UDC 629.735.051:681.513.5 (045)

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OPTIMAL SUPPRESSION OF AN ELECTROMAGNETIC INTERFERENCE EFFECT ON AN ACCELERATION MEASURING DEVICE

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Abstract**—***The algorithms of the synthesis of optimal structure and parameters of the filter transfer functions matrix which enables to extract the vector of useful signals with the noise correction as well as without it from the acceleration measuring data on the random electromagnetic interference hum.*

Index Terms—Filter; optimal; synthesis; noise; correction; factorization.

I. INTRODUCTION

The modern systems of navigation instruments and flight control systems testing require the maximum approximation of conditions of their certification to the real conditions of a flight. Therefore, creation of mobile platforms for a simulation of such flight conditions is a relevant task. One of the best options for this assignment is a hexapod. The main problem in the way of using the hexapod as a simulator of an angular and linear movement of the object is a need to clarify its dynamics models.

Hexapod is a mechanism with the parallel kinematics [1], [2]. It consists of a fixed base and a movable platform, interconnected by means of the of the six control rods. The rods are pivotally mounted. When one changes the length of the rod the platform moves relatively to the base. Its movement is accompanied by the forces of resistance in the joints. Therefore, we can assume hexapod as a moving object of control with two multidimensional inputs and one multidimensional output. Such multi-dimensional output is characterized by the vector which is composed from three movable platform accelerations.

Measurement of such accelerations is usually accompanied by an electromagnetic interference under the actual operating conditions. Since the interferences usually are random, there is a need to reduce their impact on the results of accelerations measuring.

One of the possible ways to reduce the influence of electromagnetic interference on the results of accelerations measuring is optimal filtering of measuring data.

Taking into account that under conditions of using hexapod as a simulator of the object movements, the dynamics of sensors is known and we can measure the mixture "signal-noise" and interrelated with the mixture electromagnetic interference. The objective of the article is the development of algorithms of synthesis of optimal multidimensional filter which marks out the vector of random useful signal with prior unknown characteristics.

II. PURPOSE AND TASK OF RESEARCH

In order to meet the objective we considered two structural schemes of signals' passages (Figs 1 and 2) and undertook the following task of the optimal control systems synthesis.

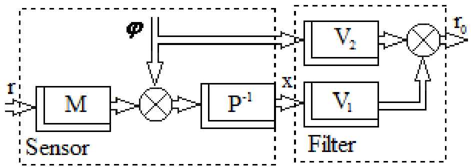

Fig. 1. Bending of a line of the sight due to useful loading

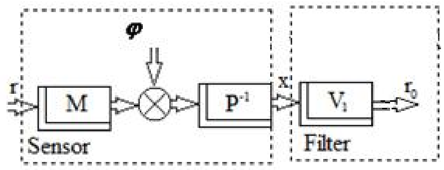

Fig. 2.The Structure of Multivariable Filter

Suppose, the dynamics of the sensor is characterized by the system of differential equations with constant coefficients of the following type

$$\mathbf{P}x = \mathbf{M}u + \varphi, \tag{1}$$

where **P**, **M** are the matrices of dimensions $n \times n$, the elements of which are polynomials from the operator of differentiation $s = d/dt$; x is the n-dimensional vector of sensor's output signals that form centered stable random process with the known matrix of spectral densities S_{xx}; r is then-dimensional vector of measuring signals which spectral densities matrix is not given; φ is the n-dimensional vector of random noises of measuring with the zero universal mean and the known matrix of spectral densities.

Suppose that vectors **x** and φ are interconnected as they act in accordancewith the structural schemes on Figs 1 and 2. The interrelation of these vectors characterizes the known matrices of the mutual spectral densities $\mathbf{S}_{\varphi x}$ and $\mathbf{S}_{x\varphi}$.

The task of the synthesis of the optimal filter is that according to the known matrices **P**, **M**, $\mathbf{S}_{xx}$, $\mathbf{S}_{\varphi x}$ and $\mathbf{S}_{x\varphi}$ to find the matrix of transfer functions of the filter V (Fig. 1) or the block matrix [6] of transfer functions of the filter $V = (V_1, V_2)$, which provides the assessment of the vector of measuring signals r_0 and supplies the minimum to the weighted average sum of errors variances

$$I = \left\langle \mathbf{E}^{/}\mathbf{R}\mathbf{E} \right\rangle, \tag{2}$$

where I is the sum of weighted average variances of the error vector **E**; <> is the mean estimation symbol; ′ is the symbol of transposition; **R** is the weighted matrix, the elements of which can be set according to the methodology [7]; **E** is the n-dimensional vector of assessment errors which can be calculated according to the formula

$$\mathbf{E} = \mathbf{\Phi} r - r_0, \tag{3}$$

where**Φ** is the matrix of the wishful transformation of the signal [4]. The quality criterion (2), is presented in frequency domain according to the methodology from the article [5]

$$I = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left(RS^{/}_{\varepsilon\varepsilon}\right), \tag{4}$$

where j is a complex unit; tr is the sign of the matrix trace calculation; ′ is the sign of matrix transposition; $S_{\varepsilon\varepsilon}$ is the error spectral densities matrix which can be found according to the theorem of Wiener–Khinchin.

