UDC 629.735.051.025.36 – 519(045)

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THE SYNTHESIS OF THE ROBUST FLIGHT CONTROL WITH SIMPLIFIED STRUCTURE FOR SMALL UAV UNDER ACTION OF TURBULENT WIND

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The article represents results of H_∞ – optimization of one-dimensional flight control system on the example of longitudinal flight stabilization system for small UAV using only one sensor – altimeter. Results of controlled flight dynamics modeling with taking into account the action of noise in altimeter and stochastic disturbances (modeled by means of Dryden filter) are given. The possibility and reasonability of practical implementation of considered question is shown and justified.

Introduction. It is evident that fully autonomous and cheap UAV of new generation performing take-off from catapult or aircraft-carrier – is the privileged direction for all diverse applications of UAV in Ukraine.

Thus it is natural that the design cost of UAV should be as low as possible. At the same time creation of automatic control system with complete state vector measurement for the small UAV implies usage of expensive inertial system (including inertial sensors: micromechanical accelerometers and gyroscopes) along with air data system (ADS).

This article exposes quasioptimal flight control system synthesis for small UAV (longitudinal channel) that uses measurements from only one ADS sensor, notably, barometric altimeter, interconnecting obtained data with information from GPS receiver. Such approach essentially simplifies flight control system, decreases the cost of UAV and increases reliability.

Problem formulation. The problem is the creation of a quasioptimal controller that the closed-loop system «controller-plant» will possess satisfactory dynamic characteristics from the viewpoint of practical realization. With the purpose of approaching the modeling situation to real conditions a measurement noise has to act at the altitude channel. In addition to this the action of the turbulent wind on the longitudinal motion of flying vehicle should be taken into account.

The model of UAV under consideration is given by the quadruple of matrices in the state space form [1; 2]. Aircraft is flying with true airspeed equal to 250 km/h. Its wingspan is 2 meters, standard quadratic deviation of noise in altimeter is set to be 2 meters, the control task is to climb on 100 meters.

It easy to mention that matrix A has zero-column and consequently it has one zero eigenvalue. The object of control is serially connected to an actuator. Actuator is represented by the inertial element of the 1-st order with the time constant equal to 0,5 seconds.

The stated above task is interesting in the essence of onboard equipment quantity minimization, and as the result – achieving the reduction of UAV cost, increasing its commercial load facility, increasing its reliability by means of simplification of the system. Usually, the stabilization of longitudinal motion in flight control is performed with the help of three, or at least – two sensors. In addition to altitude the pitch angle and (or) pitch rate are measured. For the task of stabilization the aircraft longitudinal motion, while measuring above mentioned dynamic characteristics, algorithms are well-known [3 – 5]. But there is the question: is it possible to control the longitudinal motion of an aircraft measuring only its true altitude? In order to receive the answer on this question we would apply the H_∞ – synthesis approach.

Quasioptimal controller. Solution of any robust problem lies in finding out the compromise between the robustness and the performance of a system. Graphically robustness could be represented by the sensitivity function. Performance could be imagined as the complementary sensitivity function. Figure 1 shows possible view of aforementioned functions [6].

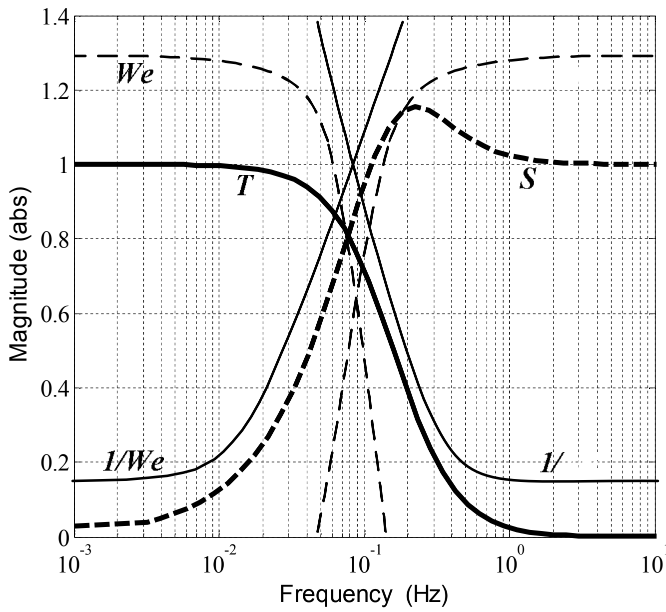

Fig. 1. Sensitivity (S) and complementary sensitivity (T) functions

These functions could be given by the following equations:

$$S = (I + PC)^{-1}, \tag{1}$$

$$T = PC(I + PC)^{-1}. \tag{2}$$

For sensitivity and complementary sensitivity functions it is true that $T + S = 1$, and it is clearly visible on the fig. 1 [7].

In our case also weight transfer function for sensitivity of control, that is given by the equality (3), was used:

$$KS = C(I + PC)^{-1}. \tag{3}$$

To achieve some concrete degree of robustness and performance in a system, so-called weight transfer functions could be applied. Such an approach is the extension of H_∞ – optimization method. Weight transfer functions play role of filter, filtering out investigated signals to obtain desired dynamic characteristics of the resultant closed-loop "controller-plant" system after the optimization procedure. Figure 1 also reveals weight transfer functions view for the considered above example. W_e and W_p are weight transfer functions for sensitivity and complementary sensitivity functions respectively. On fig.1 we can see, that their inverse values form desired shapes of S and T. That is how it works.

In accordance with H_∞ – optimization approach H_∞ norms of S, KS and T should be less than some predetermined value γ [2; 6], which would influence on performance and robustness indexes of closed loop system. Thus we can state the following inequality:

$$\left\| \begin{matrix} S \\ KS \\ T \end{matrix} \right\|_\infty \leq \gamma. \tag{4}$$

This method is known as the staked sensitivity problem [7]. After including weight transfer functions to (4) and taking $\gamma = 1$ we will obtain:

$$\left\| \begin{matrix} W_e S \\ W_u KS \\ W_p T \end{matrix} \right\|_\infty \le 1. \quad (5)$$

The inequality (5) is one of the main rules that should be fulfilled while searching for the satisfying suboptimal controller. For our case the following weight transfer functions were taken:

$$W_e = \frac{0,1s + 0,025}{s + 2,5 \cdot 10^{-5}}; \quad W_u = \frac{166,7s + 11.11}{s + 0,8333}; \quad W_p = \frac{1000s + 4}{s + 40}.$$

Synthesis of suboptimal controller is based on the solution of two Riccati equations [6] (one – is for deterministic controller and another – is for stochastic observer) having the following structure:

$$A'X + XA + XRX - Q = 0$$

where X is the searched solution, A, Q and R – are the real $n \times n$ matrices, Q and R – are symmetric. These matrices define the $2n \times 2n$ Hamiltonian matrix of following view:

$$H = \begin{bmatrix} A & R \\ Q & -A' \end{bmatrix}.$$

The H_∞ – controllers that are under synthesis then could be mathematically represented as follows:

$$K = \begin{bmatrix} \hat{A}_\infty & -M_\infty N_\infty \\ L_\infty & 0 \end{bmatrix}$$

where

$$\hat{A}_\infty = A + \gamma^{-2} B_{r1} B'_{r1} X_\infty + B_{r2} L_\infty + M_\infty N_\infty C_2; \quad (6)$$

$$L_\infty = -B'_{r2} X_\infty, \; N_\infty = -Y_\infty C'_2 \text{ and } M_\infty = (I - \gamma^{-2} Y_\infty X_\infty)^{-1}. \quad (7)$$

In equations (6) and (7) X_∞, Y_∞ are solutions for each of two algebraic Riccati equations that are to be solved. Let H_∞ be a Hamiltonian function for Riccati equation with solution X_∞ and J_∞ – is that for the equation with Y_∞ solution. $\begin{bmatrix} A & B_1 \\ C_1 & D_{12} \end{bmatrix}$ and $\begin{bmatrix} A & B_2 \\ C_2 & D_{21} \end{bmatrix}$ – are quadruples of matrices for deterministic controller and stochastic observer.

