СИСТЕМИ АВТОМАТИЧНОГО КЕРУВАННЯ

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[1]**M. M. Komnatska**, Post-Graduate stud.,
[2]**O. I. Poddubchenko**, stud.,
[3]**O. V. Serdyuk**, stud.

OPTIMIZATION OF UNMANNED AERIAL VEHICLE ROBUST FLIGHT CONTROL SYSTEM WITH INCOMPLETE STATE VECTOR MEASUREMENTS

Aerospace Control System Department, National Aviation University
e-mail: [1]martakomnatska@gmail.com, [2]pussy_cat2007@ukr.net, [3]Enigma89@bk.ru

This paper is devoted to the robust flight control law design for lateral channel of small Unmanned Aerial Vehicle (UAV) based on $\mathbf{H_2/H_\infty}$*-robust optimization technique. The task is divided into two stages. On the first stage a linear quadratic regulator is synthesized based on Luenberger observer. The second stage is devoted to the robust optimization procedure of the constructed regulator. The robust optimization technique applies genetic algorithms. It is known that the genetic algorithm optimization is more robust than traditional optimization procedures, also it has the propriety to converge to the global minimum and well suited to seek a compromise between contradictory requirement expressed in terms of multi-objectives function. The simulation results prove the efficiency of the proposed procedure.*

Key-words: robust flight control, composite performance index, $\mathbf{H_2/H_\infty}$-optimization, genetic algorithm.

Introduction. External stochastic disturbances suppression and providing necessary robust stability and performance with respect to internal parametrical disturbances during the flight is the main task of flight control system (FCS). The solution for this problem can be robust control, especially in the area of the Unmanned Aerial Vehicles (UAV). This can be explained by the fact, that parameters of the small UAV are very susceptible towards changing of atmosphere conditions. The UAV are used as well as for agricultural, metrological, ecological and others fields of our economy. It can execute a wide variety of functions, namely remote sensing, scientific research, search and rescue, etc. Small Unmanned Aerial Vehicle (UAV) Aerosonde is designed for easy motion in different environments. In addition, such kind of UAVs should easily move in a wide range of velocity and altitude changes. Therefore, the manipulation of such UAVs requires a necessary stability, performance and robustness. It is known, that various modern control methods require full information about the states of the UAV; and it is important to admit that measuring process of these states is difficult. Even in a case when all state measurements are possible, they are distorted by noises. In order, to make the flight control system for small UAVs simple enough and cheap from the point of view to power consumption, weight, low cost design and size, the limited number of navigation sensors are used. The limited number of sensors restricts the number of measured states. It is known, that traditional optimal control procedure supposes that all components of the state space should be available for measures [1 – 3]. In case, when the full measurements are not available for measuring, a state observer is necessary in order to restore them. After the restoration the full state vector a linear quadratic technique could be applied. The same control law is applied to the perturbed models in order to achieve the robust properties both to the nominal and parametrically perturbed closed loop models under the external disturbances. Moreover, it is necessary to hold the desired performance level which is diametrically opposed to the robustness. To satisfy the contradictive requirements to performance and robustness the procedure of $\mathbf{H_2/H_\infty}$-robust optimization based on genetic algorithm is used. The main goal of this procedure is to achieve the compromise between the robustness and performances of the closed loop system and defining the controller that could assure the stability and acceptable performance for the family of nominal and parametrically perturbed models [1 – 8]. A global optimization method based on genetic algorithms (GA) is used to achieve the above-mentioned compromise. GAs have the ability to explore huge data sets and find the global optimum. The genetic algorithms operate on a wide space of search, in different directions, simultaneously. Their successful application to real-world problems confirms the conclusions that GAs are powerful robust optimization technique [11]. The efficiency of the proposed method is proven in the case study of Aerosonde UAV's lateral channel.

The 1st Stage: Complete State Space Vector Restoration via Reduced-Order State Observer Luenberger. It has to be noted, that Flight Control System for small UAV should provide nominal performance and robust stability. Thus, such conclusion can be made: because of small number of navigation sensors in UAV and limited abilities of airborne computer, the control law should be simple for its implementation in the airborne equipment. First of all, it is necessary to use state observer to restore the full state space vector, as FCS is provided by incomplete measuring of the state space vector. In our case, we use Luenberger observer.

Let the control plant (UAV) would be denoted by state space model:

$$\dot{X} = Ax + Bu;$$
$$y = Cx + Du.$$

where dimension of state vector **x** equals **n**, and dimension of variable y, which is observed, equals $\boldsymbol{l}$, $\boldsymbol{l < n}$. Thus, number of measuring variables 1 less than number of all phase coordinate n, so we must define observer's gain matrix F that minimize the error $\varepsilon = \chi - \tilde{\chi}$.

