UDC 629.733.33-519:629.7.015.3(045)

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STRUCTURAL-PARAMETRIC SYNTHESIS OF THE FLIGHT CONTROL SYSTEM WITH THE PREVIOUS DESIGN OF UNMANNED AERIAL VEHICLE MODEL

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Abstract. *The paper is devoted to the structural – parametrical design of the control systems for unmanned aerial vehicles. The structural – parametrical design of control systems was carried out with use of the separation theorem. The optimum law of control with use of Kalman filter and the optimum deterministic regulator was synthesized. The technique of simplification of obtained solution for the purpose of a possibility of its realization in the simple onboard computer is performed. Aforementioned synthesis procedure was applied for design of unmanned aerial vehicles autopilots with limited amount of the navigation sensors in presence of stochastic disturbances. The aerodynamic coefficients that compose the stat space matrices and used for synthesis are obtained via DATCOM software application.*

Keywords: unmanned aerial vehicle, aerodynamic coefficients, DATCOM, longitudinal motion, structural-parametrical synthesis, separation theorem, Kalman filter, the optimal determined regulator, simplification of obtained solution, flight control system, autopilot.

Introduction

Contemporary science develops in the direction of technological miniaturization as the finding in such an aspect gives the possibility to solve new kinds of tasks that were unsolvable without miniaturized means. Another aim of technological development – is automation of tasks fulfilment. Both these branches of scientific progress are successfully implemented in the field of aircrafts design in general, and – the unmanned aerial vehicles (UAV) design in particular. New technological achievements make possible involving unmanned aviation to the solution of tasks of higher and higher complexity. The growing complicacy of problems to solve requires steep growing of automated control systems accuracy. Thus the characteristics of robustness and performance come to the front of scientific investigations, as the most reliable way to acquire scientifically proved optimal control system in the sense of its effectiveness [1], [2].

Modern autopilots use computer software to control the aircraft. The software reads the aircraft's current position, and controls a Flight Control System to guide the aircraft. In such a system, besides classic flight controls, many autopilots incorporate thrust control capabilities that can control throttles to optimize the air-speed, and move fuel to different tanks to balance the aircraft in an optimal attitude in the air. Although autopilots handle new or dangerous situations inflexibly, they generally fly an aircraft with a lower fuel-consumption than a human pilot.

In order to provide competitive ability and efficiency of autopilots use in small UAVs, the number of fitted affordable sensors is decreasing, in this case not all needed values can be measured. And the measured ones might include noises. It complicates synthesis of effective control system, and use of standard rules becomes impossible.

Taking into account all listed above, and trying to avoid use of slow and expensive adaptive systems, the necessities of robust control systems synthesis are inevitable. They allow keeping stability and controlling quality in permissible limits, on object parameters change in wide range.

In order to provide competitive ability and efficiency of autopilots use in small aircraft, the number of fitted affordable sensors is decreasing, in this case not all needed values can be measured. And the measured ones might include noises. This can be decided with the use of structural parametric synthesis, which allows keeping stability and control quality in permissible limits, on object parameters change in wide range.

Determining vehicle geometry

Aerodynamic stability and control coefficients and derivatives are a set of numbers that are used in aircraft equations of motion for calculating aerodynamic forces and moments acting on the aircraft. These numbers are determined by aircraft geometry and directly affect aircraft performance. Several methods exist for determining an aircraft's aerodynamic characteristics. The most accurate method is to measure aerodynamic forces and moments acting on the aircraft during flight testing. This method is very expensive as it requires building a flight-worthy prototype and flying it. Another method is wind tunnel testing. This method is also expensive as it requires building a wind tunnel model of an aircraft and running a wind tunnel to measure aerodynamic forces and moments acting on the model. Both wind tunnel testing and flight testing must be run at some point in the development process.

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To be able to determine reasonable aircraft geometry prior to wind tunnel testing and flight testing, methods for analytical prediction of aerodynamic characteristics can be used [3]. Several analytical prediction methods exist. One of better known analytical methods is Digital DATCOM software developed by U.S. Air Force. A detailed description of the information that needs to be specified in Digital DATCOM input files can be found in Digital DATCOM manual. Once the input file is created, Digital DATCOM can be run to produce the output file containing estimated stability and control coefficients and derivatives for the specified aircraft geometry and flight conditions. Also it creates 3D model of the research object.

