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FLIGHT CONTROL SYSTEM DESIGN VIA LMI-APPROACH

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Abstract. *A contemporary approach of flight control system design via static output feedback design is proposed. The static output feedback is formulated in terms of linear matrix inequalities. The obtained solution guarantees stabilization of unmanned aerial vehicle during flight mission. During flight envelope the unmanned aerial vehicle is subjected to the external stochastic disturbances. The efficiency of the proposed approach is illustrated by a case study of unmanned aerial vehicle longitudinal motion.*

Keywords*:* aircraft control; disturbance attenuation; external disturbances; linear matrix inequality; robustness; static output feedback; unmanned aerial vehicle.

Introduction

During the last years, the problem of robust controller design has attracted considerable attention from the automatic control society, especially in the area of unmanned aerial vehicles (UAVs) [1]. The wide UAV application is explained by the fact that such vehicles are able to perform various tasks. Preferably, they are used in dangerous and inaccessible regions to avoid physical injuries in case of manned vehicles usage. These conditions lead to the robust flight control system design which possesses with ability to meet the contradictory requirements imposed on the UAV during the flight.

Furthermore, one should care about various problems connected with law cost design and power consumption in order to be implemented onboard computer with restricted abilities. In turn, it leads to the limited number of navigation sensors, their size and weight. These circumstances lead to the problem of a static output feedback (SOF) controller design. The main advantage of SOF design is that it requires only available signals from the plant to be controlled. The SOF problem concerns finding a static or feedback gain to achieve certain desired closed-loop characteristics. It is necessary to admit that the output feedback problem is much more difficult to solve in comparison to state feedback control problem. Nevertheless, the obtained control law is simpler and easier to be realized. Moreover, in contrast to the observer based controllers, SOF controller does not need to solve differential equations that results in a decreasing of power consumption and computational cost. A survey devoted to this problem is presented in [2].

This paper deals with the static output feedback (SOF) controller design in terms of linear matrix inequalities (LMIs) [3] – [7] for UAV control during flight envelope. The main feature of this paper is that the obtained SOF controller stabilizes the set of autonomous systems, simultaneously. To prove the efficiency of the proposed technique, the longitudinal motion of the aircraft control is considered as a case study.

Problem statement

Consider a linear time invariant system described by the following differential equation

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B}_{\upsilon}\upsilon(t); \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \end{cases}$$

where $\mathbf{x} \in \mathbf{R}^{\mathbf{n}}$ is the state space vector, $\mathbf{u} \in \mathbf{R}^{\mathbf{m}}$ is the control vector, $\mathbf{y} \in \mathbf{R}^{\mathbf{p}}$ is the output vector and $\upsilon \in \mathbf{R}^{\mathbf{n}}$ is a disturbance vector. Besides that, the state space matrices of the controlled plant have the following dimensions $\mathbf{A} \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}$, $\mathbf{B} \in \mathbf{R}^{\mathbf{m}\times\mathbf{n}}$, $\mathbf{C} \in \mathbf{R}^{\mathbf{p}\times\mathbf{n}}$. It could be seen that number of measuring variables p is less than number of all phase coordinates, n. Therefore, the control law is designed taking into account only variables that are available for measurement.
The control law is given by

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{y}(t) = -\mathbf{K}\mathbf{C}\mathbf{x}(t), \qquad (1)$$

where K is a constant output feedback gain, that minimizes performance index:

$$J(\mathbf{K}) = \int_0^{\infty} \|z(t)\|^2 dt$$
$$= \int_0^{\infty} \left(\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\right)dt < \gamma^2 \int_0^{\infty} \upsilon^T(t)\upsilon(t)dt,$$
$$\forall \upsilon(t) \neq 0,$$

where $\mathbf{Q} \geq 0$ and $\mathbf{R} > 0$ are diagonal matrices, weighting each state and control variables, respectively. Output signal $z(t)$ used for performance evaluation is defined as follows:

$$z = \begin{bmatrix} \sqrt{Q} & 0 \\ 0 & \sqrt{R} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}.$$