III. ALGORITHM

Algorithm for the synthesis of an optimal matrix transfer functions was obtained by minimizing the functional (4) in the class of a stable and physically realizable matrices of transfer functions with a help of the Wiener–Kolmogorov method.

In order to use this method, let's find $S^{/}_{\varepsilon\varepsilon}$ matrices corresponding to the schemes shown on Figs 1 and 2. Consider the structure of systems on Figs 1 and 2 and equations (1), (3) we found the following error vectors

$$\varepsilon_1 = \Phi M^{-1}\left[\left(P \quad -E_n\right) - M\Phi^{-1}V\right]\zeta, \tag{5}$$

$$\varepsilon_2 = \Phi M^{-1}\left[\left(P \quad -E_n\right) - M\Phi^{-1}\left(V \quad O_n\right)\right]\zeta, \tag{6}$$

where indices 1 and 2 are the numbers of figures which correspond to the matrix; $\mathbf{E}_n$ is an identity matrix with the dimensions $n \times n$; $\mathbf{O}_n$ is a zero matrix with the dimensions $n \times n$; ζ is a room-vector of influences that has $2n$ components

$$\zeta = \begin{bmatrix} x \\ \varphi \end{bmatrix}. \tag{7}$$

Application of the Wiener–Khinchin to the expressions (5) and (6) made it possible to determine the ratio of the two transposed matrices of errors spectral densities

$$S^{/}_{\varepsilon\varepsilon 1} = \Phi M^{-1}\left[P \quad -E_n\right]S^{/}_{\zeta\zeta}\begin{bmatrix} P_* \\ -E_n \end{bmatrix} - \Phi M^{-1}\left[P \quad -E_n\right]S^{/}_{\zeta\zeta}V_* - VS^{/}_{\zeta\zeta}\begin{bmatrix} P_* \\ -E_n \end{bmatrix}M_*^{-1}\Phi_* + VS^{/}_{\zeta\zeta}V_*, \tag{8}$$

$$\begin{aligned} S^{/}_{\varepsilon\varepsilon 2} = {} & \Phi M^{-1}PS^{/}_{xx}P_*M_*^{-1}\Phi_* - VS^{/}_{xx}P_*M_*^{-1}\Phi_* - \Phi M^{-1}PS^{/}_{xx}V_* + VS^{/}_{xx}V_* - \Phi M^{-1}S^{/}_{x\varphi}P_*M_*^{-1}\Phi_* \\ & + \Phi M^{-1}S^{/}_{xf}V_* - \Phi M^{-1}PS^{/}_{\varphi x}M_*^{-1}\Phi_* + VS^{/}_{\varphi x}M_*^{-1}\Phi_* + \Phi M^{-1}S^{/}_{\varphi\varphi}M_*^{-1}\Phi_*, \end{aligned} \tag{9}$$

where $\mathbf{S}_{\zeta\zeta}$ is a spectral densities matrix of the vector (7)

$$\mathbf{S}^{/}_{\zeta\zeta} = \begin{bmatrix} \mathbf{S}^{/}_{xx} & \mathbf{S}^{/}_{\varphi x} \\ \mathbf{S}^{/}_{x\varphi} & \mathbf{S}^{/}_{\varphi\varphi} \end{bmatrix}, \tag{10}$$

$_*$ is the sign of the matrices Hermitical conjugation. Matrices (8), (9) which are found in such a way make it possible to start the minimization of the quality functional (4). For this purpose we found two the first variations of the quality functional

$$\delta I_1 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left\{\left[RVS^{/}_{\zeta\zeta} - R\Phi M^{-1}\left(P \quad -E_n\right)S^{/}_{\zeta\zeta}\right]\delta V_* + \delta V\left[S^{/}_{\zeta\zeta}V_*R - S^{/}_{\zeta\zeta}\begin{pmatrix} P_* \\ -E_n \end{pmatrix}M_*^{-1}\Phi_*R\right]\right\}ds, \tag{11}$$

$$\delta I_2 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\left[R\left(VS^{/}_{xx} + \Phi M^{-1}S^{/}_{x\varphi} - \Phi M^{-1}PS^{/}_{xx}\right)\delta V_* + \delta V\left(S^{/}_{xx}V_* + S^{/}_{\varphi x}M_*^{-1}\Phi_* - S^{/}_{xx}P_*M_*^{-1}\Phi_*\right)R\right]ds \tag{12}$$

