MATHEMATICAL MODELING OF PROCESSES AND SYSTEMS

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ALGORITHM FOR ESTIMATING THE AIRCRAFT ACCURACY LANDING DURING OPERATIONAL CONTROL

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***Abstract*—*A Bayesian algorithm is considered to determine the statistical accuracy factor, which can be used to estimate the accuracy characteristics of on-board automatic landing system on the stage of operational control.*

Index Terms—Landing accuracy; Bayesian estimation; a priory density of distribution; a posteriory density of distribution; confidence bound; confidence probability.

I. Problem Definition

The estimation of the airplane landing accuracy may be realized with using binary representation of the measurement results of defining parameters (side and longitudinal deviations of the airplane in a touch point). In this case the simple algorithm is used by means of calculation of events frequency. However, such method does not take into account the distance between the parameter value and the limit value of the accessible region. In deed, the values which are settled not far from the accessible region bound, and the values settled far off the found have the same weight. It is essential disadvantage which defines slow efficiency and accuracy (slow coufidence coefficient) on condition that the number of flight test have insufficient size.

Below the algorithm of accuracy control with use of Bayesian estimation procedure, taking into account statistical characteristics of the defining parameters measured deeoing control, and also a priore information for the control beginning is offered.

II. Solution of a Problem

Let the defining parameter Z be distributed according to the normal law. We will designate its allowed value Z_{al}. The information about the parameter with the corresponding interval of a priory uncertainty is known to the beginning of operational control: for the average of distribution and the mean square deviation such intervals are $m_z^{(a)} \pm \Delta_m$ and $\sigma_z^{(a)} \pm \Delta_\sigma$.

To determine a posteriory density of distribution $f\left(m_z, \sigma_z / z_1, z_2, ..., z_n\right)$ we may use the Bayesian decision based on prior density, joined with the normal likelihood kernel [1], [2]. In this case the normal – gamma distribution is used as a prior density:

$$h\left(m_z, \tau_z\right) = h_\tau\left(\tau_z\right) h_m\left(m_z / \tau_z\right), \qquad \tau_z = \frac{1}{\sigma_z^2},$$

where

$$h_\tau(\tau_z) = \left(\frac{\nu'(\vartheta')^2}{2}\right)^{\frac{\nu'}{2}} \frac{(\tau_z)^{\frac{\nu'}{2}-1}}{\Gamma\left(\frac{\nu'}{2}\right)} \exp\left\{-\frac{1}{2}(\nu')^2(\vartheta')^2\tau_z\right\},$$

$$h_m(m_z / \tau_z) = \frac{\left(\tau_z n'\right)^{\frac{1}{2}}}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\tau_z n'(m_z - m')^2\right\}.$$

Parameters of a priory density are defined by means of prior given numerical characteristics $\left\{m_z^{(a)}, \Delta_m\right\}$ and $\left\{\sigma_z^{(a)}, \Delta_\sigma\right\}$ by equating of the theoretical moments of distributions $h(\tau_z)$ and $h(m_z / \tau_z)$ to the corresponding calculated values (a priory upper bound of deviation is one sixth squared of tolerance range, that is explained by known characteristics of dispersions because of instability):

$$m' = m_z^{(a)}, \quad n' = \frac{\left(\sigma_z^{(a)}\right)^2}{s_m^2} = \left| s_m^2 = \left(\frac{\Delta_m}{3}\right)^2 \right| = \frac{9\left(\sigma_z^{(a)}\right)^2}{\Delta_m^2},$$

$$\vartheta' = \left(\sigma_z^{(a)}\right)^2, \quad \nu' = \frac{\left(\sigma_z^{(a)}\right)^2}{2s_\sigma^2} = \frac{9\left(\sigma_z^{(a)}\right)^2}{2\Delta_\sigma^2}.$$

Jet's introduce the concept of the accuracy reserve coefficient $k = \frac{m_z}{Z_{al}}$ and define the probability of an inequality $z < Z_{al}$ or $\left(z - Z_{al}\right) < 0$ it is known that for a difference of two normal random values we may write:

$$P\{(Z - U) < 0\} = 1 - \Phi\left\{\frac{m_z - m_u}{\sqrt{\sigma_z^2 + \sigma_u^2}}\right\},$$