The output file received for Global Hawk Unmanned Aerial Vehicle is composed of 8 similar sections, each of them containing the values of the same stability and control coefficients and derivatives for each of the specified 8 flight conditions. After estimates of stability and control coefficients and derivatives are obtained, needed section was find and take numbers for analysis and for further use in modelling flight vehicle dynamics and control system design. In our case it's section with next parameters: altitude 70.000 ft, velocity 388.32 ft/s, Mach number 0.4. The best aerodynamic coefficients are produced during such flight conditions, where angle of attack is ideal. As for Global Hawk it's equal to 6 degrees.

DATCOM outputs a significant amount of useful data. Not all will be used or presented here. The major components of the coefficients of Lift, Drag, and Moment are listed in table 1.

Table 1

Component Lift, Drag, and Moment Coefficients

Coefficient	Value
C_D	0.031
C_L	0.763
C_M	0.0187

Two methods exist to determine inertia. The first method uses mathematical calculations based on physical measurements to predict inertia values. The second method also uses mathematical calculations based on both physical measurements and experimental data. First method was chosen since it doesn't require using the actual flight test vehicle and less time consuming [4].

The longitudinal states and controls are [3]–[5]

$$\mathbf{x}^T = [V_t \quad \alpha \quad \theta \quad q], \qquad \mathbf{u}^T = [\delta_{th} \quad \delta_e],$$

The longitudinal coefficient matrices are given by:

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & V_T - Z_{\dot\alpha} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -M_{\dot\alpha} & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} X_{\delta_{th}} \cos\alpha_e & X_{\delta_e} \\ -X_{\delta_{th}} \sin\alpha_e & Z_{\delta_e} \\ 0 & 0 \\ M_{\delta_{th}} & M_{\delta_e} \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} X_V + X_{T_V} \cos\alpha_e & X_\alpha & -g_0 \cos Y_e & 0 \\ Z_V + Z_{T_V} \sin\alpha_e & Z_\varepsilon & -g_0 \sin Y_e & V_T + Z_q \\ 0 & 0 & 0 & 1 \\ M_v + M_{T_V} & M_\alpha & 0 & M_q \end{bmatrix}.$$

Where primed moment derivatives have been defined, according to the common convention, by

$$L'_\beta = \mu L_\beta + \sigma_1 N_\beta,\ L'_p = \mu L_p + \sigma_1 N_p,$$
$$L'_r = \mu L_r + \sigma_1 N_r,\ N'_\beta = \mu N_\beta + \sigma_2 L_\beta,$$
$$N'_p = \mu N_p + \sigma_2 L_p,\ N'_r = \mu N_r + \sigma_2 L_r,$$
$$L'_{\delta a} = \mu L_{\delta a} + \sigma_1 N_{\delta a},\ L'_{\delta r} = \mu L_{\delta r} + \sigma_1 N_{\delta r},$$
$$N'_{\delta a} = \mu N_{\delta a} + \sigma_2 L_{\delta a},\ N'_{\delta r} = \mu N_{\delta r} + \sigma_2 L_{\delta r}.$$

The **E** matrices for the two sets of equations are nonsingular (for other than hovering flight) and easily invertible in both cases. Therefore, although the original nonlinear equations were assumed implicit, the linear equations can be made explicit. The coefficient matrices depend on the steady-stare angle of attack and pitch attitude in both cases. Although they nominally apply to small perturbations about a wings-level steady-state flight condition, the equations can be used satisfactorily for perturbed bank angles of several degrees.

The linear equations were derived in wind axes in order to obtain the beta-dot state derivative, and then beta was set to zero. Because the steady-state value of beta is zero, stability-axes aerodynamic derivatives can be used in the equations. Also, the moments of inertia used in the equations are the stability-axes values. The subscript *W* on the rates *P*, *Q*, and *R* could equally well be replaced by a subscript *S*. In the definitions of stability derivatives in the remainder of this chapter, the subscripts will be dropped altogether because the stability derivatives are normally specified in the stability axes.

