is that to prevent the violation of the space-time passing
(STP) of intermediate route points (IRP) and the deviation from the desired altitude. For this is currently developed a number of ways the UAV navigation. This paper proposes an algorithm for improving of navigation method. The essence of the method is to use the standard maps prepared by a known method before driving UAV, choosing dimensional plot area of the reference card and the division of the site into a number of intermediate route points. Then, the reference trajectory of the UAV is composed by using the IRP. The current map is drawn on base of measuring parameters dimensional plot of the reference card by using radio waves. The next operation is to compare the values obtained from dimensional plot of the current and reference cards. The calculation of correction signal is made on basis of definition of measurement results, parameters of the motion control UAV by adjusting its location as it passes of dimensional plot. The disadvantages of this method include the fact that it does not describe an algorithm for adjusting the trajectory of motion of UAV.

II. PROBLEM FORMULATION

Let's an UAV is performing a flight in the horizontal plane on the altitude ($H < 100$ meters) with a constant velocity V and it is necessary with a given accuracy provide the space-time passing of the intermediate routing points. Let is given the displacement of an UAV itself (point E) and great circle coordinates (GCC) of two nearest intermediate routing points: intermediate routing point "*A*", which is passed and intermediate routing point "*B*", which is necessary to pass. Also let is given the great circle coordinates of intermediate routing point "*C*", which follows the rout "*B*" (Fig. 1). The main problem is focused on the investigation of the ability to return an UAV with minimal time to the given routing line (GRL) and also determine the range values and direction of the deviation from the GRL. As well it is necessary to develop the algorithm of the trajectory motion correction of an UAV to provide its return on GRL with a minimal time, taking into account the dynamic possibility of an UAV.

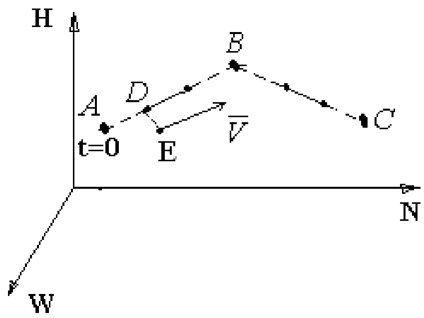

Fig. 1. The position of an UAV relativity points "*A*", "*B*" and "C"

III. SOLUTION OF THE PROBLEM

For strict solving of the return problem of an UAV on the GRL with a minimal time it is useful apply the calculus variation apparatus. Below is given the approximated iterated algorithm of the given problem solution.

Before the determination of the optimal motion trajectory of an UAV, observe the simplest problem of approaching intermediate routing point "B" for an UAV, which has the velocity vector V, parallel to the line AB and is placed in the point "*E*". Obviously that from the $\Delta\, ABE$, that minimal distance from point "*E*" to the line AB –the quantity DE we can calculate by the formula:

$$DE = \frac{2S_{\Delta}}{AB},$$

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where S_Δ is an area of the triangular $\Delta\, ABE$.

As is known that at the coordinated turn of an UAV centrifugal force R_n, lift force Y and weight G create the triangular which is shown on the figure (Fig. 2). Is known also the relation between angle of roll (γ) and the value of the normal overload (n_y), which is presented in the Table 1.

TABLE 1

THE RELATION BETWEEN ANGLE γ AND THE VALUE N_Y

γ, deg	0	30	50	55	60	65	70	75
n_y	1	1.15	1.55	1.74	2.00	2.37	2.92	3.86

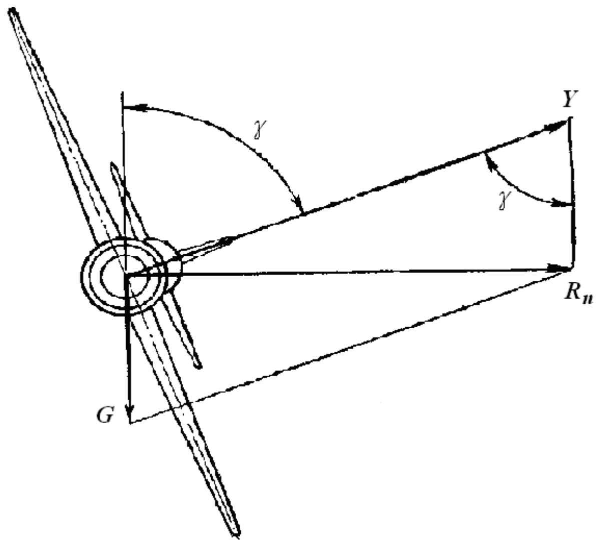

Fig. 2. The coordinated turn of an UAV

The turn radius is proportional to the value V^2 according to the formula:

$$R_t = \frac{V^2}{g\sqrt{n_y^2 - 1}},$$

where g is Gravity force acceleration.