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where $\Phi(\cdot)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{(\cdot)}\exp\left\{-\frac{t^2}{2}\right\}dt.$

If the variable U is the constant value, equaled, for example Z_{al}, then $m_u=m_{z_{al}}=Z_{al}$, $\sigma_u^2=\sigma_{z_{al}}^2=0$. In this case we may rewrite the obtained above expression as:

$$P\{(Z-Z_{al})<0\}=1-\Phi\left\{\frac{m_z-Z_{al}}{\sigma_z}\right\}$$

$$=1-\Phi\left\{\frac{\frac{m_z}{Z_{al}}-1}{\frac{\sigma_z}{Z_{al}}}\right\}=1-\Phi\left\{\frac{\frac{m_z}{Z_{al}}-1}{\frac{\sigma_z}{Z_{al}}}\right\}.$$

From a condition

$$1-\Phi\left\{\frac{k_{\lim}-1}{\frac{\sigma_z}{Z_{al}}}\right\}=P_{req}$$

it is possible to find the smallest (limit) value of coefficient $k_{\lim}$ providing fulfillment of the requirement for accuracy

$$\frac{k_{\lim}-1}{\frac{\sigma_z}{Z_{al}}}=u_{req},$$

where u_{req} is a quantile of the normal distribution for probability $1-P_{req}$, P_{req} is required probability of a condition $Z<Z_{al}$.

Thus

$$k_{\lim}=1+u_{req}\frac{\sigma_z}{Z_{al}}.$$

Next we will consider that for the given probability P_{req} of the condition $Z<Z_{al}$ it is necessary to construct Bayesian algorithm to control the condition

$$P_j=P_j\left(z_{(j)}\right)\geq P_{req}$$

for each stage of operational control, where $z_{(j)}$ is sempling $(z_1,z_2,...,z_j)$ of the measurement result. Jet's assume that Bayesian algorithm low confidence bound $\underline{P}_\gamma$ is the astimation P_j. Then, according to of monotonicity property of the function Bayesian lower, in view of monotony of function

$$P\{k,\sigma_z,Z_{al}\}=1-\Phi\left\{\frac{k-1}{\frac{\sigma_z}{Z_{al}}}\right\}.$$

According to the variable k we may write

$$P\left\{\underline{P}_\gamma\geq P_{req}\right\}=P\left\{\overline{k}_\gamma;\sigma_z\right\}\geq P\left\{k_{\lim};\sigma_z\right\},$$

where from it follows

$$\overline{k}_\gamma(z)\leq k_{\lim}(z)$$

Then, the problem consists in the calculation of Bayesian upper confidents bound $\overline{k}_\gamma(z)$ for the accuracy reserve coefficient, where the value γ is the given confidence probability.

The operational control procedure of the definite parameter Z includes the calculation of a posteriori Bayesian upper γ – confidential bound $\overline{k}_\gamma=\overline{k}_\gamma(z)$ and the comparison of the obtain value with the coefficient $k_{\lim}$.

To find the value $\overline{k}_\gamma=\frac{\overline{m}_{z\gamma}}{Z_{al}}$ it is necessary to know a posteriory density $f\left(m_z/\tau_z^*\right)$, where τ_z^* is a posteriory value of the parameter τ_z. According to the joined priory distributions theory [1] the required posteriory density for the normal law has the same distribution as a priory density, but with other parameters:

$$f\left(m_z/\tau_z^*\right)=\left(\frac{\tau_z^*n''}{2\pi}\right)^{\frac{1}{2}}\exp\left\{-\frac{1}{2}\tau_z^*n''\left(m_z-m''\right)^2\right\},$$

$$n''=n'+n,\qquad m''=\frac{n'm'+nm^*}{n'+n},$$

$$\tau_z^*=\frac{\nu'+n}{\nu'(\vartheta')^2+n'(m')^2+n\left(m^*\right)^2+nD^*-n''(m'')^2},$$

$$m^*=\frac{1}{n}\sum_{i=1}^{n}z_i,\qquad D^*=\frac{1}{n}\sum_{i=1}^{n}\left(z_i-m^*\right)^2.$$