Bounded L_2 gain design problem

The system L_2 gain is said to be bounded or attenuated by γ if [2] – [5]:

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$$\frac{\int_0^\infty \|z(t)\|^2 dt}{\int_0^\infty \|d(t)\|^2 dt} = \frac{\int_0^\infty \left(x^T Q x + u^T R u\right) dt}{\int_0^\infty \left(d^T d\right) dt} \le \gamma^2 .$$

Therefore, it is necessary to find constant output feedback gain matrix K that stabilizes the control plant such that the infinity norm of the transfer function referring exogenous input to performance output $z(t)$ approaches minimum. The minimum gain is denoted by γ^*.

The output feedback gain matrix K (1) could be found by solving the following iterative LMI

$$\begin{bmatrix} \mathbf{P}_n\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_n + \mathbf{Q} & \mathbf{P}_n\mathbf{B}_i & \mathbf{P}_n\mathbf{B}_{\upsilon_i} & \mathbf{L}_n^T \\ \mathbf{B}_i^T\mathbf{P} & -\mathbf{R} & 0 & 0 \\ \mathbf{B}_{\upsilon_i}^T\mathbf{P} & 0 & -\gamma^2\mathbf{I} & 0 \\ \mathbf{L}_n & 0 & 0 & -\mathbf{R} \end{bmatrix} \le 0, \quad (2)$$

where $i = 1,\ldots,N$ in (2) denotes the set of models associated with certain operating conditions within the flight envelope.

The main advantages of the proposed approach: there is no necessity to define the initial matrix $\mathbf{K}$; an opportunity to find solution to the set of matrices that proves the robustness properties of the system.

The matrices $\mathbf{K}$ and $\mathbf{L}$ for each n th iteration are updated as follows:

$$\mathbf{K}_{n+1} = \mathbf{R}^{-1}\left(\mathbf{B}^T\mathbf{P}_n + \mathbf{L}_n\right)\mathbf{C}^T\left(\mathbf{C}\mathbf{C}^T\right)^{-1};$$

$$\mathbf{L}_{n+1} = \mathbf{R}\mathbf{K}_{n+1}\mathbf{C} - \mathbf{B}^T\mathbf{P}_n .$$

On the last stage a convergence is checked, namely if $\|\mathbf{K}_n - \mathbf{K}_{n+1}\| \le \varepsilon$ (if K_{n+1} and K_n are close enough to each other) than terminate and set $K = K_{n+1}$, otherwise set $n = n+1$ and solve the inequality (2).

Case study

To demonstrate the efficiency of the proposed approach a longitudinal channel of the UAV is used as a case study. The state space vector of the longitudinal channel is $x = [V_t,\ \alpha,\ \theta,\ q,\ h]^T$, where V_t is the true airspeed of UAV, α is the angle of attack, θ is the pitch angle, q is the pitch rate and h is the altitude. The control input vector $\mathbf{u} = [\delta_{thl},\ \delta_e]^T$ is represented by the throttle and elevator deflections, respectively.

It is considered two operating modes with true airspeed at V_t= 33.9 m/s and V_t= 38.8 m/s. Thus, we have two mathematical models that correspond to these airspeeds. The linear models in the state space are represented by the matrices $[\mathbf{A},\ \mathbf{B}]$:

– nominal model

$$\mathbf{A}_n = \begin{bmatrix} -0.2123 & 25.6346 & -9.81 & 0 & 0 \\ -0.0442 & -3.0958 & -0.2875 & 0.9734 & 0 \\ 0 & 0 & 0 & 1{,}0 & 0 \\ 0.5404 & -114.9272 & 0.5970 & -10.1163 & 0 \\ 0 & -33.9910 & 33.9910 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B}_n = \begin{bmatrix} 1045.1375 & -0.9856 \\ 0 & -0.5171 \\ 0 & 0 \\ 0 & -258.7056 \\ 0 & 0 \end{bmatrix};$$