If the motion between points E and B is executed with constant radius of curvature (ROC), then for minimally acceptable ROC a center of the osculating circle will be located on straight, which is perpendicular to tangent of trajectory in point E. In this case an angle between a tangent to osculating circle in point B and line, wich is collinear to AB (Fig. 3), has a maximum value $\theta_{T_0} = 2\theta_T$.

The values of the segment OEB:

$$OD = R_t \cdot \cos(2\theta_T),\ OE = R_t\,,$$

and

$$L_2 = R_t - R_t \cdot \cos(2\theta_T). \quad (1)$$

So, it follows from (1) for this case of turn

$$R_t = L_2/(1 - \cos(2\theta_T)). \quad (2)$$

If we denote the value $OD = x$ then, according to the *Pythagorean Theorem*, it is possible to write:

$$(L_2 + x)^2 = x^2 + L_1^2. \quad (3)$$

From the equation (3) follows:

$$x = \frac{(L_1^2 - L_2^2)}{2L_1},$$

and the turn radius R_t is determined by the formula:

$$R_t = L_2 + \frac{(L_1^2 - L_2^2)}{2L_1}. \quad (4)$$

Obviously that the minimal possible turn radius will happen during the maximal possible overload of an UAV.

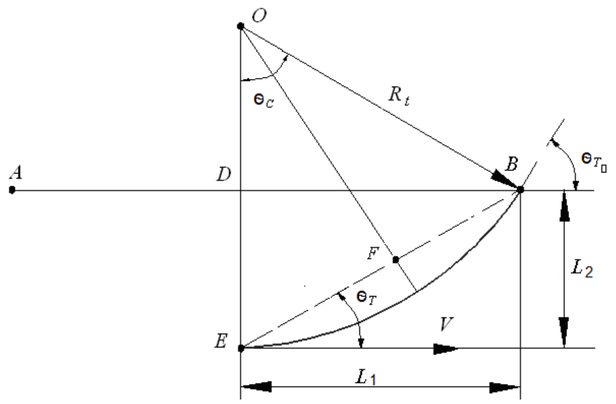

Fig. 3. The scheme of turning the UAV

This turn apply expediently in case, when the point C is located at the left of AB and with great removing from it, i.e. an angle θ_{T_0} is great. If the point C is at the left of AB, but is removing short, then an angle θ_{T0} must be small. In this case may use two variants of UAV motion: 1) increase the turn radius and decreasing an angle θ_{T_0} , accordingly; 2) approach to point B with double turn. The latter variant is suited for location of point C on the right from AB also.

At first variant the limit angle of intersecting line AB in point B is θ_T at $R_t \to \infty$.

At constructing of the double turn (Fig. 4) the radius of each turn is defined by help of equations (2) or (4). The trajectory of double turn may describe with help of the cubic polynomial from t.

For solving problems of this kind are devoted whole chapters of Computational Mathematics [2], [3]. We will confine ourselves to the submission of curves parameterized type of the third power. In this case we have

$$\vec{r}(t)=\sum_{i=0}^{3}\vec{a}_i t^{(i)}, \quad 0\le t\le 1,$$

where $\{t^{(i)}\}=\{t^3 \; t^2 \; t^1 \; 1\}$, $i=0,1,2,3$ is the vector whose components are degree of parameter *t*; $a_{\bar{i}}$ are vectors of coefficients in the representation $\vec{r}$ through *t*.

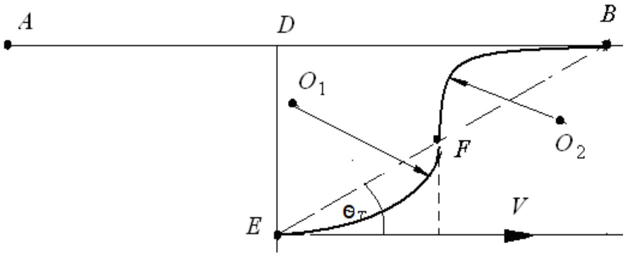

Fig. 4. The constructing of the UAV double turn

Meanwhile parameters of these vectors of coefficients are defined by help of points coordinates $E(\vec{r}_E)$, $B(\vec{r}_B)$, $F(\vec{r}_C)$ and the tangent to UAV vector velocity.