and determine three stable together with inverse fractional rational matrices $\mathbf{\Gamma}$, $\mathbf{D}_1$, $\mathbf{D}_2$

$$R = \mathbf{\Gamma}_*\mathbf{\Gamma}, \tag{13}$$

$$S'_{\zeta\zeta} = \mathbf{D_1 D_{1*}}, \tag{14}$$

$$S'_{xx} = \mathbf{D_2 D_{2*}}, \tag{15}$$

where δV is the first variation of the V. The search of representations (13) – (15) was done in the quotient of factorization [8].

Substituting equations (13) – (15) into the formulas (11), (12) allowed us to find the following dependencies

$$\delta I_1 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\Big\{\Gamma_*\Big[\Gamma V D_1 - \Gamma\Phi M^{-1}\big(S'_{x\varphi} - PS'_{xx}\big)\times D_{1*}^{-1}\Big]D_{1*}\delta V_* \\ + \delta V D_1\Big[D_{1*}V_*\Gamma_* - D_1^{-1}\big(S'_{\varphi x} - S'_{xx}P_*\big)\times M_*^{-1}\Phi_*\Gamma_*\Big]\Gamma\Big\}\,ds, \tag{16}$$

$$\delta I_2 = \frac{1}{j}\int_{-j\infty}^{j\infty} tr\Big\{\Gamma_*\Big[\Gamma V D_2 - \Gamma\Phi M^{-1}\big(S'_{x\varphi} - PS'_{xx}\big)\times D_{2*}^{-1}\Big]D_{2*}\delta V_* \\ + \delta V D_2\Big[D_{2*}V_*\Gamma_* - D_2^{-1}\big(S'_{\varphi x} - S'_{xx}P_*\big)\times M_*^{-1}\Phi_*\Gamma_*\Big]\Gamma\Big\}\,ds. \tag{17}$$

Taking into account the expressions (16), (17) the condition of the functional (4) minimum is defined by the following matrix equations

$$\mathbf{\Gamma} V\mathbf{D_1} = -\big(N_{10} + N_{1+}\big), \tag{18}$$

$$\mathbf{\Gamma} V\mathbf{D_2} = -\big(N_{20} + N_{2+}\big), \tag{19}$$

where $N_{10} + N_{1+}$ is the stable result of separation [8] of the following matrix

$$N_{10} + N_{1+} + N_{1-} = -\mathbf{\Gamma\Phi M}^{-1}\big(S'_{x\phi} - PS'_{xx}\big)\mathbf{D_{1*}^{-1}}, \tag{20}$$

$N_{20}+N_{2+}$ is the stable result of separation of the following matrix

$$N_{20} + N_{2+} + N_{2-} = -\mathbf{\Gamma\Phi M^{-1}}\big(P \quad -E_n\big)\mathbf{D_2}. \tag{21}$$

In accordance with equations (18), (19) one can write down the following rule for the calculation of the optimal filter transfer functions matrices

$$V = -\mathbf{\Gamma}\big(N_{i0} + N_{i+}\big)\mathbf{D_i^{-1}}, \tag{22}$$

where i is a number of structures in Figs 1 or 2.

To assess the effect of using algorithms of optimal filtering presented above we get expressions for calculating transposed matrices of spectral densities of filtering errors. The trace of matrix of spectral error planes with consideration of equations (13) – (15) and (20) – (22) is the following

$$tr\big(RS'_{\varepsilon\varepsilon}\big) = tr\Big[R\Phi M^{-1}\big(PS'_{xx}P_* - PS'_{\varphi x} - S'_{x\varphi}P_* + S'_{\varphi\varphi}\big)\times M_*^{-1}\Phi_* - \big(N_{i0}+N_{i+}\big)\big(N_{i0}+N_{i+}\big)_* \\ - N_{i-}\big(N_{i0}+N_{i+}\big)_* - \big(N_{i0}+N_{i+}\big)N_{i-*}\Big]. \tag{23}$$

IV. AN EXAMPLE OF IMPLEMENTATION

As an example of implementation of the new algorithms of optimal filtering (13) – (15), (20) – (23) we consider the case of signal interference filtration at the output of a single component accelerometer created on the basis of a MEMS technology. In this case $n = 1$, and the matrices $\mathbf{P}$, $\mathbf{M}$, $\mathbf{\Phi}$, $\mathbf{S_{xx}}$, $\mathbf{S_{\varphi x}}$, $\mathbf{S_{\varphi\varphi}}$ and R transform into the following functions

$$M = ks^2,\ P = 1,\ S_{\varphi\varphi} = \frac{\sigma_\varphi^2}{\pi},\ S_{xx} = \frac{\sigma_x^2}{\pi},\ S_{\varphi x} = S_{x\varphi} = \alpha\frac{\sigma_x\sigma_\varphi}{\pi},\ R = 1,\ \Phi = 1. \tag{24}$$

The task is to find expressions for the transfer functions of optimal filters without a noise φ correction and with the noise correction.