The resulting "dimensionless derivatives" have the advantage that they are less dependent on the specific aircraft and flight condition, and more dependent on the geometrical configuration of an aircraft. Methods have been developed lo estimate the dimensionless derivatives and they can be used to compare and assess different design configurations.

The longitudinal-directional dimensionless stability and control derivatives correspondingly are given in the table 2 [4].

Table 2

Longitudinal dimensional versus dimensionless derivatives

$X_V = -\frac{\bar{q}S}{mV_T}(2C_D + C_{D_V})$	$C_{D_V} \equiv V_T \frac{\partial C_D}{\partial V_T}$	$Z_{\delta e} = -\frac{\bar{q}S}{m} C_{L_{\delta e}}$	$C_{L_{\delta_e}} \equiv \frac{\partial C_L}{\partial el}$
$X_\alpha = \frac{\bar{q}S}{m}(C_L - C_{D_\alpha})$	$C_{D_\alpha} \equiv \frac{\partial C_D}{\partial \alpha}$	$M_V = \frac{\bar{q}S\bar{c}}{J_Y V_T}(2C_M + C_{m_V})$	$C_{m_V} \equiv V_T \frac{\partial C_M}{\partial V_T}$
$X_{\delta e} = -\frac{\bar{q}S}{m} C_{D_{\delta e}}$	$C_{D_{\delta_e}} \equiv \frac{\partial C_D}{\partial el}$	$M_\alpha = \frac{\bar{q}S\bar{c}}{J_Y} C_{m_\alpha}$	$C_{m_\alpha} \equiv \frac{\partial C_M}{\partial \alpha}$
$Z_V = -\frac{\bar{q}S}{mV_T}(2C_L + C_{L_V})$	$C_{L_V} \equiv V_T \frac{\partial C_L}{\partial V_T}$	$M_{\dot\alpha} = \frac{\bar{q}S\bar{c}}{J_Y}\frac{\bar{c}}{2V_T} C_{m_{\dot\alpha}}$	$C_{m_{\dot\alpha}} \equiv \frac{2V_T}{\bar{c}}\frac{\partial C_M}{\partial \dot\alpha}$
$Z_\alpha = -\frac{\bar{q}S}{m}(C_D + C_{L_\alpha})$	$C_{L_\alpha} \equiv \frac{\partial C_L}{\partial \alpha}$	$M_q = \frac{\bar{q}S\bar{c}}{J_Y}\frac{\bar{c}}{2V_T} C_{m_q}$	$C_{m_q} \equiv \frac{2V_T}{\bar{c}}\frac{\partial C_M}{\partial Q}$
$Z_{\dot\alpha} = -\frac{\bar{q}S\bar{c}}{2mV_T} C_{L_{\dot\alpha}}$	$C_{L_{\dot\alpha}} \equiv \frac{2V_T}{\bar{c}}\frac{\partial C_L}{\partial \dot\alpha}$	$M_{\delta e} = \frac{\bar{q}S\bar{c}}{J_Y} C_{m_{\delta e}}$	$C_{m_{\delta_e}} \equiv \frac{\partial C_M}{\partial el}$
$Z_q = -\frac{\bar{q}S\bar{c}}{2mV_T} C_{L_q}$	$C_{L_q} \equiv \frac{2V_T}{\bar{c}}\frac{\partial C_L}{\partial Q}$	–	–

Then we have all need information we can use the DATCOM environment to acquire the matrices for longitudinal motion. The received model are described below.

Structural-parametric synthesis of control system using Kalman estimator

The methodology of structural - parametrical design of robust control systems with use of the separation theorem consists of two stages: synthesis of the optimum law of control with use of Kalman filter and the optimum deterministic regulator at the first stage; robust optimization of the received optimal control law at the second stage was elaborated. The technique of simplification of obtained solution for the purpose of a possibility of its realization in the simple onboard computer is offered [5], [6].