Unobservable part of the state vector of the system could be expressed by vector p, which dimension is $(n - l)$:

$$p = \mathbf{C_0} x$$

where $\mathbf{C_0}$ is matrix of variables which are necessary to restore.

The full state x of the system could be described by the expression:

$$\mathbf{x} = \begin{pmatrix} \mathbf{C} \\ \mathbf{C_0} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{y} \\ \mathbf{p} \end{pmatrix}.$$

It is convenient to rewrite, $\begin{pmatrix} \mathbf{C} \\ \mathbf{C_0} \end{pmatrix}^{-1} = (\mathbf{L_1}, \mathbf{L_2})$, thus $x = L_1 y + L_2 p$.

In order to find the vector p it is necessary to take into account that p satisfies such differential equation:

$$\dot{p} = C_0 A L_1 y + C_0 B u. \tag{1}$$

To receive the low-order observer, it is necessary to get information about p, suppose, that

$$q = \hat{p} - Ky. \tag{2}$$

Using equations (1), (2) it's possible to determine that q satisfies to differential equation:

$$\dot{q}(t) = \left[C_0 A L_2 - KCAL_2\right] q(t) + \left[C_0 A L_2 K + C_0 A L_1 - KCAL_1 - KCAL_2 K\right] y(t) + \left[C_0 B - KCB\right] u(t).$$

Therefore, the restored vector $\hat{x}$ of the system is determined via expression:

$$\hat{x} = L_2 q + \left(L_1 + L_2 K\right) y$$

The 2nd Stage: Procedure of Linear Quadratic Regulator (LQR) Design. The procedure of optimal controller design supposes that all state space variables are available for measuring. Thus, after the full state space vector restoration, it is possible to apply the LQR technique. The problem of optimal controller construction includes in minimizing a discrete PI:

$$J_d = \sum_{n=1}^{\infty}\left(\tilde{X}^T(n)Q\tilde{X}^T(n) + U^T(n)RU(n)\right),$$

where Q and R are diagonal matrices weighting each state and control variables, respectively. This controller uses static state feedback:

$$U(n) = -F_0\tilde{X}(n),$$

where gain matrix F_0 is determined by the following expression:

$$F_0 = (R + B^T PB)^{-1} B^T PA,$$

where P – is a solution of the discrete Algebraic Riccati Equation, given in the following way:

$$P = Q + A^T\left(P - PB\left(R + B^T PB\right)^{-1} B^T P\right)A.$$

The 3rd Stage: The Procedure of H_2/H_∞-Robust Optimization Technique Using Genetic Algorithm. After achievement of desired performance by combining the state observer with linear quadratic regulator (LQR) for nominal model, it is necessary to use the same control law for parametrically perturbed models. This procedure is needed to test the robustness of the closed loop system relatively to the parametric uncertainties. For this purpose, the performance of the closed loop system for the nominal and parametrically perturbed models is estimated in the presence and/or absence of external disturbances. This performance may be estimated using $\mathbf{H_2}$-norm. On the next stage robustness is estimated, using $\mathbf{H_\infty}$-norm. The last stage consists of construction a composite performance index (CPI) by summing the above-mentioned norms to form a cost function, to be optimized. The creation of CPI is necessary to find a compromise between the performance and robustness of the nominal and perturbed closed loop systems. This procedure is called $\mathbf{H_2} / \mathbf{H_\infty}$-robust optimization [7 – 8].

The composite performance index (CPI) is given as follows:

$$\begin{aligned} J_1 = {} & \lambda_{dn}\left\|H_{UZ}\right\|_2^{dn} + \lambda_{sn}\left\|H_{uz}\right\|_2^{sn} + \lambda_{\infty n}\left\|T_{wZ}\right\|_\infty^{n} + \lambda_{dp1}\left\|H_{UZ}\right\|_2^{dp1} + \lambda_{sp1}\left\|H_{UZ}\right\|_2^{sp1} + \lambda_{\infty p1}\left\|T_{wZ}\right\|_\infty^{p1} + \\ & + \lambda_{dp2}\left\|H_{UZ}\right\|_2^{dp2} + \lambda_{sp2}\left\|H_{UZ}\right\|_2^{sp2} + \lambda_{\infty p2}\left\|T_{wZ}\right\|_\infty^{p2}, \end{aligned} \tag{3}$$

where in equation (3) H_{UZ} – is the matrix of transfer function of the closed loop system; T_{wZ} – is complementary sensitivity function. The subscripts d and s stand for the deterministic and stochastic, respectively. $\lambda_{\mathrm{dn}}, \lambda_{\mathrm{dp}}, \lambda_{\mathrm{sn}}, \lambda_{\mathrm{sp}}, \lambda_\infty, \lambda_\infty^{\mathrm{p}}$ – are the corresponding LaGrange weight coefficients. Increasing or decreasing the weights it is possible to reach the trade-off between the performance and robustness of closed loop system.

