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BIAS COMPENSATION IN DIFFERENTIAL CORIOLIS VIBRATORY GYRO

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Abstract. *This work presets one of the ways to compensate for the bias in differential mode of Coriolis vibratory gyro operation. To compensate for the bias it is necessary to know both channels scale factors, SF_x and SF_y of differential Coriolis vibratory gyro, which, as a rule, are known by the results of calibration procedure and elastic wave angle θ relative to one of the two drive axes which can be established in advance. This paper presents the different variants of bias compensation algorithms which can be applied to compensate for initial gyro bias and bias drift in motion. Experimental results on differential Coriolis vibratory gyro key parameters temperature influence are also presented.*

Keywords: differential Coriolis vibratory gyro; bias; standing wave angle; phases difference.

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I. INTRODUCTION

The differential mode of operation, which is analyzed in this paper, can be implemented in the ring-like Coriolis vibratory gyro (CVG) resonator by keeping a standing wave between the electrodes by applying two stable amplitude signals on X and Y drive axes. In this case two magnitudes of angular rates with opposite signs can be read-out from X and Y sense axes [1]. The resulting angular rate can be obtained by signals subtraction of two angular rate channels and at proper alignment of standing wave cross damping bias component is compensated. Moreover, the sum of these two signals gives information about the bias drift components without angular rate that can be used to estimate certain bias components for on-line compensation [2].

II. PROBLEM STATEMENT

Differential CVG gives many possibilities to estimate and/or to compensate for bias drift. Some of bias compensation algorithms are derived and discussed in this paper. Experimental results on differential CVG key parameters temperature influence are also presented.

III. DIFFERENTIAL CVG MEASUREMENT EQUATIONS

Suppose that under applying control signals f_x and f_y to the X and Y electrodes the standing wave is aligned between them, as it is shown by the dashed line in Fig. 1. The dynamic equations of the standing wave in this case can be written down as follows [3]:

$$\ddot{x} + d_{xx}\dot{x} + k_{xx}x + k_{xy}y = (2k\Omega - d_{xy})\dot{y} + f_x,$$
$$\ddot{y} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y = (-2k\Omega - d_{xy})\dot{x} + f_y, \quad (1)$$

where k is the Brian coefficient, approximately equal to 0.4; $d_{xx} = 2/\tau + h\cos 2\theta_\tau$ is X axis damping coefficient $2/\tau = 1/\tau_1 + 1/\tau_2$, $h = 1/\tau_1 - 1/\tau_2$; τ_1 is the minimum resonator damping time; τ_2 is the maximum resonator damping time; $d_{xy} = h\sin 2\theta_\tau$ is the damping cross-coupling; d_{yx} = d_{xy}; $k_{xx} = \omega_1^2 - \omega\Delta\omega\cos 2\theta_\omega$ is the resonator rigidity coefficient along X axis normalized by the mass, $\omega\Delta\omega = (\omega_1^2 - \omega_2^2)/2$, where ω_1 and ω_2 are the maximum and minimum resonant frequencies; $k_{xy} = -\omega\Delta\omega\sin 2\theta_\omega$ is the rigidity cross-coupling; k_{xy} = k_{yx}; $d_{yy} = 2/\tau - h\cos 2\theta_\tau$ is Y axis damping coefficient; $k_{yy} = \omega_2^2 + {}+\omega\Delta\omega\cos 2\theta_\omega$ is the resonator rigidity coefficient along Y axis normalized by the mass; f_x, f_y are normalized by the mass control signals; θ_ω is an angle between minimum frequency axis and standing wave (antinode) axis; θ_τ is an angle between the minimum damping axis and the standing wave (antinode) axis.