Thus for the synthesis of control system using Kalman filter it is necessary to define four matrices in state-space description of plant. It is also necessary to have characteristics of sensors noises and stochastic disturbances, which are acting on the plant.

The condition of procedure of Kalman filter synthesis use is the white noises influencing the plant. Turbulence of the atmosphere – is the colored noise. Therefore the peculiarity of the plant state space description is a necessity of forming filter (Dryden filter) including in its structure, which input is being disturbed by the white noise, and on an output we have the colored noise which characterizes turbulence of atmosphere. The model of forming filter is standardized in American practice [5], [6]. Thus the inputs of the extended plant in state-space will be disturbed by the white noise that is corresponds to the terms of the plant description for the synthesis of Kalman filter. And the colored noise will act directly on plant.

The model's true airspeed is 650 km/h. State space matrix **A** and the control **B** for linear motion with input control **u**, state vector **x** and the output vector **y** are look like following:

V = 180.5556 m/s (650 km/h).

$$\mathbf{A}_0 = \begin{bmatrix} -0.0160 & 0.0168 & 0.6797 & -32.17 & 0 \\ -0.1364 & -1.619 & 448.5 & 0.0558 & 0 \\ 0 & -0.0242 & -0.7189 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -\mathrm{Vt} & \mathrm{Vt} & 0 & 0 \end{bmatrix};$$

$$\mathbf{B}_0 = \begin{bmatrix} -3.188 & 0.0014 \\ -45.34 & 0 \\ 0 & 0 \\ -22.68 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_{g0} = \begin{bmatrix} -0.0160 & 0.0168 & -32.17 \\ -0.1364 & -1.619 & 0.0558 \\ 0 & -0.0242 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\mathbf{C}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{D}_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

The following variables are used as the state **x**: Vt is velocity; α is angle of attack; θ is pitch angle; q is pitch rate; h is height. Researching the stochastic case we have assumed that there is side turbulent wind acting on UAV. Winds root-mean-square deviation of the instantaneous velocity is equal to 2.5 m/s. Also it is necessary to designate the model of the elevator and throttle. For the given object the model of the elevator is described by state-space matrices:

$$\begin{bmatrix} \mathbf{A}_{el} & \mathbf{B}_{el} \\ \mathbf{C}_{el} & \mathbf{D}_{el} \end{bmatrix} = \begin{bmatrix} -1/T_{el} & n_{el}/T_{el} \\ 1 & 0 \end{bmatrix}.$$

where $T_{el} = 0.5$ is time constant of the aileron; $n_{el} = 1$ is elevator's amplification coefficient. The model of the throttle is described by the same state-space matrices.

Synthesis is performed for the series connection of the mechanism and the plant.

Except of the system that is described in state space, for the synthesis of the Kalman filter it is necessary to set the meaning of the noises covariance matrices $\mathbf{V}_1$, that is acting on the state of the system, and $\mathbf{V}_2$ noises of the observations. For our case they have following meaning:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{V}_2 = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 25 \end{bmatrix}.$$

The result of the synthesis is the optimal observer of the 11 th order.

After system state fully restored, procedure of optimal deterministic controller synthesis can be used. The matrices of the weight coefficients $\mathbf{R}_1$, $\mathbf{R}_2$, that is considering the influence of the state space and the control for our are following:

$$\mathbf{R}_1 = [1\ 10\ 1\ 10\ 0.01\ 1\ 100\ 10\ 0.01\ 0.01\ 0.01];$$

$$\mathbf{R}_1 = \text{diag}(\mathbf{R}_1); \mathbf{R}_2 = \text{eye}(2).$$

The optimal control law of 11th order is obtained as a result. Realizing such control law on a simple onboard computer is rather difficult. Therefore the order of the obtained optimal controller needs to be decreased. The decrease of the order is executed by means of balance models [5]. As a result an equivalent realization was obtained. For obtained model the controllability and observability gramians are equal and their diagonal entries forming the vector M. Small entries in M indicate states that can be removed to simplify the model. The order of the synthesized system was reduced to four. Optimal controller of the 4 th order is described by the matrices:

$$\mathbf{A}_r = \begin{bmatrix} 0.6357 & -0.4352 & -0.02171 & 0.01373 \\ 0.4352 & 0.08823 & -0.006917 & 0.006649 \\ 0.0217 & -0.006902 & 0.9984 & 0.03149 \\ 0.01383 & -0.006682 & -0.03148 & 0.9981 \end{bmatrix};$$

$$\mathbf{B}_r = \begin{bmatrix} -0.06273 & 0.0004517 & -0.04623 \\ -0.01082 & -0.0001469 & -0.008212 \\ 0.001378 & 0.0002299 & 0.001067 \\ 0.002292 & -0.0002021 & 0.001838 \end{bmatrix};$$

$$\mathbf{C}_r = \begin{bmatrix} 0.07793 & -0.0135 & 0.0017 & -0.002 \\ 0.00013 & -4.3\text{e-}005 & 0.0002 & -0.0003 \end{bmatrix};$$

$$\mathbf{D}_r = \begin{bmatrix} -0.001431 & -3.574\text{e-}005 & -0.001118 \\ 1.211\text{e-}005 & 1.938\text{e-}006 & 1.053\text{e-}005 \end{bmatrix}.$$

Performance indexes and standard deviations of our system are shown in Table 3, 4 respectively.

Table 3

Maximum standard deviation of the UAV outputs in a stochastic case

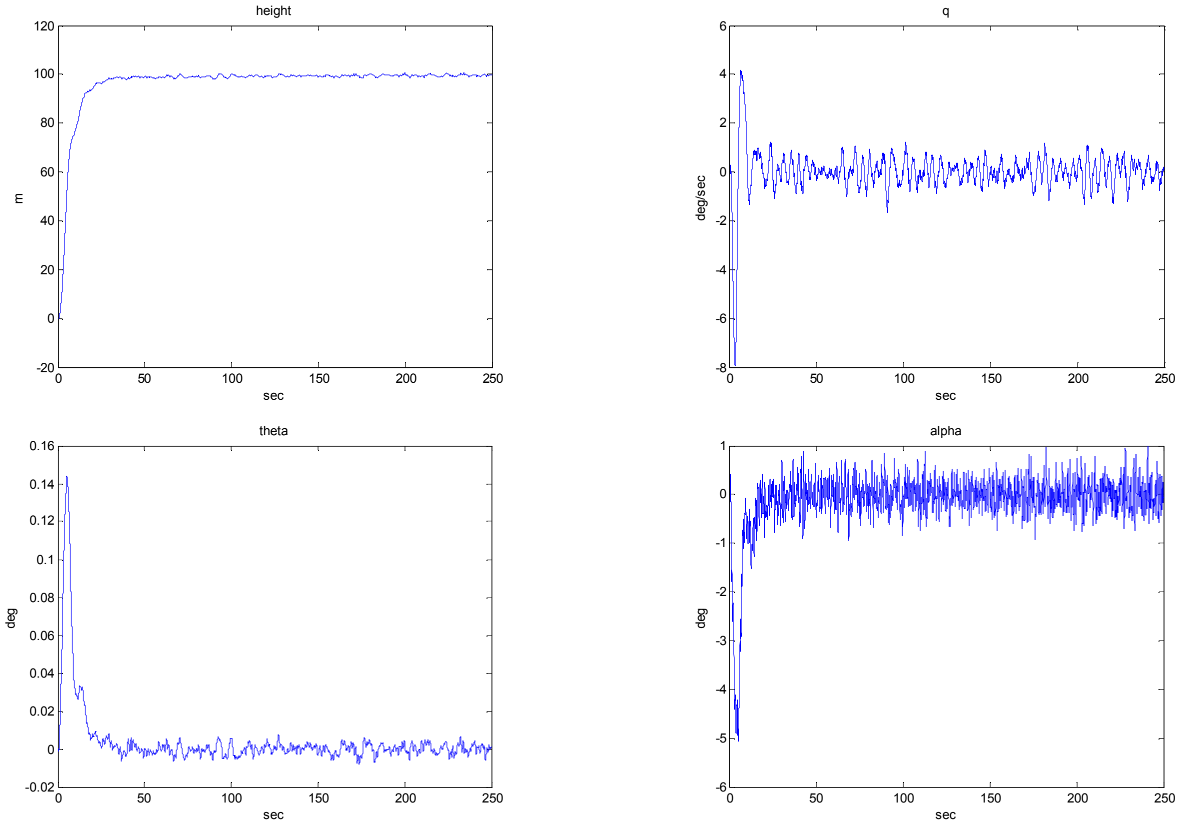
Standard deviation in a stochastic case						
σ_v, m/sec	σ_α, deg	σ_ϑ, deg	σ_q, deg/sec	σ_h, m	σ_{el}, deg	σ_{th}, deg
6.7260	0.0657	0.0013	0.2010	0.3873	0.0115	0.0231