$\mathbf{H_2}$-norm for nominal and perturbed systems in the deterministic case is given by the following expression:

$$\left\|H_{UZ}\right\|_2^{dn} = \sqrt{\sum_{n=0}^{\infty}\left(\tilde{X}^T(n)QX(n) + U^T(n)RU(n)\right)}$$

with corresponding diagonal weight functions Q, R for each state space and control variables. In the stochastic case $\mathbf{H_2}$-norm is computed for the nominal model and the perturbed models as follows:

$$\left\|H_{UZ}\right\|_2^{sn} = \sqrt{E\left(\sum_{n=0}^{\infty}\left(\tilde{X}^T(n)QX(n) + U^T(n)RU(n)\right)\right)},$$

where E is the symbol of the expectation operator, produced by the ensemble averaging.

The $\mathbf{H_\infty}$-norm is computed for the complementary sensitivity function and is given by the following expression:

$$\left\|T_{wZ}\right\|_\infty = \sup\bar{\sigma}\left(T_{wZ}(j\omega)\right),$$

where σ – is maximum singular value of the transfer matrix $T_{wz}(j\omega)$ at the current frequency ω.

Moreover, the total cost function (3) for running optimization procedure has to include some penalty function (PF) for violation of the location's area of the closed loop system poles in the complex plane [7].

Thus, the total cost function to be optimized is represented as:

$$J = J_1 + PF$$

For achievement aforementioned compromise the global optimization method is used based on genetic algorithms.

Genetic algorithms. The origin of the genetic algorithms was an effort to mimic some of the processes taking place in natural evolution. Genetic algorithms are currently the most prominent and widely used computational models. It may be explained by further facts. Genetic algorithms always consider a population of solutions. It keeps in memory more than a single solution at each iteration and can recombine different solutions to get better ones. Genetic algorithms can optimize multi-modal, discontinuous and nondifferentiable functions. They are shown to solve linear and nonlinear problems by exploring all regions of the state space. Genetic algorithm requires the determination of six basic issues: chromosome representation; selection function; the genetic operators making up the reproduction function; the creation of the initial population; termination criteria; evaluation function [11].

Case study. Consider the lateral channel of small UAV. The state space vector of the model has the following components: $\mathbf{X} = [\mathbf{v}, \mathbf{p}, \mathbf{r}, \varphi, \psi]$, where v is velocity, p – roll rate, r – yaw rate, φ – roll angle, ψ – yaw angle [9-10]. The control variables are the positions of ailerons and rudder. During work performance we dealt with the nominal model corresponded at the true air speed $V_n = 30\,\text{m/s}$, the first perturbed model at $V_{p1} = 25\,\text{m/s}$ and the second one at $V_{p2} = 35\,\text{m/s}$. Thus, we have three models, which are represented in the state space, with the following matrices:

$$\mathbf{A_n} = \begin{bmatrix} -0.8297 & 0.5669 & -29.9943 & 9.7843 & 0 \\ -5.4777 & -26.9761 & 12.9834 & 0 & 0 \\ 0.8890 & -3.4991 & -1.3570 & 0 & 0 \\ 0 & 1.0 & 0.0189 & 0 & 0 \\ 0 & 0 & 1.0002 & 0 & 0 \end{bmatrix} \qquad \mathbf{B_n} = \begin{bmatrix} -2.1338 & 5.4466 \\ -187.3534 & 3.3711 \\ -7.3871 & -34.4140 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{A_{p1}} = \begin{bmatrix} -0.6909 & 1.2123 & -24.9704 & 9.8089 & 0 \\ -4.5543 & -22.4221 & 10.7916 & 0 & 0 \\ 0.7389 & -2.9084 & -1.1279 & 0 & 0 \\ 0 & 1.0 & 0.0485 & 0 & 0 \\ 0 & 0 & 1.0012 & 0 & 0 \end{bmatrix} \qquad \mathbf{B_{p1}} = \begin{bmatrix} -1.4780 & 3.7726 \\ -129.7714 & 2.3350 \\ -5.1167 & -23.8370 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{A_{p2}} = \begin{bmatrix} -0.9666 & 0.0537 & -34.9996 & -9.7881 & 0 \\ -6.3785 & -31.4095 & 15.1171 & 0 & 0 \\ 1.0351 & -4.0742 & -1.5800 & 0 & 0 \\ 0 & 1.0 & 0.0015 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B_{p2}} = \begin{bmatrix} -2.8985 & 7.3987 \\ -254.5012 & 4.5793 \\ -10.0346 & -46.7481 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The measured variables are $\overline{\mathbf{X}} = [\mathbf{p} \quad \mathbf{r} \quad \psi]^{\mathrm{T}}$, thus the observation matrix is given as follows:

$$\mathbf{C}=\begin{bmatrix}\mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1}\end{bmatrix}.$$