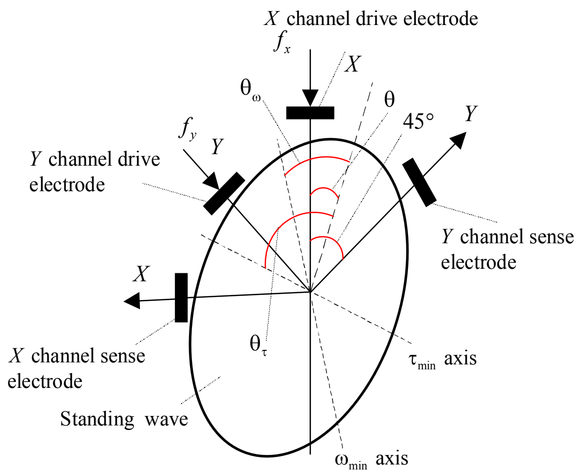

Fig. 1. Standing wave in ring-like differential CVG resonator

Control signals f_x and f_y should be applied on X and Y axes to make the rigidity equal $k_{xx} = k_{yy}$. It means that $\omega_1 = \omega_2 = \omega_r$.

Let's find stationary solution of equations (1) in view of the following expressions

$$x = r\cos 2\theta \sin(\omega_r t),\ y = r\cos 2\theta \sin(\omega_r t + \varphi), \quad (2)$$

where r is oscillation amplitude; θ is vibration angle relative to X axis; φ is phase difference between X and Y sense axes signals that should be constant to provide $\omega_1 = \omega_2 = \omega_r$ [2].

Suppose that $\theta \neq n\pi/4$, $n = 0, 1, 2, \ldots$, then for X and Y sense signals z_x and z_y in voltages after demodulation by reference signals $\sin\omega_r t$ and $\cos\omega_r t$ and transformations, the following expressions can be obtained [2]

$$\begin{aligned} -2k\Omega D_y \mathrm{tg}2\theta\cos\varphi + d_{xy}D_y\mathrm{tg}2\theta\cos\varphi + D_x d_{xx} = z_x; \\ 2k\Omega D_x\cot 2\theta + d_{xy}D_x\mathrm{ctg}2\theta + D_y d_{yy}\cos\varphi = z_y, \end{aligned} \quad (3)$$

where D_x and D_y are transformation coefficients of mechanical deformation into the X and Y electrodes voltages.

IV. Bias compensation by switching control

Coefficients at angle rate Ω variable are scale factors of the X and Y sense channels

$$\begin{aligned} SF_x &= 2kD_y\mathrm{tg}2\theta\cos\varphi; \\ SF_y &= 2kD_x\cot 2\theta. \end{aligned} \quad (4)$$

Let $\varphi = 0$, then (3) can be rewritten as follows

$$\begin{aligned} -SF_x\Omega + d_{xy}D_y\mathrm{tg}2\theta + D_x d_{xx} = z_x^0; \\ SF_y\Omega + d_{xy}D_x\mathrm{ctg}2\theta + D_y d_{yy} = z_y^0. \end{aligned} \quad (5)$$

When $\varphi = \pi$, then (3) is

$$\begin{aligned} SF_x\Omega - d_{xy}D_y\mathrm{tg}2\theta + D_x d_{xx} = z_x^\pi; \\ SF_y\Omega + d_{xy}D_x\mathrm{ctg}2\theta - D_y d_{yy} = z_y^\pi. \end{aligned} \quad (6)$$

After summing and subtracting (5) and (6) the following expressions can be obtained

$$\begin{aligned} z_x^0 + z_x^\pi &= 2D_x d_{xx};\ z_y^0 - z_y^\pi = 2D_y d_{yy}; \\ z_y^0 + z_y^\pi &= 2SF_y\Omega + 2d_{xy}D_x\cot 2\theta; \\ z_x^\pi - z_x^0 &= 2SF_y\Omega - 2d_{xy}D_y\tan 2\theta. \end{aligned} \quad (7)$$

Solution of (7) relative to Ω yields

$$\begin{aligned} z_y^0 + z_y^\pi + z_x^\pi - z_x^0 = 2(SF_x + SF_y)\Omega \\ + 2d_{xy}(D_x\cot 2\theta - D_y\tan 2\theta); \end{aligned} \quad (8)$$