Table 4

The performances criteria indexes

H_2 stoch. case	H_2 det. case	H_∞
0.1638	0.0685	1.4871

The simulation was executed taking into account all nonlinear functions that correspond to the real autopilot, under turbulent wind ($\sigma_w = 2.5$ m/s) and the results are shown on Figure. From the Figure we can make a conclusion that received characteristics are satisfy the requirements for the flight.



Transient processes of system: height is height of the aircraft; *q* is pitch rate; theta is pitch angle; alpha is angle of attack

Conclusions

The aerodynamic coefficients, moments of inertia and then state-space matrices are obtained via DATCOM software application. These parameters are initial data for structural-parametric synthesis of control low. The optimal discrete control system for the longitudinal channel of the UAV is synthesized and order lowering of the obtained control laws is carried out. With the purpose of verification of the obtained results, the design of the closed loop system for longitudinal channel control of the UAV is executed taking into account nonlinearity, appropriate to the real systems including models of the concordance mechanisms and other systems, which are wasn't taken into account of synthesis procedure.

The obtained results satisfy the requirements but the robust optimization is recommended as the next step of design.

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Received 23 October 2013

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Т. А. Галагуз. Структурно-параметричний синтез системи управління з попереднім розрахунком моделі безпілотного літального апарату

Розглянуто структурно-параметричний синтез системи управління безпілотного літального апарату. Синтез виконувався з використанням теореми розділення. Було синтезовано оптимальний стохастичний фільтр Калмана та оптимальний детермінований регулятор. Виконано пониження порядку отриманого закону управління для можливості подальшої реалізації на бортовому комп'ютері. З використанням описаної процедури синтезовано автопілот повздовжнього каналу безпілотного літального апарату з обмеженою кількістю навігаційних датчиків та за наявності випадкових збурень. Аерокосмічні коефіцієнти, необхідні для опису об'єкта у просторі станів отримано з використанням програми DATCOM.

Ключові слова: безпілотний літальний апарат; аеродинамічні коефіцієнти; DATCOM; повздовжній рух; структурно-параметричний синтез; теорема розділення; фільтр Калмана; оптимальний детермінований регулятор; пониження порядку; автопілот.

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Т. А. Галагуз. Структурно-параметрический синтез системы управления с предварительным расчетом модели беспилотного летательного аппарата

Рассмотрен структурно-параметрический синтез системы управления беспилотным летательным аппаратом. Синтез осуществлялся с использованием теоремы разделения. Было синтезировано оптимальный стохастический фильтр Калмана и оптимальный детерминированный регулятор. Выполнено понижение порядка оптимального закона управления для обеспечения возможности реализации на бортовом компьютере в дальнейшем. С использованием вышеописанной процедуры синтезирован автопилот продольного канала беспилотного летательного аппарата с ограниченным количеством навигационных датчиков и в условиях действия случайных возмущений. Аэрокосмические коэффициенты получены с использованием программы DATCOM.

Ключевые слова: беспилотный летательный аппарат; аэродинамические коэффициенты; DATCOM; продольное движение; структурно-параметрический синтез; теорема разделения; фильтр Калмана; оптимальный детерминированный регулятор; понижение порядка; автопилот.

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