For finding of the trajectory curvature more convenient represent of trajectory coordinates, for example, for two-dimensional case, as

$$x=a_0t^3+b_0t^2+c_0t+d_0, \quad y=a_1t^3+b_1t^2+c_1t+d_1. \qquad (5)$$

As known from theoretical mechanics [4], if we have equations (5), than we can easy calculate the radius of the trajectory curvature in the given point by formula

$$R=V^2/a_n, \qquad (6)$$

where $V=\sqrt{(V_x)^2+(V_y)^2}$ is the quantity of velocity, $V_x=dx/dt$, $V_y=dy/dt$; $a_n=\left|\frac{a_xV_y+a_yV_x}{V}\right|$ is the centripetal acceleration, $a_x=\frac{dV_x}{dt}$, $a_y=\frac{dV_y}{dt}$.

It isn't difficult to calculate the radius curvature for whole of trajectory also.

The algorithm of the trajectory motion correction is that to determine from choosing area of route the point which has a minimum radius curvature $R_{\min}$.

Next step of algorithm is to compare values $R_{\min}$ and R_t.

If in point $F(x, y)$, which lies between point *E* and *B*, $R_{\min} > R_t$, then we get over to forming the segment trajectory from point *B*.

Otherwise, we design the segment trajectory between points *E* and *M*.

Further it is necessary to design the trajectory from point *A* to *B* and so on.

But, not always possible to provide given conditions of task, namely, if the angle between vector velocity and vector **AB** has great value, for example more $\pi/2$, then to provide the turn of UAV to IRP "*B*" very difficult. In this case, the UAV must make a turn on π and continue a motion in direction of IRP "*B*".

Decreasing of velocity and correspondingly decreasing of trajectory radius curvature is not desirable because of possibility of stall and spin entry.

Strictly speaking, we have here the 3-rd order polynomial

$$y=f(x)=ax^3+bx^2+cx+d, \qquad (7)$$

for approximation of trajectory and it is necessary to compose and decide four equations with four unknown quantities.

Two equations are composed due to two known points, in our case points *B* and *E*, rest equations – due to known tangents of trajectory in these points.

Next variant of trajectory construction is that three equations are composed due to three known points and forth equation – due to known tangent of trajectory in point *E* [3].

Let's write the system of equations for this case:

$$\begin{cases} y_E=ax_E^3+bx_E^2+cx_E+d; \\ y_F=ax_F^3+bx_F^2+cx_F+d; \\ y_B=ax_B^3+bx_B^2+cx_B+d; \\ 0=3ax_E^2+2bx_E+c, \end{cases} \qquad (8)$$

direction of axes "*OX*" coincides with direction of vector velocity.

By solving this system of equations by anyone from the known methods for example by the Kramer's rule or by method of the step by step exclusion of unknown values it is possible to find coefficients *a, b, c, d* of the cubic polynomial.

IV. Example of calculation of returning the UAV to a predetermined route line

The task is to explore the possibility of returning the UAV to a predetermined route line. Let the flight of UAV with velocity $V=V_0$ (m/s) is executed from point $A(\lambda_a,\varphi_a,\rho_a)$ to point $B(\lambda_b,\varphi_b,\rho_b)$ and at the present it has coordinates $(\lambda_e,\varphi_e,\rho_e)$. Assume also that the vector velocity is collinear to vector *AB* and the turn of UAV is executed in plane, which defined by vector **AB** and point *E*. The maximal possible normal overload n_y of UAV is $n_{y_{\max}}$ The average radius of the Earth R_{Earth} is 6371032 m.

For case, when $\rho_a=\rho_b=\rho_e$, the dependence of required overload (n_{yr}) of UAV at various distances (d_d) from the middle distance between points *A* and *B* is shown in Table 2.

TABLE 2

THE DEPENDENCE OF REQUIRED OVERLOAD FOR TURN OF UAV ON IRP "*B*", WHEN IT IS LOCATED AT VARIOUS DISTANCES FROM THE MIDDLE LINE *AB*=10 KM

d_d, km	V, m/s			
	100	150	200	250
0.5	1.12	1.53	2.28	3.35
1	1,23	1,89	3,02	4,57
1.5	1,64	3,84	5,28	8,17

The dependence of required overload for turn of UAV on IRP "*B*", when it is located at various distances d_d from the middle line *AB* = 5 km is shown on Fig. 5. At decreasing of value d_d the quantity of required overload for turn of UAV on IRP "*B*" is increased considerably.

Let us construct the curve with double turn, which passes points *E*(0; 0), *F*(2,5; 0,75), *B*(5; 1,5). If we set coordinates point *E* to equation (7), then we receive $d = 0$. By analogy, if we set a value of tangent for this curve in point *E*, a value $x_e = 0$ in last equation of system (8), then we receive $c = 0$. Then we can find next two coefficients: $a = -0{,}024$; $b = 0{,}18$. So, we have the 3-rd order polynomial $y = x^2(0{,}18–0{,}024x)$ that is shown on the Fig. 6.

