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APPROXIMATION OF STRUCTURE OF DECISION OF HYDRODYNAMIC'S EQUATIONS IN CYCLONE

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Abstract—*A problem of reducing expressions, which are received while solving the system of Navies -Stokes equations is discussed. An algorithm of equivalent reducing is proposed, based on the theory of chain fractions.*

Index Terms—Integral transforms, iterative procedure, convective-diffusion transform, Navies—Stokes equations, chain fractions.

I. INTRODUCTION

Cyclones are widely used in air pollution control and gas-sold separation for aerosol sampling and industrial applications. With the advantages of relative simplicity to fabricate, low cost to operate and well adaptability to extremely harsh the cyclone separators have become one of the most important particle removal devises which are preferably utilized in scientific and engineering fields.

II. THE ANALYSIS OF THE LATEST RESEARCHES AND PUBLICATIONS

The convective-diffusion transfer of substance in water objects is described by the system of nonlinear partial differential equations with responded initial and boundary conditions [1]. This system describes a motion of incompressible fluid flow in cyclone:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=\mu_T\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\partial \tilde{p}}{\partial x}-\frac{2}{3}\rho\frac{\partial K}{\partial x}; \qquad (1)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}\right)=\mu_T\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\partial \tilde{p}}{\partial z}-(\rho-\rho_\infty)g-\frac{2}{3}\rho\frac{\partial K}{\partial z}; \qquad (2)$$

$$c_p\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=\lambda_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right). \qquad (3)$$

An equation of turbulence energy K is determined as [1]:

$$\rho\left(\frac{\partial K}{\partial t}+u\frac{\partial K}{\partial x}+w\frac{\partial K}{\partial z}\right)=G-\rho\varepsilon$$
$$+\frac{\partial}{\partial x}\left(\mu_t\frac{\partial K}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu_t\frac{\partial K}{\partial z}\right)+\rho\frac{g}{T}\frac{\partial T}{\partial z}. \qquad (4)$$

Decision of this system we received by application of iterative digital-analytic's method of decision of nonlinear partial equations [2].

$$u_x^{(1)}(x,y,z,t)=u_x^{(0)}(x,y,z,t)$$
$$+\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}X_i(\alpha_i,x)Y_j(\beta_{i,j}y)Z_k(\gamma_{i,j,k}z) \qquad (5)$$
$$\times[U1_{i,j,k}^{(1)}+U2_{i,j,k}^{(1)}]e^{-\delta_{i,j,k}t}.$$

According to general iterative scheme of decision of nonlinear partial equations a linear part of the system is received as

$$u_x^{(0)}(x,y,z,t)=$$
$$\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}X_{u_x}(\alpha_i,x)Y_{u_x}(\beta x_{i,j}y)Z_{u_x} \qquad (6)$$
$$(\gamma x_{i,j,k}z)[U1_{i,j,k}^{(0)}+U2_{i,j,k}^{(0)}]e^{-\delta x_{i,j,k}t},$$

here $X_{u_x}(\alpha_i,x)$, $Y_{u_x}(\beta_{i,j}y)$, $Z_{u_x}(\gamma_{i,j,k}z)$ are self-functions of linear part of responsible boundary problem [3]. Numbers N_x,N_y,N_z determinate the numbers of row's members in decision of system of equations, which are response for required accuracy of decision. The members $U1_{i,j,k}^{(0)}$, $U2_{i,j,k}^{(0)}$ are determined by initial and boundary conditions.

Decision for components of velocity u_y, u_z can be presented in similar way:

$$u_y^{(0)}(x,y,z,t)=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}X_{u_y}(\alpha y_i,x)Y_{u_y}(\beta y_{i,j}y)Z_{u_y}(\gamma y_{i,j,k}z)[V1_{i,j,k}^{(0)}+V2_{i,j,k}^{(0)}]e^{-\delta_{i,j,k}t}.$$

$$u_z^{(0)}(x,y,z,t)=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}X_{u_z}(\alpha z_i,x)Y_{u_z}(\beta z_{i,j}y)Z_{u_z}(\gamma z_{i,j,k}z)[W1_{i,j,k}^{(0)}+W2_{i,j,k}^{(0)}]e^{-\delta_{i,j,k}t}.$$

III. TASK STATEMENT

The next stage consists in calculating of integral transforms for product of functions, which should be fined in convective part of Navies–Stoke equations – $u_x\partial u_x/\partial x$, $u_y\partial u_x/\partial y$, $u_z\partial u_x/\partial z$ in first equation, $u_x\partial u_y/\partial x$, $u_y\partial u_y/\partial y$, $u_z\partial u_y/\partial z$ – second and third equation $u_x\partial u_z/\partial x$, $u_y\partial u_z/\partial y$, $u_z\partial u_z/\partial z$ of system.