Then for requirement (5) necessary and sufficient conditions for existence of an admissible controller are following:

1) H_∞ belongs to Riccati domain and X_∞ – is the solution of Riccati equation for matrices of H_∞; $X_\infty \ge 0$;

2) J_∞ belongs to Riccati domain and Y_∞ – is the solution of Riccati equation for matrices of J_∞; $Y_\infty \ge 0$;

3) $\rho(X_\infty Y_\infty) < \gamma^2$;

4) (A, B_2) should be stabilizable and (C_2, A) detectable;

5) D_{12} and D_{21} should have full rank;

6) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ should have full column rank for all $\omega \in \mathbf{R}$;

7) $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ should have full raw rank for all $\omega \in \mathbf{R}$.

From the practical point of view the most appropriate way for finding controller which would suit the aforementioned requirements, and therefore quasioptimal one in H_∞ – optimization domain we can use **hinfsyne.m** operator. It has the following structure [8]:

[C, CP, gfin]=hinfsyne(P, nmeas, ncon, gmin, gmax, tol)

where names of variables were assigned for storing matrix of controller «C» in them and also closed loop system of controled object and controller «CP» which was obtained by means of «gfin» – estimation H_∞ – criterion. Variable of received object «P» was obtained for optimization procedure; the number of measured outputs «nmeans» and the number of controlled inputs «ncon» were introduced. «gmin» and «gmax» variables corresponds to the lower and upper limit where the procedure of finding suboptimal solution would be performed. For the longitudinal channel the most appropriate values are 0,1 and 15. «tol» is the operator which is responsible for relative error of optimization process. In our case it was taken to be equal 0,0001.
As the result, the quasioptimal controller was obtained. Figure 2 represents the total aggregate of worked out system. It is essential to mention that the optimization procedure doesn't take into account the existence of turbulent wind, which is added after the controller synthesis to test the influence of such disturbance on the designed flight control system, where V, α, θ, q, h, elevator – are elements of state vector of the system representing vehicle velocity, its attack angle, pitch angle, pitch rate and the angle of elevators deflection correspondingly; n – is the signal of noise in altimeter of standard quadratic deviation (SQD) = 2 meters.

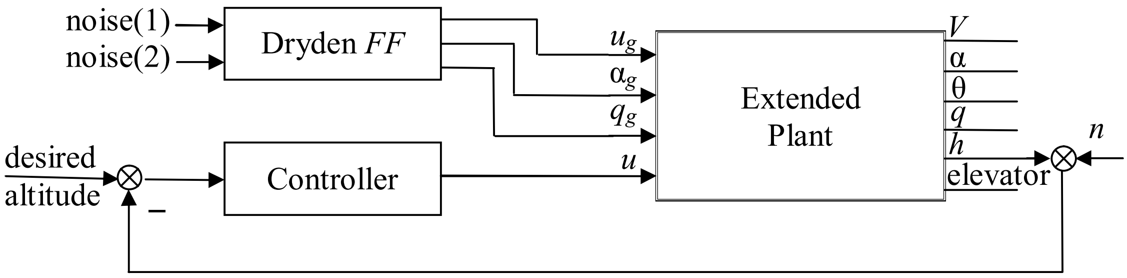

Fig. 2. Flight control system

Dryden filter. On figure 2 extended plant is the system for which quasioptimal controller has been designed, but with increased amount of inputs. Additional inputs were obtained by means of modifying the control matrix. Observation matrix and matrix of transfer were also changed to be corresponding to new system dimensions. u_g, α_g, q_g and u are velocity of turbulent wind longitudinal component, turbulent attack angle, turbulent pitch rate and control signal correspondingly.