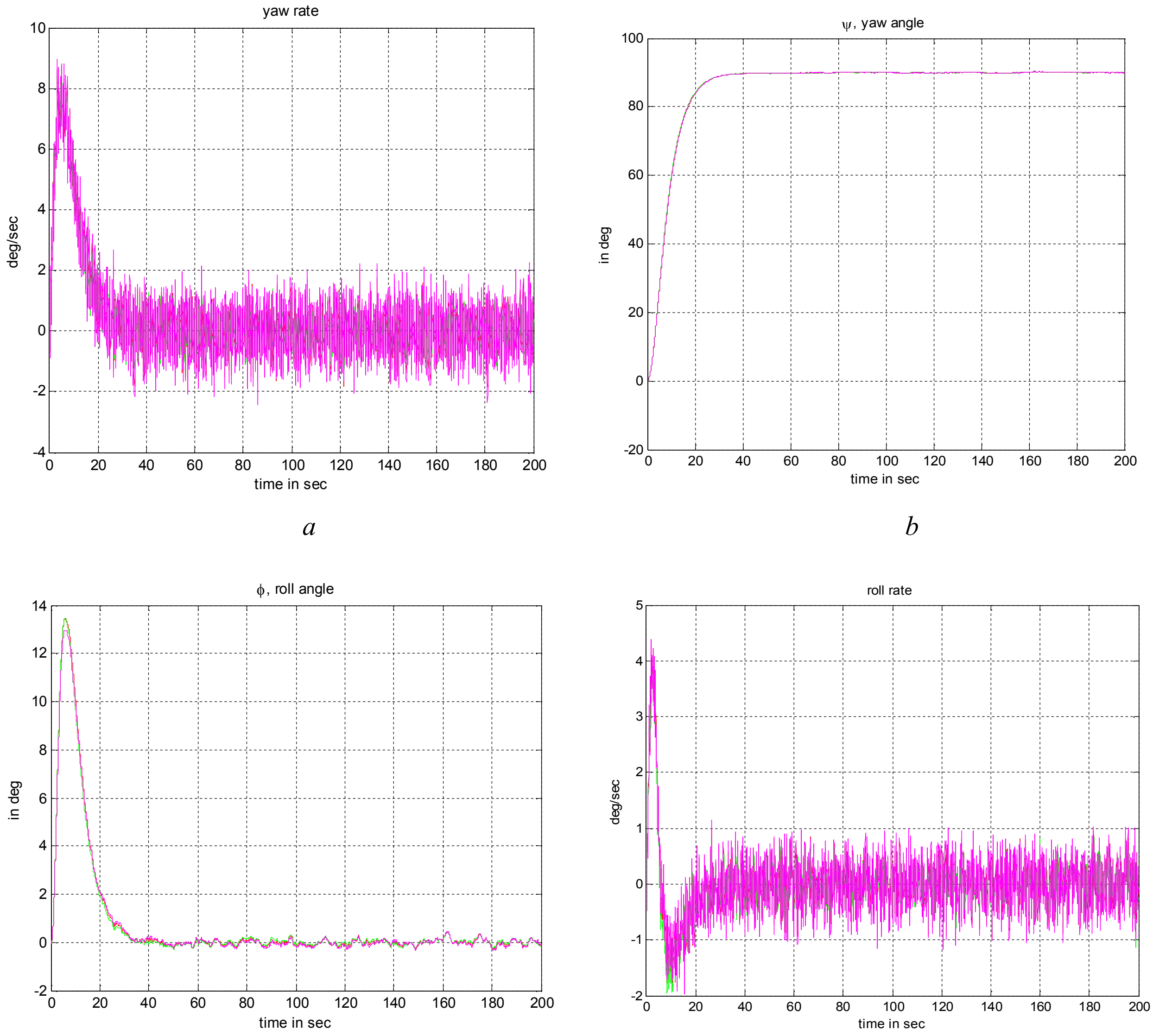
The state space description of actuator connected in series with UAV model is the following:

$$\begin{bmatrix}\mathbf{A_a} & \mathbf{B_a}\\ \mathbf{C_a} & \mathbf{D_a}\end{bmatrix}=\begin{bmatrix}-\mathbf{1/T_a} & \mathbf{1/T_a}\\ \mathbf{1} & \mathbf{0}\end{bmatrix}$$

where T_a is time constant.