According to the concept of Bayesian confidential estimation [3], [4] upper bound $\overline{m}_\gamma$ with the confidence level γ is defined by the equation

$$\int_{-\infty}^{\overline{m}_\gamma}f\left(m_z/\tau_z^*\right)dm_z=\gamma.$$

Then

$$\frac{\sqrt{\tau_z^*n''}}{\sqrt{2\pi}}\int_{-\infty}^{\overline{m}_\gamma}\exp\left\{-\frac{1}{2}\frac{(m_z-m'')^2}{\frac{1}{\tau_z^*n''}}\right\}dm_z=N\left[\overline{m}_{z\gamma},m'',\frac{1}{\sqrt{\tau_z^*n''}}\right]$$

$$=N\left[\frac{\overline{m}_{z\gamma}-m''}{\frac{1}{\sqrt{\tau_z^*n''}}};\ 0;\ 1\right]=\Phi\left[\frac{\overline{m}_{z\gamma}-m''}{\frac{1}{\sqrt{\tau_z^*n''}}}\right]=\gamma.$$

Wherefrom it follows

$$\left(\overline{m}_{z\gamma} - m''\right)\sqrt{\tau_z^* n''} = u_\gamma,$$

where u_γ is a quantile of the normal distribution for the probability γ.

Thus, we get the expression for the upper confidence bound $\overline{k}_\gamma$

$$\overline{k}_\gamma = \frac{\overline{m}_{z\gamma}}{Z_{al}} = \frac{m''}{Z_{al}} + u_\gamma \frac{1}{Z_{al}\sqrt{\tau_z^* n''}}.$$

Example. Measurements of the plane a lateral deviation in touch point at a stage of operational control are carried out:

$$\{|z_i|\} = 5;\ 3;\ 6;\ 9;\ 9;\ 12;\ 15;\ 12;\ 5;\ 4.5;\ 3;\ 3;\ 4\ 4;\ 2.$$

For allowed value of the defining parameter $|Z_{al}| = 8,2$ m the required level of probability $P_{req} = 0,95$. A priory information is given by the values: $m_z^{(a)} = 3$ m; $\Delta_m = 1$ m; $\sigma_z^{(a)} = 2$ m; $\Delta_\sigma = 0,8$ m.

The limit value of the accuracy reserve coefficient is defined by the expression:

$$k_{\lim} = 1 + u_{req}\frac{\sigma_z^{(a)}}{Z_{al}} = 1 - 1.645\frac{2}{8.2} = 0.599.$$

It is seen from an example condition the given accuracy is provided before operational control, because

$$\frac{\left|m_z^{(a)}\right|}{|Z_{al}|} = 0.366 < k_{\lim}.$$

In process of receipt of experimental information change of estimation $\overline{k}_\gamma$ for the confidence probability $\gamma = 0,9$ $(u_\gamma = 1.282)$ is calculated according to the obtained pasteriory parameters m'', n'' and τ_z^* on each step.

III. Conclusions

The offered technique of the accuracy reserve coefficient estimation allows to take into account the required probability of finding of the defining parameter in the given allowed area, and also the statistical characteristics of the parameter, obtained by results of operational control. The technique can be used in the software for a database to maintain an operational phase of airplanes, equipped with on-board automatic control systems.

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О. А. Зеленков, А. П. Голік. Алгоритм оцінки точності приземлення літака на етапі експлуатаційного контролю

Розглянуто байєсівський алгоритм визначення статистичного коефіцієнта точності, який можна використовувати для оцінки точності бортових систем автоматичного приземлення на етапах експлуатаційного контролю.

Ключові слова: точність приземлення; байєсівське оцінювання; апріорна щільність розподілу; апостериорна щільність розподілу; довірча межа; довірча ймовірність.

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А. А. Зеленков, А. П. Голик. Алгоритм оценки точности приземления самолета на этапе эксплуатационного контроля

Рассмотрен байесовский алгоритм определения статистического коэффициента точности, который можно использовать для оценки точностных характеристик бортовых систем автоматического приземления на этапах эксплуатационного контроля.

Ключевые слова: точность приземления; байесовское оценивание; априорная плотность распределения; апостериорная плотность распределения; доверительная граница; доверительная вероятность.

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