The Dryden forming filter (Dryden FF) [9] is also given as the system in state space, to the input of which noises of different power (2 and 1 unit correspondingly) are sent. At the result turbulent signals of type that corresponds to the task statement were obtained. Referencing of received signals was provided by the «Dryden filter» block in SIMULINK environment. At the output of the mentioned block we have longitudinal, lateral and vertical components of the turbulent wind. From the last one it is easy to obtain turbulent attack angle α_g and turbulent pitch rate q_g:

$$\alpha_g = \arctan(w_g / U_0), \; q_g = -\dot{\alpha}_g$$

where w_g is the vertical component of turbulent wind, U_0 is the velocity of flying vehicle.

Atmospheric turbulence SQD: $u_g = 0{,}383$ m/s, $\alpha_g = 0{,}2282$ deg, $q_g = 1{,}0542$ deg/sec.

Estimation of obtained results. At the result of modeling the following transient processes were obtained:

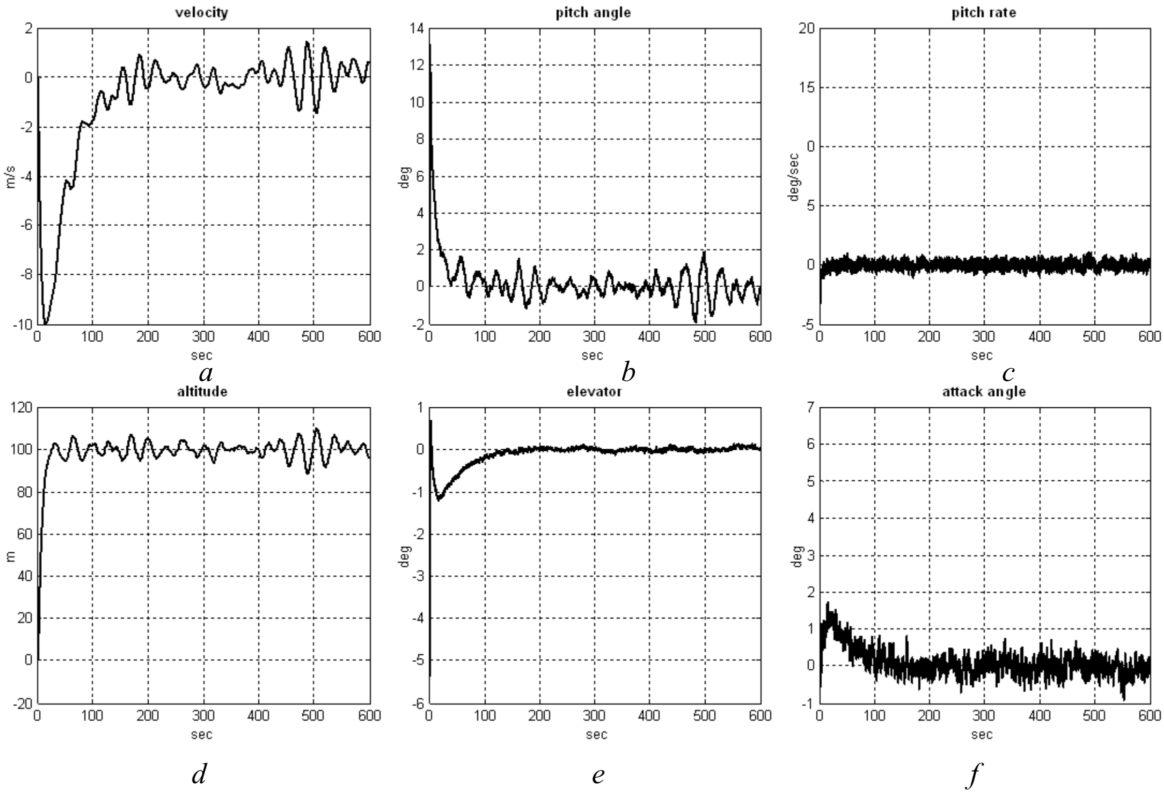

Fig. 3. Finally obtained transient processes of *a* – velocity; *b* – pitch angle; *c* – pitch rate; *d* – true altitude; *e* – elevators deflection; *f* – attack angle of the UAV

From the shown above graphs it is evident that results are quite satisfying. Deflections of UAV angular characteristics are possible from the practical point of view and lower than the stall angle. Altitude varies in ± 10 meters boundaries.