In order, to take into account the influence of the atmospheric disturbances the Dryden model is used [1; 9; 10]. The simulation results for the lateral channel of UAV motion is given on a figure below.

Simulation results



a *b* *c* *d*

Simulation results of the lateral channel: *a* – yaw rate of UAV nominal and perturbed models; *b* – yaw angle of UAV nominal and perturbed models; *c* – roll angle of UAV nominal and perturbed models; *d* – roll rate of UAV nominal and perturbed models

The performance and robustness indices are given in table 1. The standard deviations for the set of nominal and parametrically perturbed models are reflected in table 2.

Table 1

Estimated performance and robustness of the nominal and perturbed closed loop systems

Plant	$H_{2\det}$	H_{2s}	$H_{\infty y}$	$H_{\infty z}$
Nominal	0,4200	1,0280	2,7639	0,8712
Perturbed1	0,7041	0,9084	3,0149	0,7674
Perturbed2	0,2794	1,1426	2,6837	0,9791

Table 2

The values of standard deviation of the nominal and perturbed closed loop systems

Plant	Standard deviation						
	σ_β, ^{o}	σ_p, deg/sec	σ_r, deg/sec	σ_φ, ^{o}	σ_ψ, ^{o}	σ_{Ail}, ^{o}	σ_{Rud}, ^{o}
Nominal	2,2045	0,3033	0,5008	0,1351	0,1167	0,0336	0,4503
Perturbed1	2,6483	0,2780	0,4385	0,1368	0,1225	0,0334	0,4405
Perturbed2	1,8928	0,3324	0,5780	0,1551	0,1132	0,0333	0,4687

Conclusion. The aim of the paper is robust autopilot design with incomplete state space measurements. Procedure of robust $\mathbf{H_2/H_\infty}$ design is used in order to achieve the trade-off between the robustness and performance of the nominal closed-loop model and family of the closed loop perturbed one. The efficacy of the proposed control scheme has been verified and confirmed by computer simulation. The heading is stabilized at the reference signal (90 deg.) and all flight requirements are satisfied within the acceptable bounds. The performance and robustness indices of the overall closed loop systems prove the efficiency of the synthesized controller.

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М. Н. Комнацкая, Е. И. Поддубченко, О. В. Сердюк

Робастная оптимизация системы управления полетом беспилотного летательного аппарата при неполных измерениях вектора состояния

Рассмотрена процедура синтеза робастного закона управления на примере бокового движения беспилотного летательного аппарата с использованием процедуры $\mathbf{H_2/H_\infty}$-робастной оптимизации на основании генетических алгоритмов. Первый этап работы заключался в восстановлении полного вектора состояния с помощью наблюдателя пониженного порядка Люенбергера и синтеза оптимального регулятора. Качество и робастность системы управления оценены с помощью $\mathbf{H_2}$-нормы функции чувствительности и $\mathbf{H_\infty}$-нормы комплементарной функции чувствительности. Второй этап работы состоял в робастизации полученного закона управления с помощью генетического алгоритма. В ходе оптимизационной процедуры найден компромисс между робастностью и качеством замкнутой системы управления. Результаты моделирования замкнутой системы свидетельствуют об эффективности предложенной процедуры.

М. М. Комнацька, О. І. Поддубченко, О. В. Сердюк

Робастна оптимізація системи керування польотом безпілотного літального апарату при неповних вимірах вектора стану

Розглянуто процедуру синтезу робастного закону керування на прикладі бічного руху безпілотного літального апарата з використанням процедури $\mathbf{H_2/H_\infty}$-робастної оптимізації на основі генетичних алгоритмів. Перший етап роботи полягав у відновленні повного вектора стану за допомогою спостерігача пониженого порядку Люенбергера та синтезу оптимального закону керування. Якість та робастність замкненої системи керування оцінено за допомогою $\mathbf{H_2}$-норми функції чутливості та $\mathbf{H_\infty}$-норми комплементарної функції чутливості. Другий етап полягав в робастизації отриманого закону керування за допомогою генетичного алгоритму. У процесі оптимізаційної процедури знайдено розумний компроміс між робастністю та якістю замкненої системи керування. Результати моделювання замкненої системи свідчать про ефективність запропонованої процедури.