It can be concluded from (8) that if

$$D_x\cot 2\theta - D_y\tan 2\theta = 0;$$

$$\text{or } \frac{D_x}{D_y} = \tan^2 2\theta; \to \theta_* = \frac{1}{2}\tan^{-1}\sqrt{\frac{D_x}{D_y}}, \quad (9)$$

Pure angle rate without bias can be obtained:

$$\begin{aligned} z_y^0 + z_y^\pi + z_x^\pi - z_x^0 &= 2(SF_x^{\theta_*} + SF_y^{\theta_*})\Omega; \\ SF_x^{\theta_*} = SF_x^\theta * TR;\ &SF_y^{\theta_*} = SF_y^\theta / TR; \\ TR = \frac{1+\tan 2\Delta\theta}{1-\tan 2\Delta\theta};\ &\Delta\theta = \theta - \theta_*. \end{aligned} \quad (10)$$

It follows from (4) that

$$\frac{SF_y}{SF_x} = \frac{D_x}{D_y}\cot^2 2\theta. \quad (11)$$

So, at the beginning wave can be set on arbitrary angle $\theta \neq k\pi/4$, $k = 0, 1, 2, \ldots$, more convenient value is $\theta = \pi/8$. In this case $\cot 2\theta = 1$ and

$$\frac{SF_y^{\pi/8}}{SF_x^{\pi/8}} = \frac{D_x}{D_y} \to \theta_* = \frac{1}{2}\tan^{-1}\sqrt{\frac{SF_y^{\pi/8}}{SF_x^{\pi/8}}}. \quad (12)$$

Differential CVG control system block diagram is presented in Fig. 2.

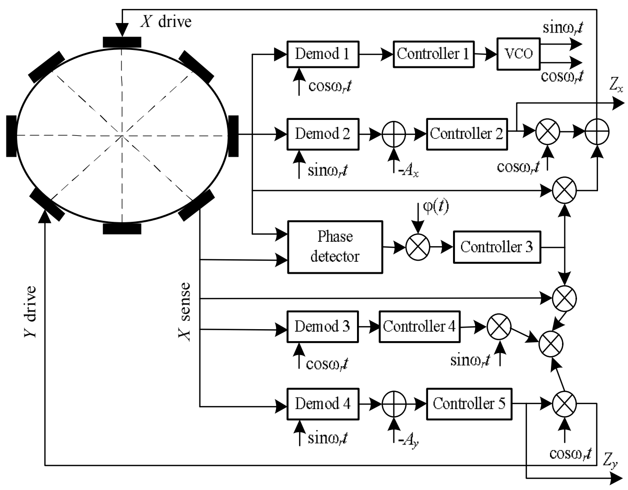

Fig. 2. Differential CVG block diagram

Phase difference $\varphi(t)$ control law to compensate for the differential CVG bias is graphically presented in Fig. 3.

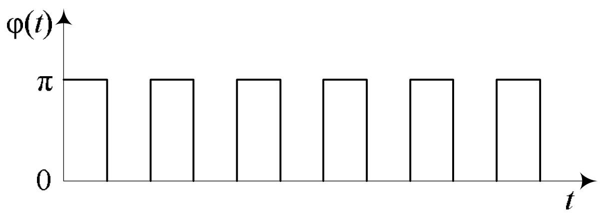

Fig. 3. Phase difference switching control law to compensate for the differential CVG bias

Because of scale factors SF_x and SF_y can change versus temperature that results in wave angle θ change, temperature sensitivity of these parameters should be studied.

Figures 4 and 5 show temperature dependencies of X and Y channels scale factors and total one $SF_{\text{total}}^{\pi/8} = SF_x^{\pi/8} + SF_y^{\pi/8}$ when wave angle is π/8 (22.5 deg). As can be seen from the Fig. 5 $SF_{\text{total}}^{\pi/8}$ temperature sensitivity is 0.016 %/°C.