– perturbed model

$$\mathbf{A}_p = \begin{bmatrix} -0.2428 & 33.5390 & -9.81 & 0 & 0 \\ -0.0442 & -3.5420 & -0.2513 & 0.9734 & 0 \\ 0 & 0 & 0 & 1{,}0 & 0 \\ 0.6184 & -150.5485 & 0.5983 & -11.5816 & 0 \\ 0 & -38.880 & 38.880 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B}_p = \begin{bmatrix} 1045.1375 & -1.2895 \\ 0 & -0.6312 \\ 0 & 0 \\ 0 & -338.1254 \\ 0 & 0 \end{bmatrix},$$

where the subscript "n" corresponds to the nominal model and perturbed model is designated by the subscript "p".

The actuator dynamics is described by quadruple matrix given by

$$\left[\begin{array}{c|c} A_a & B_a \\ \hline C_a & D_a \end{array}\right] = \left[\begin{array}{c|c} -1/T_a & 1/T_a \\ \hline 1 & 0 \end{array}\right],$$

where T_a= 0,23 s is an actuator time constant. The output vector of measured variables is given as follows $y_{est} = [V_t,\ \theta,\ q,\ h]^T$.

Disturbance, υ affecting the longitudinal motion of the aircraft involves the following components: the true airspeed, V_t, angle of attack, α and pitch rate, q, so that $\upsilon = [V_t,\ \alpha,\ q]^T$. In order to simulate the atmospheric turbulence the Dryden filter is used [8].

The attenuation level γ is found to be equal to 1.04. By solving LMI (2) the stabilizable static output feedback gain matrix is obtained. The obtained controller guarantees disturbance attenuation with predefined value of γ. The gain matrix has the following form

$$\mathbf{K} = \begin{bmatrix} 0.0125 & 0.2963 & 0.0067 & 0.0114 \\ -0.0170 & -1.3775 & -0.0292 & -0.0510 \end{bmatrix}.$$

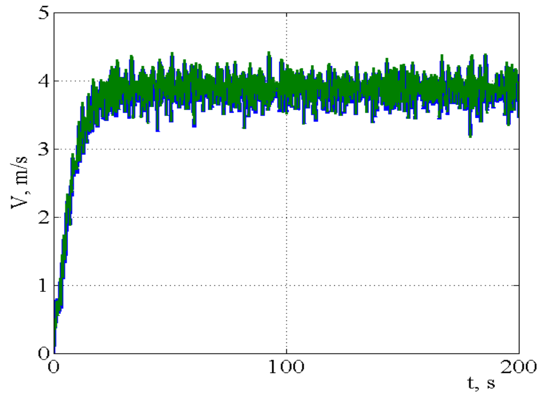
Performance indices for a set of nominal and parametrically perturbed of the closed loop systems are given in table 1.

Table 1

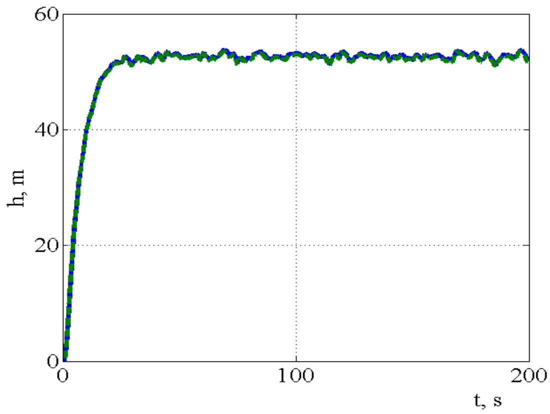
Estimated performance indices for the set of nominal and parametrically perturbed closed loop systems

Performance index		Plant	
		Nominal	Perturbed
H_2 –norm	deterministic case	0.3830	0.3795
	stochastic case	0.5936	0.6181
H_∞ – norm		0.5534	0.6917