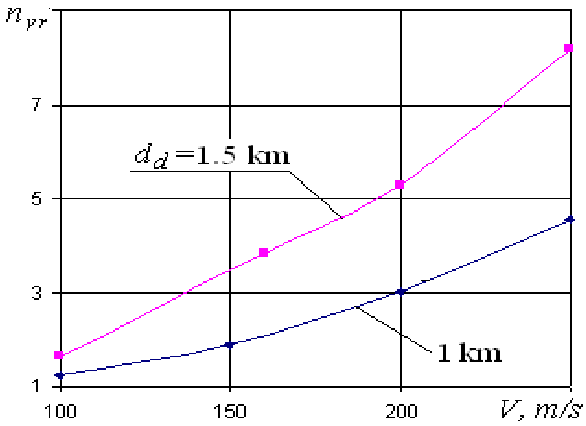

Fig. 5. The dependence of required overload for turn of the UAV on IRP "*B*", when it is located at various distances d_d from the middle line *AB* = 5 km

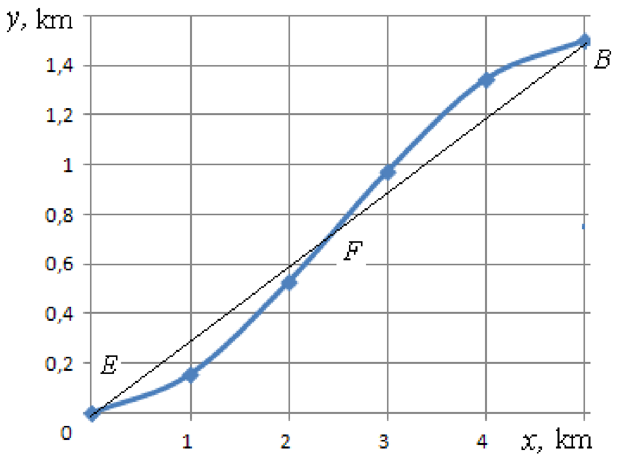

Fig. 6. The curve with double turn between points *E* and *B*

Now we can use equations (5) and (6) for calculation of radius of curvature, but analytical model with help of equations (1–4) is easier.

CONCLUSION

In this paper an approximately algorithm of the trajectory motion correction of an UAV to provide its return on GRL, taking into account the dynamic possibility of an UAV is given.

From considered example is follows conclusion: on considerable moving off from IRP *"B"*, at decreasing distance between point *E* and line *AB* the value of required overload for turn of UAV on IRP *"B"* is decreasing. But at approach to point B this rule is beginning to change.

Coordinates of UAV trajectory at return to intermediate route point may describe quadratic equation (single turn) or the 3-rd order polynomial (double turn) in dependence from position of UAV, direction its vector velocity and location of next IRP.

Wherein the definition radius of curvature for the 3-rd order polynomial boil down to same task of trajectory, which describes quadratic equation.

For determination peculiarities of this algorithm it is necessary to make additional researches.

REFERENCES

[1] Hrustalev, A. A.; Koltsov, Yu. V.; Egorov, S. N. Russian patent 2338158 dated 17.05.2007, the IPC G01C21/00. 2008.

[2] Mikhaylenko, V. E.; Vanin, V. V.; Kovalev, S. M. Engineering Graphics: Manual. for ed. V. E. Mikhaylenko. Kyiv: Karavela, 2008. 272 p.

[3] Graphics and geometric modeling and interactive systems: Manual. N. F. Tupitsyn, M. G. Lutski. Kyiv:NAU, 2011. 192 p.

[4] Dobronravov, V. V.; Nikitin, N. N. Course of theoretical mechanics. Moscow: Higher School, 2002. 336 p.

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М. Ф. Тупіцин, І.О. Степаненко, Є.С. Волощенко, С.С. Шильдський. Окремий випадок методу оперативного програмування маршруту польоту

Наведено опис алгоритму корекції траєкторії руху безпілотного літального апарату. Частину операцій алгоритму показано досить докладно. Приклад розрахунку повернення безпілотного літального апарату на задану лінію маршруту додається.

Ключові слова: навігація; проміжні точки маршруту; корекція; радіус повороту; координати великого кола; задана лінія маршруту.

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Н. Ф. Тупицин, И. А. Степаненко, Е.С. Волощенко, С.С. Шильдский. Частный случай метода оперативного программирования маршрута полета

Приведено описание алгоритма коррекции траектории движения беспилотного летательного аппарата. Часть операций алгоритма показана достаточно подробно. Пример расчета возврата беспилотного летательного аппарата на заданную линию маршрута прилагается**.**

Ключевые слова: навигация; промежуточные точки маршрута; коррекция; радиус поворота; координаты большого круга; заданная линия маршрута.

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