As can be seen from Fig. 4 $SF_x^{\pi/8}(T)$ and $SF_y^{\pi/8}(T)$ curves are almost parallel to each other. The latter means that wave angle θ calculated as a ratio of these two scale factors (see (12)) will be very stable.

Figure 6 shows wave angle θ calculated by expression (12) for different temperatures. Temperature sensitivity of this angle is sufficiently low and is equal to 0.95×10^{-4} %/°C.

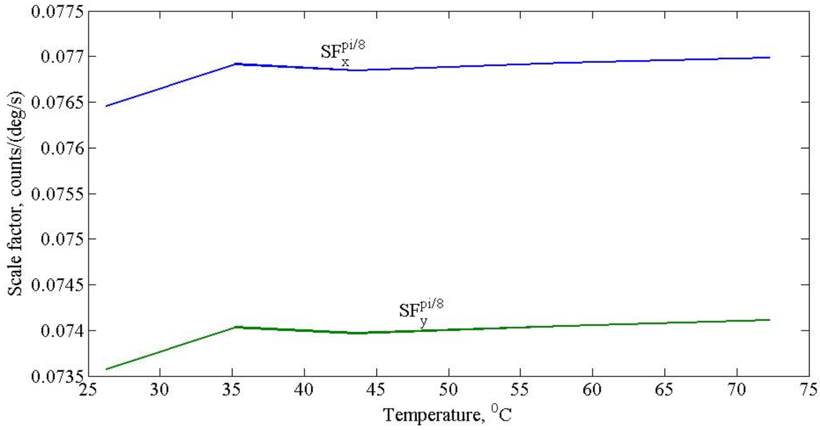

Fig. 4. Scale factors temperature dependence

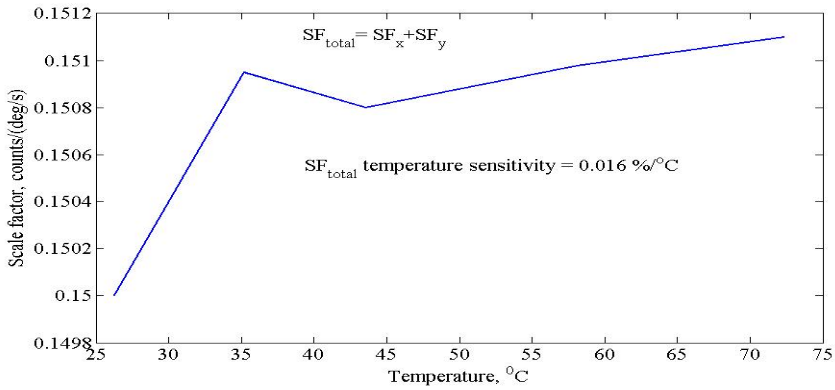

Fig. 5. Total scale factors temperature sensitivity

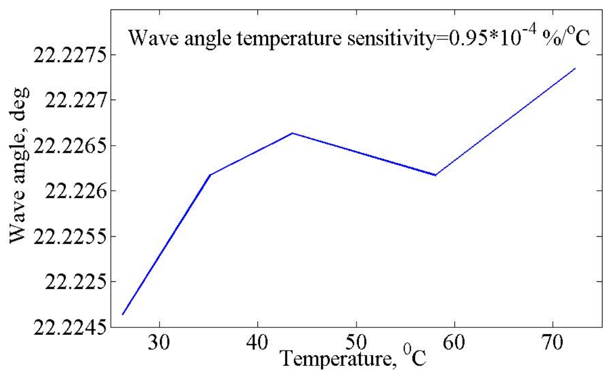

Fig. 6. Wave angle θ versus temperature

So, first step in differential CVG bias compensation procedure is to align standing wave under angle θ = π/8 and to carry out scale factors SF_x and SF_y calibration. It can be carried out at room temperature, because their ratio is almost insensitive to temperature change. The second step is to realign the standing wave to the angle θ in accordance with expression (12). The third step is to send a signal φ(t) in accordance with one presented in Fig. 3 to the input of controller 3 (see Fig. 2). Angle rate Ω should be calculated by he expression (10), where θ = π/8 and as follows from Fig. 6 Δθ = 22.5 – 22.226 = 0.274 deg. (0.00478 rad), TR = 0.99967.