Table 1 represents results of SQD calculation for each of investigated signals.

Table 1

V, m/sec	α, deg	θ, deg	q, deg/sec	h, m	*elevator*, deg
0,5290	0,2297	0,5787	0,2920	3,5916	0,0420

It is important to mention that the standard quadratic deviations were calculated for the period from the beginning of the steady state of estimated transient process till the end of simulation. From above given data we can conclude that noisy deviations of useful signals are quite low. Thus the obtained results are very good and satisfactory.

Conclusion. This article covers the method of single-variable flight control system synthesis. Finally robust controller for longitudinal channel of small UAV was synthesized. It is essential to underline that flying vehicle was in the mode of climbing, using only one sensor and influenced by wind disturbances and noise in that sensor.

Theoretical base of proposed method is formulated through the H_∞ – optimization procedure.

Synthesized suboptimal controller realizes the control of small UAV, providing stability of UAV-autopilot system in straight-forward motion under the action of longitudinal turbulent wind and adequate usage of its control surfaces (elevator).

The content of this article – results of mathematical modeling of control processes in SIMULINK environment – substantiates the reasonability of implementation of this approach in practice.

References

1. *A. A. Tunik, Hyeok Ryu and Hae–Chang Lee*. Parametric Optimization Procedure for Robust Flight Control System Design // KSAS International Journal. – Vol. 2. – № 2. – November 2001. – P. 95 –107.
2. *А. А. Тунік, Є. В. Галкін, К. В. Мельник* // H_∞-оптимізація системи керування польотом // Електроніка та системи управління. – 2006. – №2(8). – С. 107–118.
3. *Ю. П. Гуськов, Г. И. Загайнов.* Управление полетом самолётов / Под ред. Г. В. Александрова. – М.: Машиностроение, 1980. – 213 с.
4. *Ю. В. Байбородин, В. В. Драбкин.* Бортовые системы управления полетом. – М.: Транспорт, 1975. – 336 с.
5. *В. А. Павлов, С. А. Понырко, Ю. М. Хованский* Стабилизация летательных аппаратов и автопилоты. – М.: Высш. шк., 1964. – 484 с.
6. *Kemin Zhou.* Essentials of Robust Control. Prentice Hall, May 1999. – 411 p.
7. *H. Kwakernaak.* Robust Control and H_∞-Optimization. –Tutorial Paper. Automatica. – Vol.29, – №2. – 1993. – P. 255 – 273.
8. http://www.mathworks.com/
9. *Marc O. Rauw //* The Flight Dynamics and Control Toolbox // oct 11, 2000. – 263 p.

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Синтез робастного управления полётом с упрощённой структурой для малого беспилотного летательного аппарата при действии турбулентного ветра

Представлены результаты H_∞-оптимизации одномерной системы управления полётом на примере системы стабилизации продольного движения малого беспилотного летательного аппарата с использованием всего лишь одного датчика измерения – высотомера. Приведены результаты моделирования динамики управляемого движения при наличии шумов в датчике высоты и стохастических возмущений (моделируемых фильтром Драйдена). Показано и обосновано полную состоятельность практического применения и реализации поднятого вопроса.

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Синтез робастного керування польотом із спрощеною структурою для малого безпілотного літального апарата у разі дії турбулентного вітра

Подано результати H_∞-оптимізації одновимірної системи керування польотом на прикладі системи стабілізації поздовжнього руху малого безпілотного літального апарата з використанням лише одного датчика вимірювання – висотоміра. Наведено результати моделювання динаміки керованого руху за наявності шумів у датчику висоти та дії стохастичних збурень (модельованих фільтром Драйдена). Показано й обґрунтовано повну спроможність практичного застосування і реалізації порушеного питання.