A product of expressions like (5) or (6) and application of integral transforms on space variables gives as

$$\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}\sum_{l=1}^{N_x}\sum_{m=1}^{N_y}\sum_{n=1}^{N_z}U3_{i,j,k}V3_{l,m,n}e^{-\delta x_{i,j,k}t} \tag{7}$$

Carrying out of further iterations give us double sums relative on time variable. That fact reduces to expressions, which in fact makes impossible the application of proposed algorithm for decision of nonlinear system of Navies–Stokes equations.

IV. TASK SOLUTION

So it is necessary to simplify these expressions such as (7) to ensure proposed algorithm be universal while deciding the systems of quazi linear equations such Navies–Stokes equations.

An algorithm of equivalent simplifying of given expressions we apply, which is based on application of chain fraction apparatus [3].

We apply to expression (7) the integral Laplace-transform:

$$\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}\sum_{l=1}^{N_x}\sum_{m=1}^{N_y}\sum_{n=1}^{N_z}\frac{R_{i,j,k,l,m,n}}{p+\delta x_{i,j,k}+\delta_{l,m,n}}.$$

Consider partial sum such as

$$Y(p)=\sum_{n=1}^{N}\frac{R_n}{p+\delta_n}.$$

We present this expression as rational function

$$Y(p)=\frac{\sum_{n=0}^{N-1}b_n p^n}{\sum_{n=0}^{N}a_n p^n}. \tag{8}$$

An application of algorithm of equivalent simplifying to (6) gives as such an expression

$$Y(p)\approx\frac{b_0+b_1p}{a_0+a_1p+a_2p^2}. \tag{9}$$

In original-space we have

$$y(t)=e^{-\alpha t}[A\varphi_1(t)+B\varphi_2(t)], \tag{10}$$

$$\varphi_1(t)=\begin{cases}\cos\omega t, & \omega>0;\\ \cosh\omega t, & \omega<0,\end{cases}$$

where

$$\varphi_2(t)=\begin{cases}\sin\omega t, & \omega>0;\\ \sinh\omega t, & \omega<0.\end{cases}$$

The similar expressions such (10) we will receive for the rest of velocity components. The base of such an algorithm is – behaving of any system of high order can be described by the system of second order with delay [4].

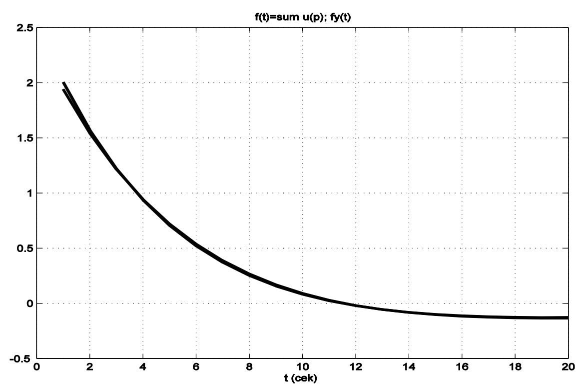
So, we have

$$\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}\sum_{l=1}^{N_x}\sum_{m=1}^{N_y}\sum_{n=1}^{N_z}U3_{i,j,k}e^{-\delta x_{i,j,k}t}V3_{l,m,n}e^{-\delta_{l,m,n}t}$$
$$\approx e^{-\alpha t}[A\varphi_1(t)+B\varphi_2(t)].$$

As a result we approximate the decision of the first equation (1) by such an expression

$$u_x^{(1)}(x,y,z,t)=u_x^{(0)}(x,y,z,t)$$
$$+\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}X_i(\alpha_i,x)Y_j(\beta_{i,j}y)Z_k(\gamma_{i,j,k}z)$$
$$\times e^{-\alpha_{i,j,k}t}[A_{i,j,k}\varphi_1(\omega_{i,j,k}t)+B_{i,j,k}\varphi_2(\omega_{i,j,k}t).$$

Algorithmic mistakes, which are appeared in this case, are not so essential (Figure), and they may be compensated by additional iterations of algorithm we apply.



The use of additional compensation algorithm iterations

Conclusions

Proposed algorithm gives us an effective approach method of decision of nonlinear partial equations, which are described the moving of gas in cyclones. This algorithm can be apply to the wide class of nonlinear systems of partial differential equations.

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К. С. Бовсуновська. Апроксимаія структури розв'язання системи рівнянь гідродинаміки

Розглянуто задачу еквівалентного спрощення виразів, які отримують при розв'язанні системи квазілінійних рівнянь Нав'є–Стокса. Запропоновано алгоритм спрощення, що ґрунтується на використанні теорії ланцюгових дробів.

Ключові слова: інтегральні перетворення; ітераційна схема; конвективно-дифузійний перенос; ланцюгові дроби; рівняння Нав'є–Стокса.

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К. С. Бовсуновская. Аппроксимация структуры решения системы уравнений гидродинамики

Рассмотрена задача эквивалентного упрощения выражений, получаемых при решении системы уравнений гидродинамики. Предложен алгоритм эквивалентного упрощения, основанный на применении теории цепных дробей.

Ключевые слова: интегральные преобразования, итерационная схема, конвективно-диффузионный перенос, уравнения Навье–Стокса.

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