V. BIAS COMPENSATION BY SINE CONTROL

Switching control law can cause transients which result in measurement errors. To reduce errors in high dynamic measurements sine function control law as depicted in Fig. 7 can be used.

$$\varphi(t) = \varphi_0 \sin \omega_\varphi t. \qquad (13)$$

Substitution (13) in (3) yelds

$$-2k\Omega D_y \mathrm{tg}2\theta \cos(\varphi_0 \sin \omega_\varphi t) + d_{xy} D_y \mathrm{tg}2\theta \cos(\varphi_0 \sin \omega_\varphi t) + D_x d_{xx} = z_x;$$
$$2k\Omega D_x \cot 2\theta + d_{xy} D_x \mathrm{ctg}2\theta + D_y d_{yy} \cos(\varphi_0 \sin \omega_\varphi t) = z_y. \qquad (14)$$

After Bessel function of the first kind expansion of function $\cos(\varphi_0 \sin\omega_\varphi t)$, the following expressions can be obtained

$$z_x = -SF_x J_0(\varphi_0)\Omega - 2SF_x J_2(\varphi_0)\Omega \cos 2\omega_\varphi t + d_{xy} D_y \mathrm{tg}2\theta J_0(\varphi_0) + 2d_{xy} D_y \mathrm{tg}2\theta J_2(\varphi_0)\cos 2\omega_\varphi t + D_x d_{xx};$$
$$z_y = SF_y \Omega + d_{xy} D_x \mathrm{ctg}2\theta + D_y d_{yy} J_0(\varphi_0) + 2D_y d_{yy} J_2(\varphi_0)\cos 2\omega_\varphi t. \qquad (15)$$

Here only second harmonic of frequency ω_φ is taking into account.

Low pass filter (LPF) output signal is

$$z_x^{LPF} = -SF_x J_0(\varphi_0)\Omega + d_{xy} D_y \mathrm{tg}2\theta J_0(\varphi_0) + D_x d_{xx};$$
$$z_y^{LPF} = SF_y \Omega + d_{xy} D_x \mathrm{ctg}2\theta + D_y d_{yy} J_0(\varphi_0). \qquad (16)$$

The difference of these two signals yields

$$z_y^{LPF} - z_x^{LPF} = (SF_x J_0(\varphi_0) + SF_y)\Omega + d_{xy}\left[D_x \mathrm{ctg}2\theta - D_y \mathrm{tg}2\theta J_0(\varphi_0)\right] + D_y d_{yy} J_0(\varphi_0) - D_x d_{xx}. \qquad (17)$$

Let's find wave angle θ and phase difference scanning amplitude φ_0 to null two last summands of (17). It means two equations should be solved for θ and φ_0

$$D_x \mathrm{ctg}2\theta - D_y \mathrm{tg}2\theta J_0(\varphi_0) = 0$$
$$D_y d_{yy} J_0(\varphi_0) - D_x d_{xx} = 0. \qquad (18)$$

Taking into account (12) the following solution can be obtained

$$J_0(\varphi_0) = \frac{SF_y^{\pi/8}}{SF_x^{\pi/8}} \frac{d_{xx}}{d_{yy}};$$
$$\theta_0 = \frac{1}{2}\tan^{-1}\sqrt{\frac{SF_y^{\pi/8}}{SF_x^{\pi/8} J_0(\varphi_0)}}. \qquad (19)$$

Ratio d_{xx}/d_{yy} is also almost insensitive to temperature, because X and Y damping are almost equally changed versus temperature [4]. So, d_{xx} and d_{yy} can be measured at room temperature. For the gyro under test calculations give the following values: $d_{xx}/d_{yy} = 0.951$, $J_0(\varphi_0) = 0.885$, $\varphi_0 = 39.408$ deg, $\theta_0 = 22.862$ deg, $\omega_\varphi > d\Omega/dt$.