Substitution of the initial data into the formulas (13) – (15) makes possible to write down the following expressions

$$\mathbf{\Gamma_*\Gamma} = 1, \qquad \mathbf{D_1 D_{1*}} = \begin{bmatrix} \dfrac{\sigma_x^2}{\pi} & \dfrac{\alpha\sigma_x\sigma_\varphi}{\pi} \\ \dfrac{\alpha\sigma_x\sigma_\varphi}{\pi} & \dfrac{\sigma_\varphi^2}{\pi} \end{bmatrix}, \tag{25}$$

After factorization of the expressions (25) it will be clear that

$$\mathbf{D}_2 = \frac{\sigma_x}{\sqrt{\pi}}, \quad \mathbf{\Gamma} = 1, \quad \mathbf{D}_1 = \frac{1}{\pi}\begin{bmatrix} \sigma_x\sqrt{1-\alpha^2} & \sigma_x\alpha \\ 0 & \sigma_\varphi \end{bmatrix}. \quad (26)$$

Substitution of a correspondent data from the relations (24), (26) into the matrices (21), (22) leads to the following results of a separation

$$N_{10} + N_{1+} = -\frac{1}{ks^2\sqrt{\pi}}\begin{bmatrix} \sigma_x\sqrt{1-\alpha^2} & \alpha\sigma_x - \sigma_\varphi \end{bmatrix};$$

$$N_{20} + N_{2+} = \frac{1}{ks^2}\left(\alpha\sigma_\varphi - \sigma_x\right). \quad (27)$$

So application the rule (22) to the results (26), (27) let us find two optimal filters transfer functions. The fist function corresponds with the scheme (Fig. 1) and is equal to

$$V = \frac{1}{ks^2}\begin{bmatrix} -1 & \alpha - \alpha\frac{\sigma_x}{\sigma_\varphi} + 1 \end{bmatrix}. \quad (28)$$

The second optimal transfer function is equal to

$$V = \frac{1}{ks^2}\left(1 - \alpha\frac{\sigma_\varphi}{\sigma_x}\right). \quad (29)$$

The sums of weighted spectral densities of error assessment with the help of optimal filters (28), (29) were obtained in the following view

$$tr\left(RS^{/}_{\varepsilon\varepsilon 1}\right) = \frac{\sigma_x^2\alpha\left(1-\alpha\right)}{k^2 s^4 \pi}, \quad (30)$$

$$tr\left(RS^{/}_{\varepsilon\varepsilon 2}\right) = \frac{\sigma_\varphi^2\left(1-\alpha^2\right)}{k^2 s^4 \pi}. \quad (31)$$

The comparison of the results (14), (15) enables to conclude that in the case of acceleration measurement which is accompanied by the influence of electromagnetic interference in the form of white noise the introduction of noise correction provides the absence of the influence on the result of assessment of electromagnetic interference intensity.

CONCLUSIONS

So, the algorithms of the synthesis of optimal structures and parameters of the matrix of transfer functions of the filter which enables to assess the vector of useful signals with the noise correction as well as without it according to the data on sensor dynamics, electromagnetic interference characteristics and vector characteristics of the measured signals were developed.

The necessary condition of implementation of the obtained algorithms is the demand for fixed signals which act in measuring circuits.

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Received 23 October 2014.

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С. І. Осадчий, В. А. Зозуля. Оптимальне пригнічення впливу електромагнітних наводок на прилад для виміру прискорень

Обгрунтовано нові алгоритми синтезу оптимальних структури та параметрів матриць передавальних функцій фільтрів, які дозволяють відокремити вектор корисних сигналів акселерометра на фоні завад від випадкових електромагнітних наводок з корекцією за шумом та без неї.

Ключові слова: фільтр; оптимальність; синтез; шум; корекція; факторизація.

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С. І. Осадчий, В. А. Зозуля. Оптимальное подавление влияния электромагнитных наводок на прибор для измерения ускорений

Обоснованы новые алгоритмы синтеза оптимальных структуры и параметров матриц передаточных функций фильтров, позволяющих выделить вектор полезных сигналов акселерометра на фоне помех от случайных электромагнитных наводок с коррекцие по шуму и без нее.

Ключевые слова: фильтр; оптимальность; синтез; шум; коррекция; факторизация.

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