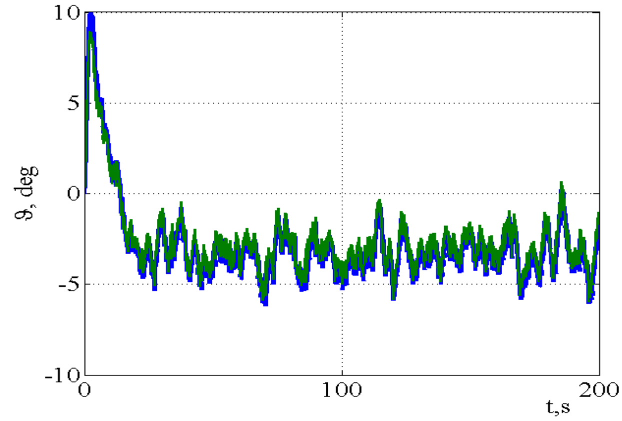
Transient processes in nominal and parametrically perturbed system, which were simulated taking into account all nonlinear functions inherent to the real autopilot as well as the influence of the random wind, simulated according to the standard Dryden model of turbulence [8]. Simulation results are shown in Fig. 1.



a



b



c

Fig. 1. Simulation results for longitudinal channel of UAV in the presence of external disturbances: *a* is velocity; *b* is altitude of UAV nominal and perturbed models; *c* is pitch angle of UAV nominal and perturbed models

Simulation results prove the efficiency of the proposed approach. It can be seen that the handling quality of the nominal and the perturbed models are satisfied.

Table 2 reflects standard deviations of the UAV outputs in a stochastic case of nominal and parametrically perturbed model with static output feedback controller in a control loop

Table 2

Standard deviations of the UAV outputs in a stochastic case

Plant	Standard deviation						
	σ_V, m/s	σ_α, °	σ_q, deg/sec	σ_θ, °	σ_h, m	σ_{el}, °	σ_{th}, %
Nominal	0.1681	1.1954	3.5785	0.9677	0.4547	0.0579	0.0010
Perturbed	0.1798	1.2056	3.9755	0.9838	0.4657	0.0601	0.0010

Conclusions

The paper presents procedure of static output feedback controller design in terms of linear matrix inequalities. The main advantages of static controller application are their simplicity; the control law directly forms basing on available information about measuring output vector.

The simulation results of longitudinal motion control with static controller in the loop including performance indices prove the efficiency of the proposed approach of flight control system design.

The maximum deflection of pitch angle is enclosed within acceptable interval: $-2 < \theta \leq 20$ deg. The altitude h and velocity V are also held at their reference signals $h_{ref} = 50\,\mathrm{m}$ and $V_{ref} = 4\,\mathrm{m/s}$ respectively with acceptable deflections.

Simulation results along with numerical results, represented in Table1 and Table 2 show that the controller possesses with robustness property.

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М. М. Комнацька. Синтез системи управління польотом за допомогою апарату лінійних матричних нерівностей

Запропоновано нову процедуру синтезу статичного зворотного зв'язку за виходом на основі апарату лінійних матричних нерівностей. Показано, що управління забезпечує стабілізацію безпілотного літального апарата під час виконання льотного завдання в умовах дії на нього зовнішніх неструктурованих збурень. Дослідження проведено на прикладі управління поздовжнім рухом безпілотного літального апарата.

Ключові слова: безпілотний літальний апарат; гасіння збурень; неструктуровані зовнішні збурення; робастність; стабілізація літального апарата; статичний зворотний зв'язок за виходом.

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М. Н. Комнацкая. Синтез системы управления полетом с помощью аппарата линейных матричных неравенств

Предложена новая процедура синтеза статической обратной связи по выходу на основе аппарата линейных матричных неравенств. Показано, что управление обеспечивает стабилизацию беспилотного летательного аппарата во время выполнения летного задания в условиях действия на него внешних неструктурированных возмущений. Исследование проведено на примере управления продольным движением беспилотного летательного аппарата.

Ключевые слова: беспилотный летательный аппарат; гашение возмущений; неструктурированные внешние возмущения; робастность; стабилизация летательного аппарата; статическая обратная связь по выходу.

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