The last two summands can be nulled by using combination of LPF and band pass filter (BPF) signals extracting the second harmonic $2\omega_\varphi$ frequency. After demodulation of this signals the following expressions for output signals of demodulators can be obtained

$$z_x^{BPFD} = -2SF_x J_2(\varphi_0)\Omega + 2d_{xy} D_y \mathrm{tg}2\theta J_2(\varphi_0);$$
$$z_y^{BPFD} = 2J_2(\varphi_0) D_y d_{yy} \tan 2\theta. \qquad (20)$$

The difference of these two signals yields

$$z_y^{BPFD} - z_x^{BPFD} = 2SF_x J_2(\varphi_0)\Omega + 2J_2(\varphi_0) D_y \tan 2\theta (d_{yy} - d_{xy}). \qquad (21)$$

After combination with the second equation of (16) the following equation can be obtained

$$z_y^{LPF} - z_y^{BPFD} - z_x^{BPFD} = \left[SF_y + 2SF_x J_2(\varphi_0)\right]\Omega + d_{xy}\left[D_x \cot 2\theta - 2D_y J_2(\varphi_0)\tan 2\theta\right] + D_y d_{yy}\left[J_0(\varphi_0) - 2J_2(\varphi_0)\tan 2\theta\right]. \qquad (22)$$

As in the previous case let's find angle θ and φ_0 to null the two last summands of (22) which give the following two equations

$$D_x \cot 2\theta - 2D_y J_2(\varphi_0)\tan 2\theta = 0;$$
$$J_0(\varphi_0) - 2J_2(\varphi_0)\tan 2\theta = 0.$$

So, taking into account (12) it can be obtained

$$\frac{J_0^2(\varphi_{00})}{2J_2(\varphi_{00})} = \frac{SF_y^{\pi/8}}{SF_x^{\pi/8}};$$
$$\theta_{00} = \frac{1}{2}\tan^{-1}\left[\frac{1}{J_2(\varphi_{00})}\frac{SF_y^{\pi/8}}{SF_x^{\pi/8}}\right].$$

For the testing gyro the following values can be obtained: φ_{00}=76.748 deg θ_{00}=39.156 deg.

VI. Conclusion

Some on-line algorithms to compensate for rate CVG bias have been proposed. To implement these algorithms into practice one should be realized the possibility to modulate phase difference between oscillations along X and Y drive axes and to align standing wave angle θ at the prescribed value. It has experimentally been shown that the prescribed wave angle θ that is able to null bias is almost temperature insensitive, so it can be aligned by the manufacturer and will not be changed during operation.

For more accurate applications wave angle θ should be calibrated versus temperature in order to change it during CVG operation in accordance with temperature change.

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В. В. Чіковані, Г. В. Цірук. Компенсація зміщення нуля в диференціальному коріолісовому вібраційному гіроскопі

Представлено різні варіанти алгоритмів компенсації зміщення нуля коріолісового вібраційного гіроскопа, які можуть застосовуватися для компенсації початкового зміщення нуля гіроскопа та його дрейфу. Також представлено експериментальні результати впливу температури на ключові параметри диференціального коріолісового вібраційного гіроскопа.

Ключові слова: диференціальний коріолісовий вібраційний гіроскоп; зміщення нуля; кут стоячої хвилі; різниця фаз.

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В. В. Чиковани, А. В. Цирук. Компенсация смещения нуля в дифференциальном кориолисовом вибрационном гироскопе

Представлены различные варианты алгоритмов компенсации смещения нуля кориолисового вибрационного гироскопа, которые могут применяться для компенсации начального смещения нуля гироскопа и его дрейфа. Также, представлены экспериментальные результаты по влиянию температуры на ключевые параметры дифференциального кориолисового вибрационного гироскопа.

Ключевые слова: дифференциальный кориолисовый вибрационный гироскоп; смещение нуля; угол стоячей волны; разность фаз.

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